

THE
PHYSICAL REVIEW

*A JOURNAL OF EXPERIMENTAL AND
THEORETICAL PHYSICS*

CONDUCTED WITH THE CO-OPERATION
OF THE

AMERICAN PHYSICAL SOCIETY

BY

EDWARD L. NICHOLS, ERNEST MERRITT,
AND FREDERICK BEDELL

VOL. XXXIV

THE PHYSICAL REVIEW

LANCASTER, PA., AND ITHACA, N. Y.

BERLIN: MAYER AND MUELLER

1912

PRESS OF
THE NEW ERA PRINTING COMPANY
LANCASTER, PA

CONTENTS TO VOLUME XXXIV.

CLXXXVIII. JANUARY, 1912.

The Collapse of Thick Cylinders under High Hydrostatic Pressure. P. W. BRIDGMAN..	1
Electrical Discharge from a Point to a Plane. O. HOVDA.....	25
Effect of Frequency on the Capacity of a Condenser, with Kerosene for the Dielectric. S. HERBERT ANDERSON.....	34
A Model of the Elementary Magnet. S. R. WILLIAMS.....	40
An Important Practical Problem in Gyrostatic Action. W. S. FRANKLIN.....	48
On the Mobility of Ions in Air at High Pressures. A. J. DEMPSTER.....	53
On the Production of a Helix of Rays from the Wehnelt Cathode. CHAS. T. KNIPP...	58
Proceedings of the American Physical Society.....	62
Minutes of the Fifty-Ninth Meeting; Minutes of the Sixtieth Meeting; Treasurer's Account for the year 1911; The Effect of Prolonged Illumination on Photo-electric Discharge in a High Vacuum, R. A. Millikan and J. R. Wright; The Production of Light by Cathode Rays, Gordon S. Fulcher; Rays of Positive Electricity from the Wehnelt Cathode, Chas. T. Knipp; Current Produced by Light in a Metallic Film, P. A. Ross; The Measurement of Inductances and Capacities with the Thomson Double Bridge, S. J. Barnett; A Determination of the Ratio of the Specific Heats for Air and Carbon Dioxide, H. W. Moody; The Intensity Factors in the Binaural Localization of Sound, G. W. Stewart; A Method of Measuring the Effective Capacity of High Tension Cables, G. W. Stewart and D. M. Terwilliger; The Electron Theory of Metallic Conduction, O. W. Richardson; The Charges of Thermions Produced in Air and Hydrogen at Atmospheric Pressure, J. C. Pomeroy; On the Relation be- tween Pressure Shift and Wave-Length, Henry G. Gale and Waller S. Adams.	

CLXXXIX. FEBRUARY, 1912.

Reaction Effects Produced by the Discharge of Electricity from Points in Gases and the Bearing of these Effects on the Theory of the Small Ion. EDWARD J. MOORE.....	81
The Magnetic Rotation and Ellipticity produced by Mirrors of Massive Metals. PAUL D. FOOTE.....	96
A Study of the Reversible Pendulum. Part II. Experimental Verifications. JOHN C. SHEDD, J. A. and W. N. BIRCHBY.....	110
Proceedings of the American Physical Society.....	125
The Primary Concepts of Physics, William Francis Magie; The Kerr Rotation for Transverse Magnetic Fields, L. R. Ingersoll; A New Type of Apparatus for Measur- ing Linear Expansion, Arthur W. Gray; The Spectrum of Titanium in a Partial Vacuum and the Proportionality of Displacement to Pressure at Moderate Pressures, Waller S. Adams and Henry G. Gale; The Effect of Temperatures on the Absorbed Charge in Electric Condensers, Anthony Zeleny; The Electrical Resistance and the Polarization E.M.F. of a Mixture of Clay, Feldspar and Quartz, A. A. Somerville and O. E. Buckley; Absorption of the β -Particles by Gases, Alois F. Kovarik; The Spectra of Iron and Titanium at Moderate Pressures, Henry G. Gale and Waller S. Adams; A Quantitative Measure of Development in Scientific Observation, Otto Stuhlman, Jr.; Relation between the Joule Effect and the Induction and Permea- bility in the Same Specimens of Steel, S. R. Williams; The Application of Statistical	

Principles to Photoelectric Effects and Some Allied Phenomena, *O. W. Richardson*;
 A Study of Crystal Rectifiers, *Robert H. Goddard*; A Simple Slit for the Spectroscope,
J. P. Naylor; The Distribution of the Active Deposit of Radium in an Electric Field,
E. M. Wellisch and H. L. Bronson; The Thompson Effects in and the Thermal
 Conductivities of Tungsten, Tantalum and Carbon at Glowing Temperatures, *A. G.*
Working; The Dielectric Constant, Resistivity and Electrostatic Absorption of
 Different Crystals, *H. L. Curtis*; The Influence of Temperature on the Phenom-
 ena of Phosphorescence in Zinc Sulphide, *Herbert E. Ives and M. Luckiesh*; Spark
 Spectra in the Schumann Region, *Theodore Lyman*; A New Form of Mechanical
 Vacuum Pump, *John Johnston*; The Photoelectric Effect of Phosphorescent Material,
Chester A. Butman; Note on the Ascensional Rate of the Free Balloons Used for
 Meteorological Purposes, *Wm. R. Blair*; The Vertical Temperature Gradient of
 the Atmosphere, *Wm. R. Blair*.

New Books..... 160

CXC. MARCH, 1912.

The Selective Transmission and the Dispersion of the Liquid Chlorides. H. H. MARVIN	161
The Distribution of Current in Point-plane Discharge. ROBT. F. EARNHART.....	187
The Effective Depth of Penetration of Selenium by Light. F. C. BROWN.....	201
Rays of Positive Electricity from the Wehnelt Cathode. CHAS. T. KNIPP.....	215
Note on the Measurement of the Peltier E.M.F. HAROLD C. BARKER.....	224
Proceedings of the American Physical Society.....	226
Minutes of the Sixty-First Meeting; A Sensitive Black-Body-Vacuum Thermal- Junction and a Method of Producing High Vacua, <i>A. H. Pfund</i> ; The Absorption of Gamma Rays of Radium by Air at Different Pressures, <i>Henry A. Erikson</i> ; The Stark-Doppler Effect for Hydrogen Canal Rays in Air, <i>Gordon S. Fulcher</i> ; Tempera- ture Influence upon the Refraction of Quartz, Boro-Silicate Crown Glass, and Dense Flint Glass, from 100° C. to -190° C., <i>F. A. Molby</i> ; Slit-Width Corrections in the Photometry of Black Body Spectra, <i>Edward P. Hyde</i> ; A New Radiation from Polo- nium, <i>H. A. Bumstead and A. G. McGougan</i> ; Change of State, Discussion of a General Case, <i>A. J. Lotka</i> ; The Thermo E.M.F. of the Nernst Filament, <i>J. S.</i> <i>Shearer</i> ; The Effects of a Magnetic Field on Photo-Electric Emission, <i>Albert W.</i> <i>Hull</i> ; The Theoretical and Experimental Determination of Reflection Coefficients of Absorbing Media, <i>J. T. Tate</i> .	

CXCI. APRIL, 1912.

On the Family-Tree Arrangement of the Elements and Calculation of Atomic Weights on the Corpuscular Theory of the Atom. ALBERT C. CREHORE.....	241
Comparative Studies of Magnetic Phenomena. II. Relation between the Joule Effect and Permeability and Induction in the Same Specimens of Steel. S. R. WILLIAMS	258
Note on the Optical Effects in Moving Media. Y. LAUB.....	268
A Determination of the Ratio of the Specific Heats and the Specific Heat at Constant Pressure of Air and Carbon Dioxide. H. W. MOODY.....	275
On the Passage of γ Rays of Radium through Matter. S. J. ALLEN.....	296
Proceedings of the American Physical Society.....	311
Variation of Electrical Resistance with Temperature, <i>A. A. Somerville</i> ; International Candle-Power Measurements, <i>Clayton H. Sharp</i> ; Magnetic Reactions Produced by a Copper Disc Rotating Between the Poles of a Magnet, <i>F. G. Benedict and W. G.</i> <i>Cady</i> ; The Electron Theory of Phosphorescence, <i>Chester A. Butman</i> ; On the Expansion of Alternating Current E. M. F.'s in a Fourier's Series, <i>Geo. R. Olshausen</i> .	
New Books.....	320

CONTENTS.

v

CXCII. MAY, 1912.

The Theoretical and Experimental Determination of Reflection Coefficients. J. T. TATE	321
Radiant Efficiency of Incandescent Filaments. W. E. FORSYTHE	333
One-Waveness in Wireless Telegraphy; Pseudo-Impact Excitation. SHUNKICHI KIMURA	345
An Interpretation of Curl and Divergence in Terms of Vector Growth. FREDERICK SLATE	359
A Compensating, Linear Scale, Hot-Wire Ammeter. A. H. TAYLOR	365
Application of the Selenium Cell to Photometry. A. H. PFUND	370
Proceedings of the American Physical Society	381
Minutes of the Sixty-Second Meeting; The Electron Theory of Conductors, O. W. Richardson; The Ions from Hot Salts, O. W. Richardson; Distortion in Spectral Luminosity Curves Produced by Variations in the Character of Comparison Standard and in the Surroundings of the Photometric Field, Herbert E. Ives; The Addition of Luminosities of Different Color, Herbert E. Ives; Accurate Potentiometer Method for Measuring Resistance, Walter P. White; On Secondary Ionization Produced by the Impact of Positive Ions on Solids, O. W. Richardson and Charles Sheard; The Photoelectric Effect, O. W. Richardson and Karl T. Compton; Heats of Dilution, William Francis Magie; The Relation of Osmotic Pressure to Temperature, William Francis Magie; The Law of Fall of a Drop through Air at Reduced Pressures and a Redetermination of e , R. A. Millikan; Variation of Electrical Resistance with Temperature. V.—Oxides, A. A. Somerville.	
New Books	400

CXCIII. JUNE, 1912.

Convection and Conduction of Heat in Gases. IRVING LANGMUIR	401
On the Conduction of Electricity at Contacts of Dissimilar Solids. ROBERT H. GODDARD	423
An Electrical Method of Measuring Small Intervals of Time. F. C. BROWN	452
Photo-Electric Potentials of Thin Cathode Films. PAUL H. DIKE	459
Proceedings of the American Physical Society	471
The Calculation of the Maximum Value of the Force between Two Coaxial Circular Currents, Frederick W. Grover; Direct Measurement of the Velocity of Kathode Rays and the Variation of Mass with Velocity, E. Leon Chaffee; A Method of Using the Photoelectric Cell in Photometry, Edward L. Nichols and Ernest Merritt; The Diurnal Range of Temperature and other Meteorological Elements at Different Levels above Mount Weather, Wm. R. Blair; On the Effect of Light on a Helix of Rays from the Wehnelt Cathode (Experimental), Chas. T. Knipp; A Theoretical Correction to the Drop Method of Determining " e ," Arthur C. Lunn; Magnetic Induction in a Group of Oblate Spheroids of Iron, S. R. Williams; Absolute Formulæ for the Attraction of Two Coaxial Solenoids, G. R. Olshausen.	
Index	482
Erratum	488

THE PHYSICAL REVIEW.

THE COLLAPSE OF THICK CYLINDERS UNDER HIGH HYDROSTATIC PRESSURE.

BY P. W. BRIDGMAN.

ALL metals exhibit, when exposed to stresses at all high, behavior other than the simple proportionality between stress and strain assumed in the mathematical treatment of elasticity. There are a number of effects invalidating this simple linear relation; such as elastic after effects, hysteresis, plastic yield, set, raising of the elastic limit by over strain, and one universal effect, rupture. Any complete description of the stress-resisting mechanism of a solid must provide explanations of all these effects. Conversely, from a more careful study and fuller knowledge of these obscure effects we may expect to be able to more completely characterize the internal mechanism of a solid. Our present knowledge of these effects is restricted, however, because of the narrow range of stress within which they have been studied. The reason for this is that all of these effects become of considerable magnitude only for high values of the stress, and all usual experiments with high stresses are brought to a speedy close by the rupture produced by the high stress. Thus, for example, a bar strained in tension shows, beyond a certain value of the tension, a yield point followed by an interval of stress within which the metal has the entirely new property of plasticity. But this new property of plasticity can be studied over only a comparatively narrow range, for rupture occurs very shortly after the yield point.

In this paper, experiments are described in which the applied stress is of such a nature that rupture will never occur, no matter how high the stress. In consequence, it has been possible to study these various effects over a range of stress very much higher than available under ordinary conditions of yield. Thus plastic yield has been observed over a range of stress twelve times as high as that required to produce the

first beginning of flow. It is the purpose of this paper to describe these experiments somewhat in detail, and to comment on the interesting features displayed by the various effects. The paper is intended only to be suggestive; it would be demanding too much to expect a complete theory from experiments with a single type of stress.

The experiments are tests on thick hollow cylinders, closed at the ends, and subjected to hydrostatic pressure over the entire external surface. Similar tests, in which the walls of the cylinders are comparatively thin, are of familiar occurrence in engineering practice. Under such conditions, the tube fails by collapse, folding in toward the center in one or more creases. That failure takes place in this way is due to the fact that beyond a certain value of the stress the circular figure of the tube becomes unstable, so that very slight geometrical imperfections cause collapse. So slight is the requisite geometric imperfection after the pressure of instability has been reached, that it is possible to obtain very consistent results in collapsing tests of this type. Of course if the figure of the tube were absolutely perfect, collapse by an unsymmetrical folding could never occur. In the tests to be described here, on the other hand, the walls of the cylinders are so heavy that the figure does not become unstable, and yielding to pressure can not be asymmetric. The only conceivable method of yield under these conditions is by a uniform flow in toward the center, which is what actually does happen. It is to be noticed that flow is in such a direction as to reduce the possibility of still further flow under higher pressure. In the case of a rod under tension, the flow is in the direction of elongation, which is capable of indefinite extension. Too great flow of a bar under tension is followed by rupture, but with these cylinders, too great flow can be followed only by complete closing of the central cavity; rupture can never occur.

Two systematic sets of experiments were made on this subject. One, with copper and steel cylinders, was extended to 12,000 kgm./cm.². These experiments show the relation between plastic yield and the raising of the elastic limit over this range of stress. For the copper cylinders the range is sufficient to exhaust all the possibilities, since the hole was completely closed at less than the maximum pressure. The second set of experiments, to a lower stress maximum of only 7,000 kgm., was made on a series of steel cylinders. The arrangements of this set were such that the relation between pressure and internal volume could be measured continuously during application and release of pressure, whereas for the first set only the permanent set was measured after every application of pressure. The second set gives evidence, therefore, on such questions as hysteresis, elastic-after-effects, and the linear relation between stress

and strain. The observations only of these two sets of tests will be presented at first, comment being reserved until the data are all in hand.

The first set of tests on both copper and steel was made on cylinders 2 inches long and $\frac{5}{16}$ inch outside diameter. Seven of these cylinders of each metal were used for the tests, the internal diameter varying in steps of $\frac{1}{64}$ inch from $\frac{1}{16}$ inch to $\frac{5}{32}$ inch. The holes were drilled first and then the exterior turned concentric with the hole. The copper cylinders were made from commercial rod, softened by heating to redness; the steel cylinders were made from a mild bessemer boiler plate which had been proved by special test to be particularly uniform in every direction. The cylinders were closed at the ends as shown in Fig. 1 by stoppers of hardened steel fitting into the ends, and leak was prevented by a thin rubber tube attached to the two stoppers and covering the entire cylinder. The advantage of this method of closing the ends is that after every application of pressure the arrangement may be easily taken apart, and both external and internal diameter measured. It is then possible to subject the cylinder to a higher pressure, so that a record of the relation between set and stress may be obtained after every advancing pressure step. The stoppers of course introduce an end effect, since the projections on the stoppers prevent collapse at the very end, but the effect of these was seldom sensible at a distance from the stopper greater than the original diameter of the hole. The cylinders were subjected to pressure, seven at a time, in a large pressure chamber. The fluid transmitting pressure to the cylinders was a mixture of glucose and glycerine. Pressure was measured by an absolute pressure gauge inserted directly into the same chamber with the cylinders. This is the same absolute gauge which has been described in detail elsewhere.¹

Observations were made at intervals of approximately 1,000 kgm. Pressure was pushed to the maximum and kept there for a few hours, long enough for complete disappearance of the plastic yield, and then released. The cylinders were then measured outside and inside. The outside measurements were made with an ordinary micrometer reading to 0.0001 inch. The inside measurements were made by finding the diameter of a wire which would just slip through the hole. For this purpose a complete set of drill rods of all the different sizes was provided. Intermediate sizes were obtained by filing a rod in a lathe until it would

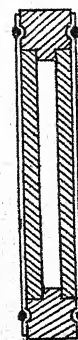


Fig. 1.

Shows the test cylinders and the manner of protecting the outside with a rubber tube so as to prevent leak.

¹ Proc. Amer. Acad., 47, 1911, pp. 319-343.

just fit. The smaller sizes were tested with wires drawn through a special draw plate. The internal diameter of the hole was uniform except at the very ends, and in general it remained perfectly round after flow, so that it was possible to measure the diameter of the hole to at least 0.0005 inch. After the measurements for one pressure, the cylinders were reassembled again with the stoppers, ready for a run to a pressure 1,000 kgm. higher. At every new higher pressure, two or three fresh cylinders were included with the original seven. The purpose of this was to determine whether the previous tests to lower pressures had affected the behavior at the higher pressures.

The results of the tests for the copper are shown in Table I. and are plotted in Fig. 2. The columns headed O.D. (outside diameter) contain

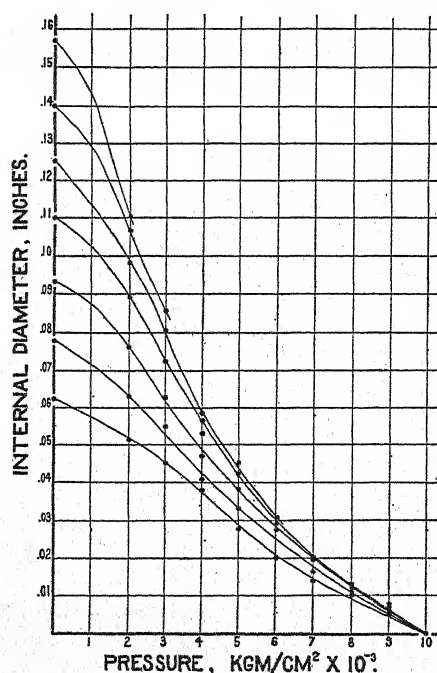


Fig. 2.

The results of the collapsing tests on the copper cylinders. The internal diameter is given as a function of the maximum pressure to which the cylinder has been subjected.

two figures corresponding to each pressure. The upper of these gives the mean external diameter of the cylinder after subjecting to pressure. The lower figure gives the eccentricity produced in the cylinder by the pressure, that is, the difference between the least and greatest external diameters. The results are somewhat irregular, as they always are for tests of this character, but there is sufficient regularity to show well the

TABLE II.

Collapsing Tests on Iron Cylinders.

Max. Pressure kgm./cm. ²	Cylinder No. 1.		Cylinder No. 2.		Cylinder No. 3.		Cylinder No. 4.		Cylinder No. 5.		Cylinder No. 6.		Cylinder No. 7.		Results with Cylinders Fresh for Each Application.		
	O. D.	I. D.	O. D.	I. D.	O. D.	I. D.	O. D.	I. D.	O. D.	I. D.	O. D.	I. D.	O. D.	I. D.	Original internal diameter	Original internal diameter	Original internal diameter
0	.3105	.0625	.3112	.078	.3121	.093	.312	.110	.312	.125	.311	.140	.311	.157	.0625"	.093"	.140"
2000	.3105	.0625	.3112	.078	.3121	.093	.312	.110	.312	.124	.310	.139	.310	.154	.0625"	.093"	.140"
3000	.3105	.0625	.3112	.078	.3121	.093	.311	.106	.308	.117	.305	.129	.304	.142	The final internal diameter is re-recorded.		
4000	.3105	.060	.311	.0755	.311	.088	.310	.101	.306	.112	.302	.110	.295	.111			
5000	leak		.310	.0725	.309	.083	.308	.095	.303	.103	.298	.105	.298		.0612	.085	.099
6000	.3095	.0515	.308	.0675	leak		.306	.0895	.3007	.0965	.2930	.090	.2930		.0582	.080	.089
7000	.308	.050	.307	.063	.307	.0715	.304	.0825	.300	.084	.298	.0725	.298		.055	.074	.062
8000	.308	.046	.3065	.059	.305	.066	.302	.0725	.296	.0725	.279	.055	.279		.053	.067	.034
9000	.3065	.0425	.3065	.056	.3035	.060	.301	.066	.294	.064	.277	.0425	.277		.051	leak	.0355
10000	.3070	.040	.3055	.050	.303	.054	.295	.0565	.292	.0565	.273	.032	.273		.046	.046	Collapse
11000	.3067	.037	.3047	.045	.3025	.043	.2985	.053	.2905	.049	.270		.270		.041	.046	.023
12000	.3065	.033	.3045	.041	.3015	.035	.248	.037	.289	.0415					.035	.046	.023
	.0005		.0015		.0005		.0040		.0072						.034	.0415	.023

general tendency of the results. The two cylinders with the thinnest walls showed the collapse characteristic of thin walled tubes, and the results with these two cylinders are not plotted in the figure. The observations were continued, however, on one of these cylinders in which the hole was initially 0.140 inch diameter, because the collapsing did not increase with rising pressure, but rather became less as the walls thickened under continued flow toward the center. The table shows how the eccentricity of this cylinder rose to a maximum and then decreased again. The results with the three cylinders which were freshly subjected to pressure each time were more irregular than the above. Within the limits of error, however, there were scarcely perceptible variations between the old and the fresh cylinders. If anything, the tendency was for the fresh cylinders to show the greater yield. The tests on the original six copper cylinders were not continued beyond 10,000 kgm., because at this pressure the hole had closed completely at the ends, and in order to find whether the hole was closed completely all the way through, it was necessary to cut the cylinders in two. The cylinders were found to be closed throughout the entire length except in a few isolated places where complete closing was prevented by a layer of scale which had cracked off from the inside of the hole. The tests on fresh pieces was continued up to 12,000, however. The results at 10,000, 11,000 and 12,000 were all the same, complete closing of the inner hole throughout the entire length of the cylinder.

The corresponding tests for the soft iron cylinders are given in Table II. and Fig. 3. The results with the iron were in general character the same as for the copper. The plastic flow was less, however, so that it was possible to push the pressure on more of the cylinders to the pressure of instability. Only the two heaviest cylinders have failed to begin the collapsing process at a pressure of 12,000 kgm. The results with the sets of three fresh cylinders put in for every new application of pressure were more regular than for the copper. In general, except for the irregularities attending collapse or growing eccentricity of figure, it made no difference whether the cylinder had been subjected to lower pressures previously or not.

Besides the measurements tabulated, measurements were also made of the length, both of the steel and of the copper cylinders. Only slight variations were found, not over 0.002 inch, and these were irregular. Evidently there was no change of length due to the collapsing of the cylinder, but the slight irregularities observed were due to the automatic seating of the hardened stoppers.

The second series of tests to lower pressures was made on seven steel

cylinders simultaneously. These cylinders were of the form shown in Fig. 4. The diameter is $\frac{1}{2}$ inch at *B* and $\frac{5}{8}$ inch at *A*. The lower part was exposed to pressure from *A* to *B*, while at the upper end a stem $\frac{5}{16}$

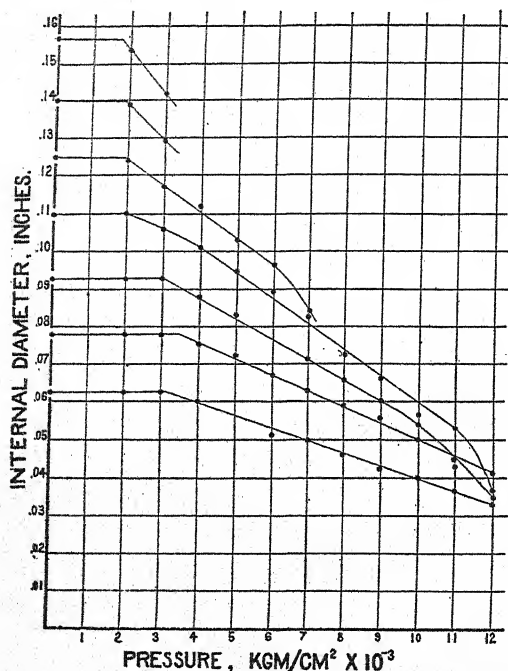


Fig. 3.

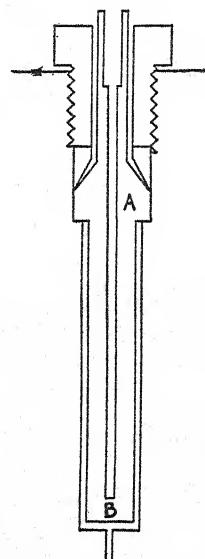


Fig. 4.

The results of the collapsing tests on the iron cylinders. Similar to the tests on the copper cylinders. The form of steel cylinder with which the change of internal volume was measured as a continuous function of the pressure.

inch in diameter projected into the air through the conical steel packing as shown. This $\frac{5}{16}$ inch stem was sealed directly to a calibrated and graduated glass capillary tube. The different cylinders differed in the material and the size of the inner hole. Two were of soft bessemer steel, with an interior hole of $\frac{1}{4}$ inch and $\frac{1}{8}$ inch respectively. The other five cylinders were of tool steel, 1.25 per cent. carbon, left soft, with holes ranging from $\frac{1}{8}$ inch to $\frac{1}{4}$ inch by steps of $\frac{1}{32}$ inch. All of these seven cylinders were placed in a single large block of steel, the different cavities being in communication with each other, so that all seven were exposed to the same pressure, and the stress history of all seven was identical. This block of steel was a particularly homogeneous piece of open hearth steel presented to the laboratory for the purposes of this research by the Bethlehem Steel Co. Grateful acknowledgment is hereby made of this courtesy. In making the cylinders, considerable care was necessary to

drill the hole accurately concentric the whole length. That sufficient accuracy was obtained was shown by the regularity of the results, and by the special examination of those cylinders which it was necessary to cut open after the tests. The size of the holes was found accurately by weighing the mercury which filled them to a known depth.

To make these readings, the entire interior of the cylinders, and the capillary up to a certain mark, were filled with mercury by exhausting the cylinder and working the mercury in through the capillary. The effect of an application of pressure to the outside of the cylinder is to decrease the internal volume, producing a rise of mercury in the capillary. The amount of this rise was recorded as a function of the pressure, and from the known dimensions of the several parts the equivalent change of internal volume or of internal radius was found. There was no change of length during the tests.

The procedure was to apply pressure in several steps to a maximum and then release pressure in steps to zero. After every change of pressure the position of the mercury in all seven capillaries was read, and recorded

TABLE III.

Pressure History of the Seven Steel Cylinders of the Second Set of Tests.

Time.			Time.			Time.		
Day.	Hour.	Successive Maxima and Minima of Pressure Kgm/cm ² .	Day.	Hour.	Successive Maxima and Minima of Pressure Kgm/cm ² .	Day.	Hour.	Successive Maxima and Minima of Pressure Kgm/cm ² .
April 24	10.05 A.M.	0	April 28	12.50 P.M.	} 6,490	Apr. 30	12.16 P.M.	1,760
	10.31	1,800		1.27			12.33	6,030
	11.00	0		3.00	0		12.41	3,680
	11.33	1,770		4.45	6,320		12.50	6,300
	12.03 P.M.	0		5.50	} 0		1.03	1,720
	12.55	3,380	April 29	10.13 A.M.			1.11	4,440
	1.21	} 0		11.47	} 6,400		1.20	} 1,530
	2.48			2.10 P.M.			1.58	
	3.18	3,050		2.44	0		2.09	0
	3.51	0		3.54	6,380			
	5.04	4,070	April 30	5.00	} 0			
	5.37	} 0		9.20 A.M.				
April 27	9.55 A.M.			9.50	6,380			
	12.43 P.M.	} 5,250		10.18	0			
	1.25			10.35	2,160			
	2.31	} 0		10.49	0			
	2.47			11.17	4,350			
	4.23	5,790		11.35	0			
	5.10	} 0		11.45	2,000			
April 28	9.44 A.M.			11.52	910			
				12.08 P.M.	4,060			

as a function of the pressure. It was necessary to wait after every change of pressure a sufficient length of time for elastic-after-effects or plastic yield to entirely subside. The pressure was measured with a mercury resistance gauge of the type described in Proc. Amer. Acad., No. 9, 1909. The pressure measurements could be made to 1/10 per cent., more than was necessary from the self consistency of the other readings. After describing a pressure cycle as above, another similar cycle was described reaching to a higher maximum pressure than the first, and then another, with a still higher maximum. In all, fourteen such cycles were described. The essential history of these cycles is shown in Table III., giving the maximum pressure of each cycle and the corresponding times. After this series of readings, the cylinders were removed from the block and the dimensions measured again. To show the complete record of the behavior of all these cylinders would take a great deal of space and is hardly necessary, because the characteristic features shown by all are the same. Two cylinders are taken as typical of the lot, and the various cycles of these shown in Figs. 5 and 6. In Fig. 5 the actual observed points are indicated, in order to show the general order of accuracy and regularity of the results. The observed points are omitted in the subsequent figures, however, as they tend to obscure the diagram. The succession of cycles described on April 30 was different from that on the other days; these cycles will be discussed in detail later.

Before discussing the results of these two sets of experiments, it will pay to examine the conditions of stress and strain produced in such a cylinder by the applied stress system. The solution is very easily obtained, and may be found in any book on elasticity. The precise formulation of the problem is as follows. Given a cylinder of external radius a , internal radius b , with a hydrostatic pressure P applied to the external surface, zero pressure to the internal surface, and a compressive stress across planes perpendicular to the axis uniform throughout the mass of the metal and of amount $P \frac{a^2}{a^2 - b^2}$. This last statement amounts simply to neglecting the end effects and supposing the entire hydrostatic pressure exerted over the closed ends to be supported uniformly by the walls of the cylinder. The solution under these conditions is found to be as follows, using the ordinary notation.

For the displacements,

$$\begin{aligned} u_r &= -\frac{Pa^2}{a^2 - b^2} \left[\frac{b^2}{2\mu r} + \frac{r}{3\kappa} \right], \\ u_\theta &= 0, \\ u_z &= -\frac{a^2 z}{3\kappa(a^2 - b^2)} P, \end{aligned}$$

where μ is the shear modulus, and κ the compressibility modulus. That is, the radial displacement is negative, in toward the center, at all points

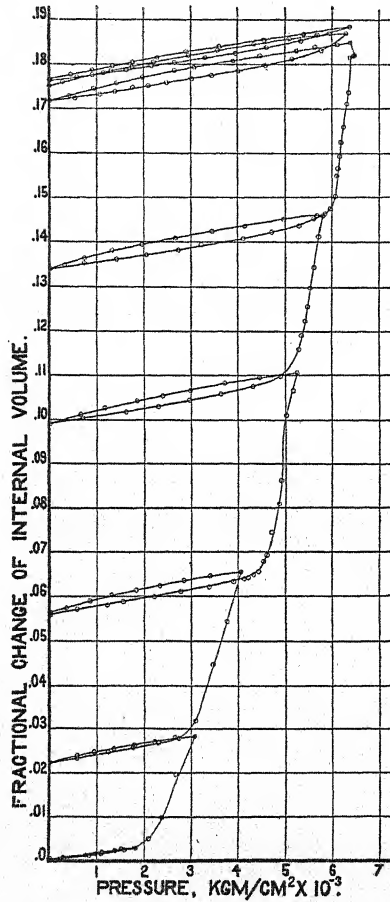


Fig. 5.

Shows the hysteresis loops obtained with a cylinder similar to that of Fig. 4. The internal diameter of this cylinder was originally 3/16 inch, the outside diameter 1/2 inch. This cylinder was of tool steel.

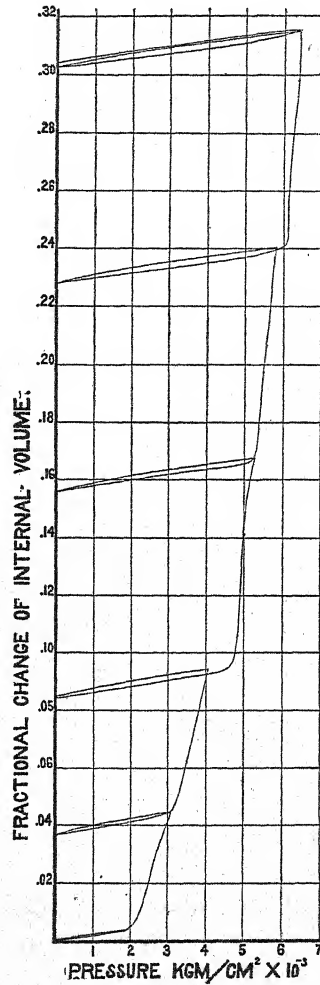


Fig. 6.

Another test like that of Fig. 5. The outside diameter of this cylinder was originally 1/2 inch, and the inside diameter 7/32 inch. This also was of tool steel.

of the cylinder, and the longitudinal displacement is a corresponding compression. The same solution gives for the strain components,

$$e_{rr} = \frac{a^2 P}{a^2 - b^2} \left[\frac{b^2}{2\mu r^2} - \frac{1}{3\kappa} \right],$$

$$e_{\theta\theta} = \frac{-a^2 P}{a^2 - b^2} \left[\frac{b^2}{2\mu r^2} + \frac{1}{3\kappa} \right],$$

$$e_{zz} = \frac{-a^2 P}{a^2 - b^2} \frac{1}{3\kappa},$$

and for the stress components,

$$R_r = -\frac{Pa^2}{a^2 - b^2} \frac{r^2 - b^2}{r^2},$$

$$\Theta_\theta = -\frac{Pa^2}{a^2 - b^2} \frac{r^2 + b^2}{r^2},$$

$$Z_z = -\frac{Pa^2}{a^2 - b^2}.$$

These are the principal strain and stress components at every point, so that the cross components such as $e_{r\theta}$ all vanish.

The circumferential strain at every point is seen to be a compression, as one would expect, increasing in amount toward the center. The longitudinal strain is similarly as one would expect, a compression uniform at every point of the cylinder. But the radial strain is perhaps a little unexpected. The precise value of this depends on the elastic constants μ and κ . For steel we may take as average values, $2\mu = 15.6 \times 10^5$ kgm./cm.² and $3\kappa = 45.6 \times 10^5$ kgm./cm.². Substituting these values shows that at the inner surface, where $r = b$, the radial strain is an extension instead of the compression that one might perhaps have been prepared to expect. The radial compression becomes less at points more remote from the center, and when r is approximately equal to $1.75b$ changes sign, and from here out becomes the expected compression. For thin steel tubes, therefore, the radial strain is throughout an extension. For copper we have for the elastic constants $2\mu = 0.94 \times 10^6$ and $3\kappa = 4.05 \times 10^6$, which, just as for iron, gives an extension as the strain at the inner surface, and $r = 2.1b$ as the location where the compression changes to an extension. And in general, whatever the material, we must always have $3\kappa > 2\mu$, for this is merely the condition that Poisson's ratio be positive, and therefore always at the inner surface the radial strain will be an extension, whatever the material.

The circumferential strain is seen to be always a compression, and is greatest at the inner surface. The longitudinal strain is a compression, uniform everywhere, so that the sections remain plane. The greatest shearing strain comes at the inner surface and is equal to $Pa^2/\mu(a^2 - b^2)$, being greatest for thin-walled cylinders.

All the three principle stresses at any point are seen to be compressions,

except the radial stress, which becomes zero at the inner surface only. The magnitude of the circumferential compression is greater at every point than the radial compression, and the magnitude of the difference is greatest at the inner surface.

These equations hold only while the metal is within the elastic limit. If this limit is exceeded, the relations break down. It is impossible to tell in our present state of knowledge what the new relation must be. It would be possible to make a variety of hypotheses and write down the consequent stress components. The only feature necessarily common to these various systems of possible stresses would be the satisfying of the conditions that at the interior surface $R_r = 0$, and at the exterior surface $R_r = -P$. We also have, by considerations of symmetry, that even after the plastic yield has begun, the directions of principle stress continue to be the same mutually perpendicular directions as before. The components of strain cease to have any definite meaning after plastic yield has continued for any time. The elastic components of stress and strain do enable us to see what to expect, however, when yield begins.

Let us inquire first what we can deduce from the simple fact that flow exists, and that plastic yield does come to an end, as shown by the first set of experiments. Yield begins at the inside surface as flow toward the center. This is evidently connected with the radial extension at the inner surface; when this radial extension becomes greater than the material can stand, there is viscous yield in the direction of the extension. It is to be noticed that at the inner surface where the yield is greatest there is no stress in the direction of yield. It is not necessary, then, that there should be stress in the direction of flow any more than that there should be stress in the direction of elastic yield below the elastic limit. The immediate cause of the flow is the existence of stresses at right angles to the direction of flow. The experiments also show the converse fact that flow which might be expected to take place in a given direction may be prevented from taking place by forces at right angles to that direction. This is shown by the fact that none of the cylinders showed any change of length under the longitudinal compressing force. This stress, in the case of the copper cylinders subjected to 12,000 kgm., is eight or ten times as much as would produce longitudinal set in a pure compression test. The longitudinal flow was evidently prevented from taking place by the force at right angles. What is a bit surprising is that one of these forces at right angles is always less than the longitudinal stress and may be zero. This is sufficient to show that the relations between the three principal stresses and direction of flow, to say nothing of the magnitude of the flow, are not perfectly simple. This relation

must be established by experiment. It may be pointed out, however, that in this case the flow takes place entirely in the plane of the greatest and least principal stress, that is, in the plane of maximum shear stress.

After questions as to the fact of flow, the next natural consideration is as to how far the flow will proceed before it stops. This is a complicated matter and the question cannot be answered as yet. A complete solution must take account of the change of geometric shape with flow, of the hardening produced in the metal by flow, which varies in different metals, and of the unknown relation between the principal stresses for metals in the plastic state, which again probably varies for the different metals. In view of all these possibilities it would be impossible even to guess whether the flow would continue indefinitely or not; the experimental fact that the hole does close up is a new contribution which could not have been predicted. The mere fact that the hole does close up, moreover, is sufficient to enable us to rule out one relation between the stresses during plasticity which has been used tentatively a number of times in lack of any more probable hypothesis. This is the condition of maximum stress difference; namely, that during yield the difference between the greatest and the least principal stresses cannot exceed a certain value. This possible stress difference may not be perfectly constant, but may be increased slightly by the hardening of the metal under flow. In any event, however, the stress difference cannot rise above some definite maximum. This condition admits of precise mathematical formulation, so that we can completely solve the problem of stress distribution. We have the equations:

$$R_r = \Theta_\theta + K,$$

which is the hypothesis of maximum stress difference, where K is the value of this difference. We also have the equation of equilibrium

$$\int_b^r \Theta_\theta dr = rR_r.$$

The solution of these two simultaneous equations is readily found to be

$$R_r = -K \log \frac{r}{b},$$

which gives at the exterior surface:

$$P = K \log \frac{a}{b},$$

where P is the external collapsing pressure, a the external and b the internal radius. That is, under any given external pressure the dimensions of the cylinder will be so changed by the hydrostatic pressure as to sat-

isfy the given relation. The equation shows that for every value of P there is a corresponding value of b . That is, the hole will never be closed up by any pressure no matter how large. The maximum stress difference criterion is not valid, therefore, for plastic flow under high stresses. The effect of hardening by flow would evidently be to decrease instead of increase the flow under any given pressure, so that we are not helped by taking account of the hardening. It must be, then, that flow beyond a certain point weakens the metal so that it is able to withstand considerably less stress difference than it could in the incipient stages of flow.

The problem cannot be attacked backwards. There are an infinite number of possible relations between Θ_0 and R_r which would give the experimental relation between pressure and internal radius. We can see roughly what the general nature of the stress distribution must be, however. The interior surface is to some extent a free surface, because yield can occur at this place. R_r at this surface is always zero, and from what we have just seen the value of Θ_0 at this surface cannot exceed a certain value, but must on the contrary tend to become less as the exterior pressure rises. But the average value of Θ_0 throughout the mass of the cylinder tends to become greater, for this must balance the external hydrostatic pressure. That is,

$$\int_b^a \Theta_0 dr = -Pa.$$

Θ_0 must rapidly increase from the center out therefore. R_r similarly is zero at the inner surface but becomes equal to $-P$ at the outside. At the outer surface, therefore, the condition approaches one of uniform hydrostatic pressure. The possibilities of supporting stress when the metal is in such a state of hydrostatic pressure are indefinite. But at the inner surface the condition becomes one of greater and greater plasticity under a one-sided stress in a direction at right angles to that of yield.

Consideration of what happens when the external pressure is removed next concerns us. The pressure is supposed to have been applied so long that flow has ceased and the metal has entirely accommodated itself to the high stress. We may consider the strain, calculated from this new state of ease, to change roughly in the same way that the strain would in a condition of perfect elasticity when external pressure is removed. This is the reverse of the strain found above when pressure is applied to the outside. That is, when pressure is removed, the circumferential strain at the inner surface becomes an extension and the

radial strain a compression; at the external surface the circumferential strain is also an extension, but the radial strain has changed from a compression to an extension. If the original strain had remained elastic under the applied pressure, the reverse strains when pressure was removed would have sufficed merely to wipe out the original strain. But here the strains are reckoned from a new state of ease at the maximum pressure, so that the metal is left with internal strains after the release of pressure. At the inner surface this strain will be a circumferential extension, which furthermore may be of considerable amount, because the pressure has been released through such a wide range. It is perfectly conceivable that this strain introduced by the release of pressure might be sufficient to exceed the elastic limit in extension, or might even be so great as to produce rupture at the internal surface.

That precisely this is the nature of the reverse strains has been verified by experiment. In an attempt to make tubing capable of withstanding high pressures, collapsing experiments like these were tried. Since it was not possible to obtain commercially drawn tubing with a hole small enough for some intended uses, it was thought that by collapsing the tube in this way it might be possible to make a tube of the requisite dimensions. None of the tubes so treated, however, were capable of standing as much pressure as the original tubes with the larger hole, and several of the treated tubes burst at practically no pressure at all. It was found on cutting these tubes open that the inside was cracked from the center practically out to the surface, evidently because of the internal strains set up by relieving pressure. The same thing was verified on several specimens of tubing which had been collapsed and then cut open without subjecting to internal pressure. These tubes were hard drawn of a steel with about 0.5 per cent. carbon. The effect was never found in the softer copper or steel tubes of the tests above.

In view of the internal strains evidently introduced after every release of pressure, one might well be prepared to expect the repeated application and release of the same pressure to be followed by progressive yield, the cylinder finally closing up under repeated applications of the same pressure. That this is not the case, however, is the first thing evident from an inspection of the diagrams of the second set of tests. With every successive application of pressure, yield is not resumed until the previous pressure maximum has been reached or exceeded. The same thing is shown in the first set of experiments by the approximate identity of the results obtained at any pressure with the fresh cylinders and with the cylinders which had been subjected a number of times previously to lower pressures.

The very fact, then, that the plastic limit can be so raised by pressure is unexpected and occasion for some thought. One might be prepared for a raising of the yield point to twice its original value. It has been shown, for example, that if the yield point of a bar be raised by over-strain in tension, the corresponding yield point in compression is lowered by the same amount, so that the total stress range, tension to compression, within which the bar behaves elastically remains constant. So here, one would be prepared for a shifting of the range from, in the first case, extension to equal compression, to, in the second case, zero extension to double the initial compression. But an increase in the range of ten or twelve times as in the case of the copper is unexpected. It points to some deep seated molecular rearrangement.

The second set of experiments shows instructive minor differences in the way in which the elastic limit may be raised. For instance, the correspondence is not perfect between the new yield point and the previous maximum; yield may occur either before or after the maximum. In Fig. 5 the old maximum was exceeded at 4,000 and 5,800 kgm., but was almost exactly reached at 3,000 and 5,000. On consulting the log of the experiment on page 9 it will be found that there is a distinct connection here with the time during which the metal has been resting. At 4,000 the cylinders had been resting under no stress for 64 hours, and at 5,800 for 16 hours, while at 3,000 and 5,000 the time of rest had been comparatively short, an hour or so. This is but a verification of common experience with tensile and similar tests, that it takes time for the hardening by over-strain to be affected. If the metal rests under no load, the elastic limit may be raised beyond the old value, whereas if stress is immediately reapplied without allowing any time for accommodation, the effect of the over-strain is to lower the elastic limit. The diagrams also show that the rapidity of recovery during rest may vary. In Fig. 6 the interval of an hour has been sufficient for complete recovery, but not in Fig. 5. The cylinder of Fig. 5 is of tool steel with a hole $\frac{3}{16}$ inch in diameter, while Fig. 6 shows the bessemer cylinder with $\frac{1}{4}$ inch hole. The yield of the tool steel cylinder to pressure is less than that of the bessemer, but the rate of recovery is slower. Corresponding to the slower yield are the heavier walls. This is only one case of the general observation that, beyond the initial stages, recovery is slower in the heavier masses of metal, irrespective of the total amount of yield, and, other things being equal, that the recovery is more rapid in the bessemer than in the tool steel. The internal structure of the bessemer is to be thought of as simpler than that of the tool steel.

The two diagrams Figs. 5 and 6 show one other characteristic of all

the cylinders; that the initial rate of yield in the neighborhood of a previous maximum is slower than normal if the metal has not completely recovered by rest, but if the yield point has been raised by prolonged resting, the yield, when it does occur, is abnormally rapid. Both of these effects result in a resumption of the old yield curve a little way beyond the previous maximum. The release of pressure to zero and reapplication does not essentially alter the character of the yield curve, therefore, but is merely a temporary incident, the effect of which speedily disappears.

There is another effect shown by all the cylinders similar in many respects to that of plastic yield, namely elastic after effect. After an increase of pressure the result of this is that yield continues slowly for a while, gradually subsiding to zero. In this respect it is similar to plastic yield, but it may occur at lower values of the stress than is required to produce what is ordinarily recognized as plastic yield. After decreasing pressure, however, the elastic-after-effect produces a gradual creep in the

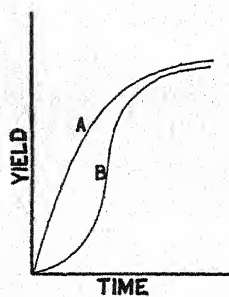


Fig. 7.

Shows two types of yield curve. *A* shows the normal type of curve; *B* the type after prolonged resting.

contrary direction to that of plastic yield. This elastic-after-effect was observed in all the cylinders after the release of pressure to zero. Of course at the maximum pressure it could not be separated from the plastic yield. The effect was in general small, too small to show on the scale of the diagrams, and remained nearly constant whatever the range of the pressure. Here, then, is one of the unusual effects which does not become very much greater for a range of stress far beyond the elastic limit. The effect was greater in the heavier cylinders which showed the least set, and was noticeably less for the bessemer than for the tool steel.

The time rate of yield was also observed a number of times. It is of course greater and extends over a longer interval of time at the higher pressures. The normal shape of the yield curve is shown at *A* in Fig. 7. But there is in addition a change in the character of the yield curves at the higher pressures. At the low pressures, the curve plotting yield against time is very steep at the origin, but as the yield pressure increases, the curves show a more uniform rise for a greater interval of time. Thus at low pressures, half the total yield might occupy only a tenth of the time required for nine tenths of the yield, but at higher pressures half the yield might consume one third of the time required for nine tenths. The same difference in character was also shown at the same pressure by different cylinders. The yield curves for bessemer

steel were very markedly more steep than those for tool steel, and among the tool steel cylinders, those with the thinnest walls showed the steepest yield curves. This shows again that these various effects, such as elastic-after-effects, time rate of recovery, and viscous yield, take more time to run to completion in the larger mass of metal. All of them would seem due to some process of molecular readjustment, which must travel through the steel from places of greater distortion to those of less. The process takes more time when there is a greater mass of metal to be readjusted.

The yield curve in one case was found to be an exception in general shape to all the others. This is shown at *B* in Fig. 7. This was at 4,000 kgm., where yield was resumed after resting for 60 hours. The yield seemed particularly reluctant to get started in this case. It ran very slowly at first, then with gradually increasing velocity, and then slowed down again to an asymptote. The curve had the form shown. Evidently the effect is due to the gradual disappearance of the hardening under the previous load.

The change in the character of the yield curve with increasing pressure has an application to ordinary testing. Frequently two points are distinguished during a tensile test, for example: a point where the material first shows set and a point later on where it shows plastic yield. It seems probable that no sharp distinction between these two points can be maintained. The set point is merely a yield point at low stresses where the process of yield takes place so rapidly as to escape notice. With increasing stress the yield curve flattens out, until it has become slow enough to observe, when we have the ordinary yield point.

There is another feature shown by all the cylinders, the loops described when pressure is released and reapplied. These loops are imperfect in Figs. 5 and 6, being complicated by hardening effects and elastic-after-effects. The fact that the loops exist is perfectly evident, however, as also the fact that these loops become wider as the pressure range increases, the increase in width being more rapid than the increase in pressure. The loops are of a width much greater than could be produced by elastic-after-effects, although these effects do tend to produce loops when the metal is carried through rapid stress cycles. These are genuine hysteresis loops. To show the character of the loops more plainly after the metal had become perfectly seasoned so as to show no further hardening effects, pressure was increased and released a number of times over the range of 6,400 kgm., as shown in the upper part of Figs. 5 and 6, until the behavior had settled down to a reproducible cycle. A number of cycles were then described similar to the cycles in making tests for magnetic hysteresis. Three of these cycles are shown in Figs. 8, 9 and

10. They are different in appearance, but were selected because they all show certain traits in common. Fig. 8 is for the tool steel cylinder with $5/32$ inch hole, Fig. 9 for the bessemer cylinder with $1/4$ inch hole, and Fig. 10 for the other bessemer cylinder with $1/8$ inch hole. The reversal in the normal direction of curvature of the cycle for the $1/4$ inch bessemer is due to two causes. This cylinder collapsed under pressure, flattening

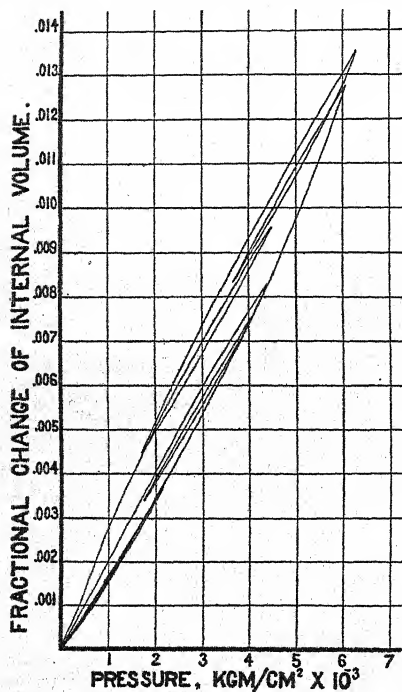


Fig. 8.

Hysteresis cycle after thorough seasoning. This cylinder was of tool steel with an internal diameter initially $5/32$ inch.

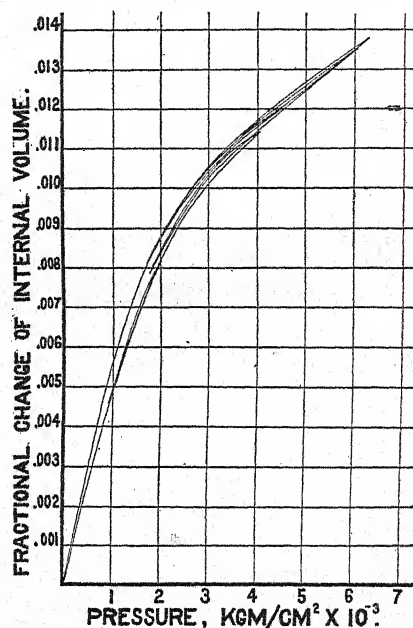


Fig. 9.

Hysteresis cycle of bessemer cylinder with hole initially $1/4$ inch.

out until it received support from the walls of the steel block, and in addition there was placed inside this cylinder a piece of iron wire, so as to reduce the volume of the mercury and so the temperature coefficient. During collapse, the walls of the cylinder received support also from this piece of wire. Fig. 9 is not illustrative at all, then, of the normal behavior of a collapsed cylinder, but is included to show that the hysteresis of even so complicated a system as this shows the same general characteristics as the simpler cases.

The characteristics shown in common by all these hysteresis loops are also the same as those shown in magnetic hysteresis cycles. The branch of the loop described with increasing pressure lies at all points below the

return branch with decreasing stress. If from any point a small cycle is described by reversing the direction in which stress has been applied before reaching that point, then on restoring pressure to the original value at the point in question, the strain will recover its initial value also. So much is common to all these cycles, in fact is common to all hysteresis cycles of whatever known cause. Beyond this, the loops show great differences of shape and size. The loops for the bessemer steel are narrower than for the tool steel, as indeed all these effects have been less in the bessemer. Among the tool steel cylinders, the widest loops are shown by the cylinders with the thickest walls, again confirming the

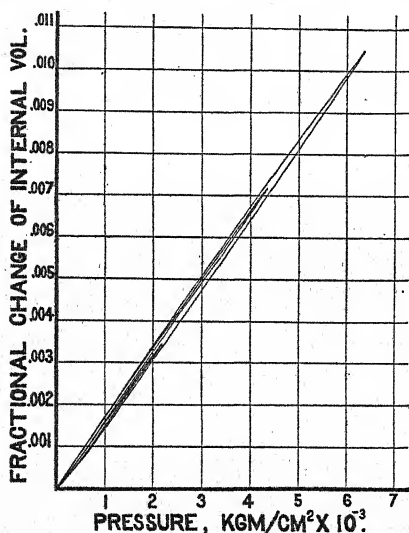


Fig. 10.

Hysteresis cycle for bessemer with hole initially 1/8 inch.

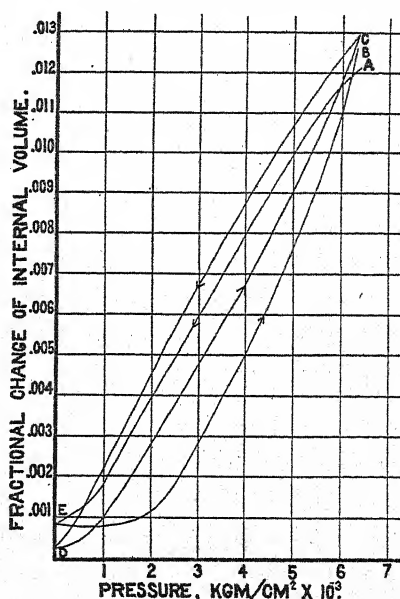


Fig. 11.

Abnormal hysteresis loops before complete seasoning for the tool steel cylinder with hole initially 1/8 inch.

experience with the other effects. The width of the hysteresis loops may be unexpectedly great. Thus in Fig. 8, the width of the loop is one seventh of the total strain under the maximum pressure, and is equal to 2/100 of the original volume, as much as the entire elastic deflection before yield began.

The behavior of one of these cylinders during the seasoning process preparatory to describing the hysteresis loops was so remarkable as to deserve special comment. This was cylinder No. 1, that of tool steel with the 1/8 inch hole. Fig. 11 shows the first cycle described during the seasoning process, and also the final shape to which the cycle settled down after several applications of pressure. The unusual features are

shown at the lower end. The initial effect of the first increase of external pressure was to increase the internal volume, instead of giving the normal decrease. This abnormal behavior gradually disappeared with successive applications, but the memory of it survived in an abnormally low slope at the lower end. The precise mechanism of this anomalous effect is not at all clear. It is significant that the effect is shown in the cylinder with the heaviest walls. The next heavier cylinder, No. 2, shows also an abnormally low initial slope, although an actual increase of internal volume under external pressure was never observed here.

All this discussion so far has been qualitative. This is about all that can be got out of the data, for there is not much quantitative regularity in the results. The first set of experiments is not adapted for a quantitative discussion, and not all the cylinders of the second set can be so used. This is because the thinner walled cylinders either collapsed or else became slightly elliptical under the maximum pressure, so that they are no longer strictly comparable among themselves.

Cylinders Nos. 1, 2, 3, and the bessemer $1/8$ inch cylinder showed no signs of collapse or of ellipticity greater than the original. No. 4 had just begun to collapse at the lower end, while No. 5 and the bessemer $1/4$ inch cylinder had collapsed as far as allowed by the retaining walls of the large steel block. Even the uncollapsed cylinders do not show the regularity of plastic flow that might be expected. Very probably slight geometrical inequalities have a considerable effect in determining the amount of flow. It is not necessary, however, that in the case of thick steel cylinders the geometric irregularity should become accentuated by flow. The first set of experiments shows several cylinders which became increasingly elliptical up to a certain point, and then, with further increase of pressure, recovered their original circular form.

The following figures may be taken for what they are worth, therefore. Table IV. for the tool steel cylinders shows the ratio of the volume set of the different cylinders to that of the thickest cylinder, No. 1. The ratio for the different cylinders is seen to decrease with rising pressure. This is evidently due to the fact that cylinder No. 1 does not acquire the normal rate of flow until the pressure has been increased for a considerable way beyond the first yield point. This is shown by all the cylinders in the gentle curvature of the lower end of the yield curve. It is shown most strikingly of all by the bessemer $1/8$ inch cylinder, where the pressure does not seem to have been pushed far enough to reach the steep part of the yield curve. The maximum yield for the $1/8$ inch bessemer cylinder was 0.012 against 0.09 for the $1/8$ inch tool steel. Evidently this first part of the curve is the location of the usual hardening effects of over-strain. It is only the initial gently rounded parts of this

curve that have been available in ordinary tests, such as tension tests. As pressure is pushed beyond this initial part of the curve, the rate of yield seems to settle down to a steady value depending on the form of the cylinder. Beyond 5,000, the yield of cylinders 1, 2 and 3 progresses at nearly the same rate for each. The increase of the ratio beyond 5,000 for cylinders 4 and 5 is evidently connected with their collapse.

TABLE IV.
Plastic Yield of Steel Cylinders.

Max. Pressure.	Volume Set of No. 1.	Ratio to Vol. Set of No. 1.			
		No. 2.	No. 3.	No. 4.	No. 5.
3,100	0.005	1.80	4.40	7.20	9.80
4,050	0.022	1.48	2.54	3.78	4.77
5,250	0.051	1.25	1.94	3.06	4.61
5,800	0.070	1.23	1.91	3.24	4.93
6,400	0.090	1.23	1.94	3.38	5.83
0		Ratio of Elastic Vol. Yield.			
		1.21	1.14	1.34	0.99

The table shows no simple relation between the rate of plastic yield and the ratios of the original elastic yield before flow began. In fact the elastic yield does not show the simple dependence on the dimensions demanded by theory. This must be due to geometric imperfections of figure.

As far as the actual value of the elastic constants goes, the plastic yield seems to have made very little difference. This may be found by comparing the slope of the cycles described after accommodation to 6,400 with the initial slope before yield had begun, taking account also of the changed dimensions. Only cylinders 2 and 3 are available for this, because No. 1 shows an anomalous cycle and 4 and 5 have been collapsed. The elastic constants of 2 and 3 have not changed over 5 per cent., one being an increase and the other a decrease.

SUMMARY.

In this paper the behavior of hollow steel and copper cylinders subjected to external pressure has been examined over a range of stress many fold greater than the original elastic limit. It is found that:

1. The yield under any given pressure does not continue indefinitely, but stops after a while. With the next application of pressure, yield is not resumed until the old maximum is reached.
2. The cylinder cannot support an indefinite pressure, but closes up tight at some fixed pressure, the same no matter what the original dimensions of the cylinder. For copper, this is about 10,000 kgm. For soft steel, judging from an extrapolation, it must be at about 20,000

kgm. The fact that the cylinder closes up shows that the maximum stress difference criterion for flow is not valid. The maximum stress difference that the material can support decreases after prolonged flow.

3. There need be no stress in the direction of flow, and a stress ordinarily great enough to produce flow need not necessarily produce such flow under all conditions. In the cases considered here, flow can take place only in the plane of greatest and least principal stress.

4. The internal adjustment, or whatever it is that enables the metal to stand this greatly enhanced pressure without yield, does not result in a general raising of the resistance to all kinds of stress, but it is an accommodation only to the particular type of stress which produced the yield. A cylinder collapsed by external pressure bursts under less than the normal internal pressure.

5. The time rate of plastic yield undergoes modification as the pressure increases. At low pressure the greater part of the yield occurs in the initial stages, but at higher pressures the yield is more evenly distributed in time. The slower rate of yield is found in larger masses of metal, even although the total yield may not be so great, and the rate is slower in tool steel than in bessemer steel. The tool steel has a more complicated internal structure.

6. The usual hardening effect of over-strain is shown. A certain interval of rest after the over-strain is necessary for the hardening. This interval is greater for large masses of metal, and is greater in tool steel than in bessemer.

7. Elastic-after-effects are about normal. They do not increase markedly with increasing pressure.

8. Hysteresis is shown by all the cylinders. It becomes rapidly greater at the higher pressures, and is much greater than under ordinary conditions of test. The breadth of a hysteresis loop may amount to the entire elastic deflection before yield began. Hysteresis shows a tendency to be greater in the larger mass of metal, and is very much greater in tool steel than in bessemer.

9. The thickest tool steel cylinder showed anomalous results during accommodation. An increase of external pressure was followed by an increase of internal volume.

All of these effects are probably connected in some way with the break-up of unstable molecular complexes, and the formation of new complexes stable under the new conditions. This process takes a longer time in large masses of metal, and produces greater changes in the properties of a material with a complex structure than in a simpler material.

JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY, CAMBRIDGE, MASS.

ELECTRICAL DISCHARGE FROM A POINT TO A PLANE.

BY O. HOVDA.

1. E. Warburg¹ investigated the effect of distance between a point and a plane upon the current flowing for a given potential, using distances up to 7 cm. In the following experiments the results have been extended to much greater distances, and the variation with distance of the minimum potential required to produce a discharge has been investigated as well as the variation of the current with the potential at the different distances. The potential gradient between the point and the plane has also been investigated and a relation found between its value at any place and the current flowing.

2. The arrangement of the apparatus and method of taking the readings were similar to those used by J. Zeleny,² except that a resistance of carbon-ruled paper³ was used to regulate the voltage from the static machine. The plane used consisted of a large circular zinc plate, 330 cm. in diameter. The discharge point was the hemispherical end of a platinum wire, 1.75 cm. long and .013 cm. in diameter, supported at the end of a brass rod 75 cm. long made up of sections of increasing diameter for the sake of rigidity. Because of the large size of the apparatus the experiment was necessarily carried on in the open air of the room. The results are reduced to 22° C. and 74 cm. pressure according to the laws of variation in these respects found by J. Zeleny.⁴

3. *Minimum Potential.*—Most of the observations were taken with a galvanometer joined to the plane to show when the current started to flow, but in some cases this was replaced by a sensitive electroscope without any change in the results. The results for the starting potential for both positive and negative discharges are represented by the curves in Fig. 1. Each circle represents the average of several readings taken on different days. It is seen that for the smaller distances the minimum potential increases quite rapidly with increase in distance, but changes very slowly with the larger distances. It appears probable therefore that for all very large distances there would be a constant minimum potential. This may have some application in the consideration of the

¹ E. Warburg, *Ann. der Phys.*, 67, p. 69, 1899.

² *PHYS. REV.*, Vol. XXV., p. 305, 1907.

³ F. Aust, *Physik. Zeitschr.*, 12, p. 732, 1911.

⁴ *Loc. cit.*, p. 326.

discharge produced from a lightning-rod point by a cloud. It is quite likely that the discharge begins when there is a certain definite electrical density on the point, and this density is reached at a lower voltage when an earthed plane is situated near by owing to the condenser effect; but

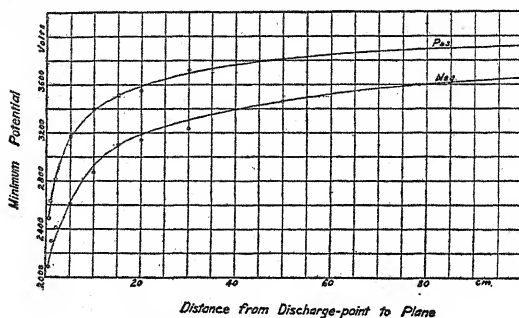


Fig. 1.

when the distance is considerable a further increase produces but a slight change in the electrical density due to a given potential. For both kinds of discharge an equation of the form

$$M = a - \frac{b}{c + \sqrt{D}}$$

was found to represent quite closely the dependence of minimum potential on distance. Expressing D in centimeters and M in volts, the experimental results are represented by the following equations:

For positive discharge,

$$M = 4260 - \frac{3740}{1.17 + \sqrt{D}}.$$

For negative discharge,

$$M = 4290 - \frac{8400}{3.06 + \sqrt{D}}.$$

The approximate equality of the constant a for the two kinds of discharge is an indication that the two starting potentials for very large distances would be nearly identical.

4. *Currents*—In order to approximate to an infinite plane, the receiving surface must be large when the distance between the point and the plane becomes considerable. According to a preliminary experiment a plane of the size used (330 cm.) should receive as much current as an infinite plane for distances up to 54 cm. between the point and the plane, while if the latter distance be as much as 100 cm., the current should not be more than 4 per cent. less than that received by an

infinite plane. The variation in current with voltage for the various distances between the point and the plane is shown by the curves of Figs. 2 and 3. These represent average results of three, and, in some

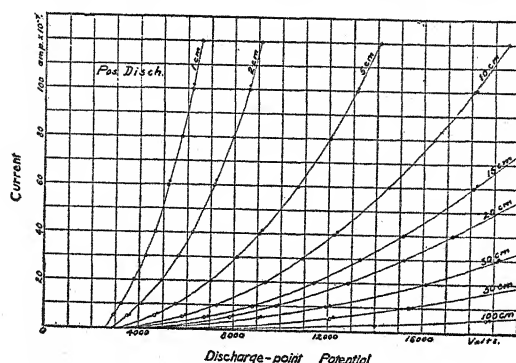


Fig. 2.

cases, four independent determinations taken at different times. It is seen that for the short distances the current increases rapidly with increase in voltage, while at the larger distances the current increases very slowly.

The results for all distances cannot be represented even roughly by an equation of the form given by Warburg,¹ who used distances up to 7 centimeters only. The experimental values of the current I may be

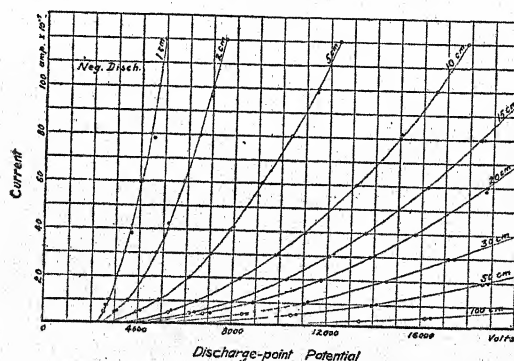


Fig. 3.

represented, however, in terms of the voltage V , minimum potential M , and distance D , by the following equations:

For positive discharge,

$$I = \frac{431 \times 10^{-15} V(V - M)}{(D + .0000589D^3)^{1.08}} \text{ amp.}$$

¹ Loc. cit., p. 83.

For negative discharge,

$$I = \frac{710 \times 10^{-15} V(V - M)}{(D - .00692D^2 + .0000659D^3)^{1.24}} \text{ amp.}$$

The points shown by the filled circles in Figs. 2 and 3, indicate the degree of agreement with the experimental results which are represented by open circles. The agreement is good in all cases except for the negative discharge at one centimeter distance.

5. *Potential Gradients.*—E. Warburg¹ investigated the state of potential at various points between a discharge point and a plane while a current was flowing between them, by means of a platinum probe-point; and the chief feature of his results is the large fall of potential which he found in the immediate neighborhood of the plane this being in some cases one fourth of the potential of the point. Since the maximum distance between the point and the plane used by Warburg in this experiment was only 6 cm. it seemed that with the large distances used in these experiments more trustworthy results could be obtained, since, especially at some distance from the point, the probe would here be less of a disturbing factor.

The probe that was used consisted of a fine platinum wire, .008 cm. in diameter, soldered to a copper wire, .025 cm. in diameter, and protruding 1.5 cm. from a finely drawn out glass tube fused about it. The drawn out glass tube formed an elongated tip of a long tapering white pine rod through the center of which ran the wire leading from the probe wire. After leaving the rod the wire was led to a voltmeter through an earthed metal tube. All the observations were made with the probe in the normal line from the discharge point to the plane, and the distance between the plane and the discharge point was 50 centimeters throughout. For each position of the probe the potential was taken for five different voltages of the discharge point.

The results are recorded in Fig. 4. The most salient feature of these curves is the fact that the points for any one distance are collinear. It is found that when the straight lines which best connect the points for the various distances are continued backward till they intersect the axis of the discharge point potential, these intersections are grouped together fairly close except in the case of the short distances where the intersections move towards the origin. It should be noted, however, that it is for these short distances that the probe disturbs the current most seriously. The introduction of the probe diminished the current about 12 per cent. for the 1.5 cm. distance, while the diminution was only 5 per cent. at 7 cm. and fell to 2 per cent. for 15 cm.

¹ Ann. der Phys., IV., 2, p. 299, 1900.

From the curves in Fig. 4, by the aid of those in Figs. 2 and 3, the potential distance curves drawn in full lines in Fig. 5 were constructed for the two values of the current there indicated. Choosing a certain current, Figs. 2 and 3 give the potentials required to maintain this current for the two kinds of discharge when the distance is 50 cm.; these discharge-point potentials are then used to obtain from Fig. 4 the corresponding probe potentials for the various distances, which are plotted in Fig. 5.

The curves in Fig. 5 show that the potential gradient between a point and a plane is much more uniform than it was before the current started to flow. There is indeed a rapid falling off of potential near the point similar to that found by Warburg; but this can hardly be called a potential fall as used in the case of vacuum tubes; for here the potential at the nearest part is actually considerably higher than it must be in the static condition before the current begins to flow. An idea of the latter case can be obtained by calculation on two concentric spheres.

The curves also show a rapid falling off near the plate; but this is very small as compared with that found by Warburg, and it seems to be, at least for the most part, in the gas and not at the immediate surface of the plate.

There is a method by which one may ascertain the distribution of potential between the point and the plane and avoid the use of the disturbing probe. For a given distance, say 50 cm., there is a certain distribution of potential for a given current. One of the equi-potential surfaces may be replaced by a metal surface and the current will remain unchanged if the same potential difference exists between the point and the surface as before; that is, if the surface introduced be now connected to earth, the potential of the point must be reduced by the previous value of the potential of the surface. The difference between the voltages on the point required to maintain the same current in the two cases gives the potential of the place in the first case before the new surface was in-

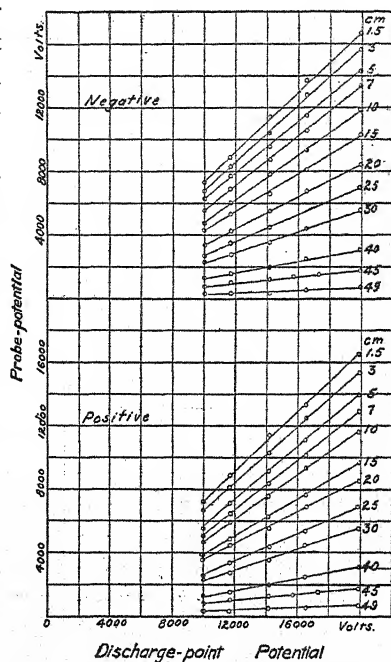


Fig. 4.

troduced. This can be repeated for various distances and a potential-distance curve plotted in this way. Supposing that a plane may replace roughly the actual equi-potential surface in question, the desired potentials can be obtained from the potential-current curves for the various distances as given in Figs. 2 and 3. It is only necessary to select a given current and find the point voltages required to maintain this current for the different distances. The differences between these give the desired data.

In using this method of differences, there is an error which may be introduced by the fact that the fall of potential close to the plate for a given current is different for different distances between the point and the plane. As a probe disturbs the conditions least when near the plate,

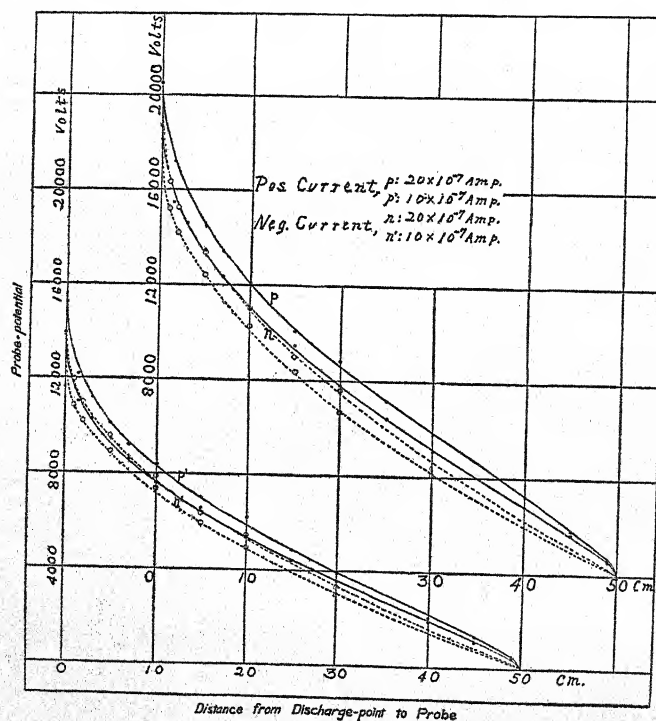


Fig. 5.

a special determination of these drops in potential at the plate was made by the probe method for the different distances and different currents, and corresponding corrections were made in taking the differences of potential in question.

The dotted curves in Fig. 5 were obtained in this way and for the sake of comparison were constructed for the same values of the current as the

other curves there shown, the upper one being for the positive discharge in each case, as before. The points thus determined are indicated by open circles, while filled circles denote the points obtained by the other method. It is seen that the two methods give results which are quite alike except that the dotted curves fall somewhat below the others.

As no current observations were taken for distances between 30 cm. and 50 cm. this portion of the curves under consideration could not be accurately determined.

The slopes of these curves, that is the value of dV'/dD for the various points, may be obtained mechanically with a fair degree of accuracy. But instead of using the mechanical method an empirical relation for V' in terms of D was obtained for the curves determined by the probe method, and the slope found by differentiation. A sample set of values of these gradients at different points is given in the following table for the two kinds of discharge, both with a current value of 10×10^{-7} amp. The ratio of these gradients for the various distances is also given in the last column.

D	$\left(\frac{dV'}{dD}\right)_+$	$\left(\frac{dV'}{dD}\right)_-$	Ratio.
1	-623	-416	1.50
2	-457	-385	1.19
5	-388	-333	1.17
10	-296	-270	1.10
15	-251	-226	1.11
20	-216	-194	1.12
30	-171	-153	1.12
40	-181	-163	1.12
45	-238	-212	1.12

It is noticed that the values of the gradients diminish as one recedes from the point, reach a minimum, and then increase again near the plate. The gradient for the positive discharge is always larger than the corresponding one for the negative discharge. Neglecting the short distances, the ratio of the two gradients is practically the same at all points and for all values of the current. The average value of the ratios for currents between 5×10^{-7} and 20×10^{-7} amp. was found to be 1.11. The ratio for a distance of one centimeter was much larger in every case.

The potential gradient at any point depends upon the value of the current flowing, and it was found that the ratio of the gradients at any point for any two given currents is practically the same for both kinds of discharge and for all places between the point and the plane. For the currents 20×10^{-7} and 5×10^{-7} amp., the average ratio found for the

gradients was 2.05; for currents 20 and 10 of the same units, the average ratio was 1.43, while that for currents 20 and 15, it was 1.16. It is seen that these ratios are but slightly in excess of the square root of the ratios of the corresponding currents.

If i denote current density along the normal where the gradients were determined, u the ionic velocity for unit gradient, and n the electric density, then

$$i = un \frac{dV'}{dD}$$

or,

$$\frac{dV'}{dD} = \frac{i}{un} = K \sqrt{i}$$

by the above results, remembering that Warburg¹ showed that the total current is proportional to the central current density. Hence $[n \propto \sqrt{i}]$; or the density of the electric charge at all points and for currents of both signs is proportional to the square root of the current flowing.

For the same current value,

$$\frac{(u_-)(n_-)}{(u_+)(n_+)} = \frac{\left(\frac{dV'}{dD}\right)_+}{\left(\frac{dV'}{dD}\right)_-} = 1.11.$$

Hence,

$$n_- = 1.11 (n_+) \left(\frac{u_+}{u_-}\right).$$

The ratio 1.11 may depend upon the ratio (u_+/u_-) , as it was determined for one value only of the latter, *i. e.*, that existing in the room air of average humidity. The relation shows however that for the same current of both signs the electric density is not greatly different, and if the value 1.11 holds for moist air,² the two values of n would be exactly equal.

6. *Summary.*—The starting potential and the currents for different voltages have been determined for a discharge from a point to a plane for distances up to 100 cm. between the latter. The distribution of potential in the air has also been investigated.

It has been found that the starting potential for the short distances between the point and the plane increases rapidly with increasing distances, while at larger distances the increase becomes small, the starting potential appearing to approach a constant value for very large distances.

The currents increase rapidly with the voltage while the distances

¹ Loc. cit.

² J. Zeleny, Phil. Trans., A. 195, p. 210, 1900.

are small but increase very slowly with increasing voltage for the larger distances.

The potential gradients along the normal from a discharge point to a plane decrease rapidly as one recedes from the point, pass through a minimum and again increase near the plate.

The gradients at any point have been found to be proportional to the square root of the current flowing. Omitting the shortest distances the potential gradient for a positive current of given value bears a constant ratio to the potential gradient for the negative current of the same value.

In conclusion I wish to acknowledge my indebtedness to Professor J. Zeleny for the kindly interest he has shown in the progress of the experiments and for his helpfulness in the preparation of this paper.

PHYSICAL LABORATORY, UNIVERSITY OF MINNESOTA,

August 30, 1911.

EFFECT OF FREQUENCY ON THE CAPACITY OF A CONDENSER, WITH KEROSENE FOR THE DIELECTRIC.

BY S. HERBERT ANDERSON.

IN carrying out some experiments with the Poulsen arc, it was necessary to have a variable condenser which should have a constant capacity for the range of frequencies and potentials used. For this purpose a variable condenser was built in the department shop. It consisted of 16 semicircular fixed plates, 19.5 cm. in diameter, with spaces of 2 cm. between the plates. Between these were 15 movable plates 15 cm. in diameter, mounted on an axis. Story,¹ in using a condenser of this type, found for a frequency of 50, a maximum capacity of 1,696 cm., and for a frequency of 1,000 a maximum capacity of 1,138 cm. (taken from the graph). As this is a difference of about 50 per cent., it seemed worth while to investigate just how the capacity varied with the frequency.

Practically every method of measuring capacity was tried but only four will be mentioned. It was necessary to avoid any method which would involve resistances or connections having a capacity as large as .000005 mfd., since this would introduce an appreciable error in the results, unless a correction could be made for such auxiliary capacities.

1. A static method was first tried. The condenser was charged for a given length of time by a potential of 150 volts and then discharged through a ballistic galvanometer. In all cases kerosene was used as the dielectric. For times of charging of less than one second, a type of Helmholtz pendulum interrupter was used. For times greater than one second a Kempe key was used. In these measurements the condenser was not allowed to stand charged before discharging through the galvanometer, but the discharge was made as nearly instantaneously upon the break from the battery as the manipulation of the apparatus would permit.

The values of the capacity found by this method for different times of discharging are as follows:

Thus we see that for a time of charge of 1 second or less, the capacity is constant. But for a time of charge of more than 1 second there is an absorption in the dielectric.

¹ Wm. E. Story, "The Poulsen Arc," *PHYS. REV.*, Vol. XXX., p. 236, 1910.

Time of Charging, Seconds.	Potential, Volts.	Capacity, Mfd.
5	150	.000806
4	150	.000744
3	150	.000707
2	150	.000689
1	150	.000676
0.2	150	.000665
0.08	150	.000671
0.008	150	.000676
0.0004	150	.000671

2. The second method used was similar to that devised by Fleming and Clinton.¹ This method is the most satisfactory of all tried, and when the atmospheric conditions are suitable for good insulation, the results can be relied upon to 1 per cent. accuracy.

A rotating commutator making a single commutation per rotation is connected with a battery, the condenser and a galvanometer so that with each commutation the condenser is charged by the battery and discharged through the galvanometer. If the period of the galvanometer is large compared with the period of charge and discharge, the galvanometer gives a steady deflection and the capacity in farads is given by

$$C = \frac{A}{nV},$$

where A is the current in amperes which gives the same steady deflection, n the frequency of discharge and V the potential in volts. Two precautions are necessary for the successful manipulation of this method: (a) the condenser, galvanometer, battery and all lead wires must be insulated just as perfectly as possible; (b) the commutator must be run at a constant speed. For this purpose a $\frac{1}{4}$ horse-power D.C. motor operated by a current from a storage battery was used. It is necessary to have a rather powerful motor in order to keep the speed constant. The values of the capacity obtained by this method are as follows:

Frequency.	Potential, Volts.	Capacity.
14.5	24.6	.000654
16.07	24.6	.000652
19.3	24.6	.000655
19.7	24.6	.000654
22.97	24.6	.000659
28.41	24.6	.000659
31.12	24.6	.000654
37.18	24.6	.000654
38.85	24.6	.000657
46.35	24.6	.000657
94.	16.4	.000660

¹ J. A. Fleming and W. C. Clinton, "On the Measurement of Small Capacities and Inductances," Phil. Mag., Vol. V., ser. 6, p. 493.

These values show that within the limits of experimental error the capacity as measured by this method is constant for frequencies from 14 to 94. It will be noticed that the mean value of the capacity as measured by this method is somewhat smaller than the value obtained by the static method. This was due to a difference in the insulation of the apparatus. The determinations with the rotating commutator were made on a day when the humidity was very low and the insulation good; while the measurements by the static method were made when the air was quite humid and good insulation impossible. It was found that

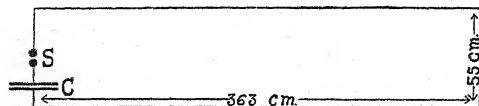


Fig. 1.

when the insulation was poor, the leakage to objects near the condenser had the effect of connecting a condenser in parallel to that whose capacity was being measured.

3. In order to test the capacity for frequencies of higher order than a hundred and of the range for which the Poulsen arc was used, the condenser was put in an oscillation circuit of the form of Fig. 1. The rectangle consisted of No. 20 copper wire. *C* is the variable condenser; *S* a spark gap of 1.8 mm. between two zinc balls well polished. The potential corresponding to this spark length is about 7,400 volts. The frequency of this oscillation circuit can be obtained by Lord Kelvin's formula:

$$n = \frac{1}{2\pi\sqrt{LC}}.$$

The coefficient of self-induction, *L*, can be computed by the dimensions of the system. By means of a Fleming cymometer, loosely coupled with the circuit, the frequency *n* can be determined. This was found to be 1,600,000, and hence the capacity .000829 mfd. Since this value was considerably higher than that obtained by the two previous methods, it seemed likely that either the cymometer was not correctly calibrated, or the inductance of the circuit was other than the value obtained by calculation, due to the proximity of iron which could not be avoided. In order to test this, the kerosene condenser was replaced by a variable air condenser. The cymometer was set at the point of resonance when the kerosene was in the circuit, and the air condenser adjusted until the circuit was in resonance with the cymometer. Then the capacity of the air condenser was measured with the rotating commutator. This was found to be .000669 mfd. This then may be taken as the value of the capacity of the kerosene condenser obtained by this method.

A second measurement was made by this same resonance method, but with the inductance increased and hence the frequency lowered by replacing the single turn of wire by seven turns. The frequency of this circuit was 380,000, and the value of the capacity .000669, the same as for a frequency of 1,600,000. This value differs from the mean value obtained by the rotating commutator method by 2.1 per cent. As the accuracy of setting the cymometer is not greater than this, we may conclude that within the limits of experimental error, the capacity of the condenser is the same for frequencies of 380,000 and 1,600,000 as for the lower frequencies.

4. As a further check upon these determinations it was desired to make some measurements for frequencies between 100 and 380,000 by the Wheatstone Bridge method. It was found that resistances of the order of 1,000 ohms must be used in order to give an accurate degree of setting, and a high potential for charging was required. For the resistances the 1,000 ohm coils in two Hartmann and Braun electrolytic bridges were used. These were wound according to the Chaperon principle so that a minimum capacity effect was introduced. These resistances were kept constant and the point of balance obtained by adjusting the variable air condenser, which had been previously calibrated by measurements with the rotating commutator. For the high potential for charging the following scheme was used: an electrically driven tuning fork of frequency 200 was connected in series with the primary of a small induction coil in place of the ordinary contact make and break. The current through the fork and primary was 0.4 ampere. The secondary of the induction coil was connected to the bridge.

As detectors of the point of balance a galena crystal rectifier in series with a galvanometer of sensitiveness 5×10^{-9} , a Duddell thermo-galvanometer of sensitiveness of 7×10^{-5} for a 10 mm. deflection, and a Siemens & Halske 400 ohm telephone receiver were tried. It was found that the telephone was about 100 times more sensitive than either of the other detectors.

The value obtained for the capacity by this method was .000680 mfd. When the 400 ohm telephone was replaced by a 1,000 ohm, a different point of balance was obtained, which gave a capacity of .000697 mfd. This difference and the fact that it was very difficult to obtain a definite point of balance shows that this method cannot be relied upon for the measurement of small capacities. Furthermore capacity effects are not altogether done away with in the resistances so that the simple formula

$$C_2 = \frac{C_1 R_1}{R_2}$$

does not hold.

SUMMARY.

1. The capacity of a parallel plate condenser with kerosene as the dielectric is not affected by frequency for a range of 1 to 1,600,000. Consequently this type of condenser is very desirable to use for work in electric oscillations, as the value of the capacity obtained for low frequencies holds for high frequencies.

2. The most satisfactory method of measuring small capacities is the Fleming-Clinton rotating commutator scheme. This is very accurate if the insulation is good.

I am greatly indebted to Prof. A. P. Carman for many suggestions and the excellent facilities for carrying on these experiments.

NOTE.

Since the above article was finished the equation for the measurement of capacity with the Wheatstone bridge, assuming that the resistance used has capacity which is not negligible in comparison with that to be measured, has been worked out. This formula shows why it is difficult to get a point of balance, and indeed that a point of balance cannot be obtained without specially wound resistances.

Let us assume that R_1 (see Fig. 2) has a capacity c_1 , and R_2 a capacity c_2 , in parallel with the resistance. Take C_1 to be a standard condenser and C_2 the condenser whose capacity is to be measured. According to

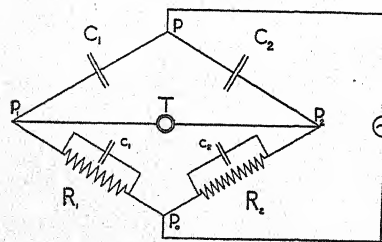


Fig. 2.

the usual manipulation, when an alternating current is applied to the bridge the resistances may be adjusted to such a ratio that P_1 and P_2 have the same potential and consequently no current goes through the telephone. Let i_1 be the entire current through R_1 and c_1 ; i_2 the current through R_2 and c_2 ; i_3 the current through C_1 ; and i_4 the current through C_2 . Then

$$i_1 z_1 = P_1 - P_0, \quad (1)$$

$$i_2 z_2 = P_2 - P_0, \quad (2)$$

$$i_3 z_3 = P - P_1, \quad (3)$$

$$i_4 z_4 = P - P_2, \quad (4)$$

where z_1 , z_2 , z_3 , and z_4 are the reactances of the respective circuits, and

$$z_1 = \frac{jR_1}{j - R_1 c_1 \omega},$$

$$z_2 = \frac{jR_2}{j - R_2 c_2 \omega},$$

$$z_3 = -\frac{j}{C_1 w},$$

$$z_4 = -\frac{j}{C_2 w},$$

where $j = \sqrt{-1}$, and $w = 2\pi n$, the angular velocity of the current vector. Subtracting (2) from (1) and (3) from (4), we get

$$P_1 - P_2 = i_1 z_1 - i_2 z_2 = 0,$$

$$P_1 - P_2 = i_4 z_4 - i_3 z_3 = 0.$$

The last two expressions are equal to zero because P_1 and P_2 must be the same when the bridge is balanced. Since $i_1 = i_3$, and $i_2 = i_4$,

$$\frac{z_2}{z_1} = \frac{z_4}{z_3}.$$

Substituting the values for z_1 , z_2 , z_3 , and z_4 , and clearing of fractions we get,

$$jR_2C_2 - R_1R_2C_2C_1w = jR_1C_1 - R_1R_2C_1C_2w. \quad (5)$$

Equating the imaginary parts we get

$$\frac{R_2}{R_1} = \frac{C_1}{C_2}, \quad (6)$$

which is the usual expression for the Wheatstone bridge. Equating the real parts we get

$$\frac{C_1}{C_2} = \frac{c_1}{c_2}, \quad (7)$$

and hence,

$$\frac{R_2}{R_1} = \frac{C_1}{C_2} = \frac{c_1}{c_2}. \quad (8)$$

This last equation expresses the conditions that must be satisfied in order to have a balance in the bridge. If we take any given ratio of the resistances, the corresponding ratio of the capacities C_1 and C_2 must be the reciprocal of this *and also the ratio of the capacities c_1 and c_2 must be the reciprocal of the corresponding ratio of the resistances.*

S. L. Brown¹ has determined the "residual" capacity of a 1,000 ohm Hartmann and Braun "Chaperon" wound coil such as the author used, and found it to be .000105 microfarad. As this is 16 per cent. of the capacity to be measured, equation (8) explains the difficulty in obtaining a balance and *shows that the Wheatstone bridge cannot be used for measuring capacities of the order of .0005 microfarad without having resistances wound to meet the conditions required by equation (8).*

¹ PHYS. REV., Vol. 29, pp. 369-391.

A MODEL OF THE ELEMENTARY MAGNET.

BY S. R. WILLIAMS.

INTRODUCTORY.

AT a meeting of the American Physical Society in December, 1910, I presented a theory of magnetism¹ in which the magnetic atom or elementary magnet was imagined as consisting of a positive nucleus (in shape an oblate spheroid), about which one or more negative electrons were revolving similar to satellites. This similarity to a planet with its attendant satellites was the reason for calling it the planetesimal theory of magnetism.

It was assumed that the positive nucleus possessed the property of permeability and since the nucleus has a major and minor axis it would tend to set itself with major axis parallel to a magnetic field when subjected to such a force. Further this nucleus possesses no permanent polarity on the removal of the imposed field but like a dielectric simply orients itself so that its greatest length is parallel to the field. The negative electrons, however, in revolving about the positive nucleus, and in the equatorial plane, produce a magnetic field which also tends to become parallel to the magnetic field in which it finds itself. There are thus two forces operative upon such a system when placed in a magnetic field. One force, due to the permeability of the nucleus, tends to set the equatorial plane parallel to the imposed magnetic field; the other, due to the field of the revolving electrons, strives to put the axis of revolution in the same position. For brevity we will call these the equatorial and axial forces respectively. The manner in which this system will be oriented in the magnetic field will depend upon its initial position and upon which force predominates. Moreover, as I pointed out in the paper referred to, upon the orientation of these elongated magnetic atoms depend the various magnetic phenomena such as magneto-striction, hysteresis, induction, the Kerr magnetic effect, the Villari effect, the Faraday effect in thin iron films and the behavior of certain crystals in a magnetic field.

Working on this hypothesis I was led to two new magneto-strictive effects, first a twist in hardened steel rods due to a longitudinal magnetic field, secondly, a change in length due to imposing a circular magnetic field upon a longitudinally magnetized rod of iron.

¹ *Phys. Rev.*, abstract, Feb., 1911.

Because of the suggestiveness of this model in opening up new fields of research and because of its usefulness in explaining so many inexplicable phenomena in magnetostriction, it seemed worth while to study the behavior of a model of this proposed magnetic atom in a varying magnetic field. The object of this paper, therefore, is to describe such a model and show some of its applications to magnetic phenomena.

APPARATUS.

An oblate spheroid of soft Swedish iron was used for the positive nucleus, which had a major axis of 1.2 cm. and a minor axis of 1.0 cm. It showed so little permanent magnetism that it would turn indiscriminately with major axis parallel to the imposed field no matter how it was oriented in the field.

To serve as the revolving electrons a current was sent through a coil of 100.5 turns of fine wire (No. 28), wound about the spheroid parallel to the equatorial plane. This system was mounted like the coil of a D'Arsonval galvanometer so that the current could be led to the coil by means of a phosphor-bronze strip, while the other end of the coil was directed downward from the lower side of the coil and terminated in a mercury cup by which the circuit was completed. In order to show the connections a vertical, cross-sectional view of the planetesimal system is given in Fig. 1. A horizontal cross-section would be the same for the coil and the core.

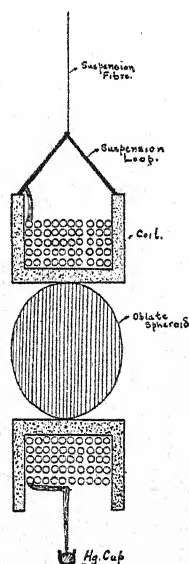


Fig. 1.

This iron spheroid with its coil was mounted inside of a large solenoid with a suspension head that allowed the equatorial plane of the spheroid to be set at any angle with respect to the axis of the inclosing solenoid. Fig. 2 shows a horizontal, cross-sectional view of the apparatus. For reading the deflections of the system a fine whip of glass was attached to the coil so that it was parallel to the minor axis of the spheroid. This pointer moved over a divided circle as shown in the figure. The large solenoid for producing the external magnetic field had 1,466 turns and was 53.7 cms. long with diameter of free space equal to 25.0 cm. The constant of the solenoid both by measurement and by calculation was 31.0 gauss-per ampere.

The first conditions studied were with the equatorial plane of the nucleus set at an angle of 45° with the axis of the large solenoid as shown in Fig. 2 and with various magnitudes of current flowing about the spheroid.

With a constant current in the coil encircling the nucleus, the orientation of the system was studied as the field in the large solenoid was varied from zero upward. For certain currents about the spheroid the axial force was at first predominant but as greater field strengths in the solenoid were attained the equatorial force became greater and a reverse rotation of the system was obtained. Again, the equatorial force could be made predominant for all field strengths if the current about the nucleus was made small enough. Similarly, by making the current large enough, the axial force could be made predominant for all field strengths.

From an inspection of Fig. 2 it is evident that if a line be drawn along the axis of the large solenoid and passing through the spheroid the length of the line in the spheroid will vary as the spheroid is rotated. If the

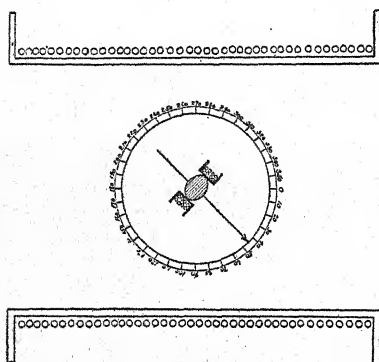


Fig. 2.

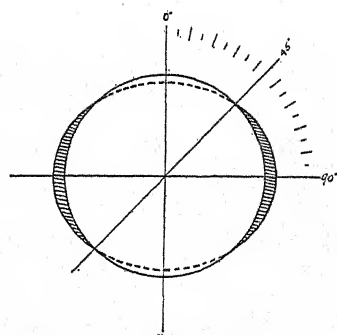


Fig. 3.

pointer moves toward 0° the length will decrease, if toward 90° it will increase. This change in length of the axial line in the spheroid was the quantity finally compared with the field in the solenoid as it was varied from zero upwards. The method of measuring this change in length was as follows: a large cross-sectional drawing of the spheroid was made to scale as shown in Fig. 3. Over this cross-section was drawn a circle whose diameter was equal to the length of the diameter of the spheroid inclined at 45° to the two axes. This diameter was taken because the equatorial plane of the spheroid was set at an angle of 45° with the axis of the solenoid and change in length of the diameter of the spheroid at this angle was taken as zero. At any point between 0° and 90° the difference in the two diameters, Fig. 3, could be measured by means of a scale. This difference gave the change in length of the axial line in the spheroid. Between 0° and 45° it was negative and between 45° and 90° it was positive. This variation in length of spheroid parallel to axis of solenoid,

as the system rotated, is shown in Fig. 4,¹ where variation in length is plotted as ordinates and degrees of rotation as abscissæ. From this curve and the readings on the divided circle, as spheroid rotated under

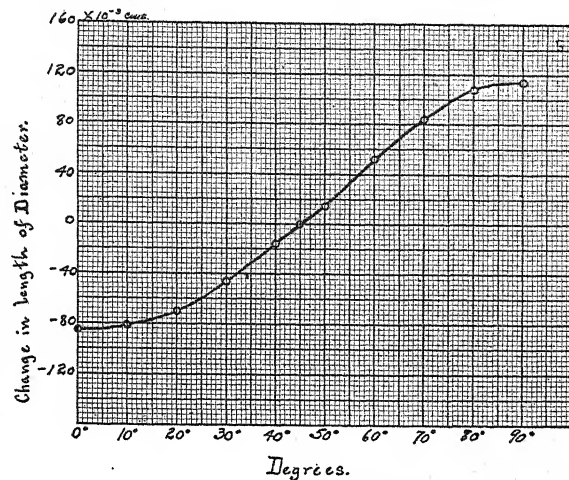


Fig. 4.

influence of magnetic field, the variation in length of diameter of spheroid parallel to axis of solenoid was measured and compared with varying field strengths as shown in Fig. 5. The object in studying this variation

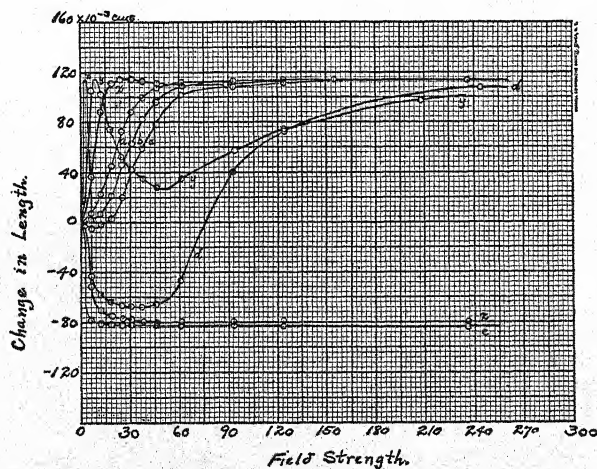


Fig. 5.

in length was because I wish to show later on that if a bar of iron for instance was made up of a great many such elementary magnets their

¹ The values as plotted were checked up by means of the equation for the diameter of an ellipse.

rotation would vary the length of the rod as we have the phenomenon in the Joule effect.

For the curves *a*, *b*, *c*, *d* and *e*, Fig. 5, the currents 0, 5, 10, 50 and 500 milamperes respectively flowed in the coil about the spheroid. The direction of the fields in the coil and the solenoid are shown in Fig. 6. On reversing the direction of the field in the coil as shown in Fig. 7,

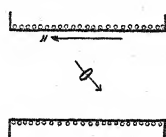


Fig. 6.

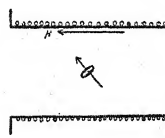


Fig. 7.

the curves *x*, *y* and *z*, Fig. 5, were obtained with currents of 20, 50 and 500 milamperes respectively flowing about the spheroid.

Two features of the curves catch the eye immediately, first, they have the characteristics of curves obtained in magnetic observations (Fig. 4 also shows this characteristic), and secondly, there are four distinct types of curves showing changes in length just as in the Joule magneto-strictive effect we have four distinct types. These are:

1. Curves which are negative and become asymptotic.
2. Curves which are positive and become asymptotic.
3. Curves which are first positive and then become asymptotic negatively.
4. Curves which are first negative and then become asymptotic positively.

By proper variation of the current flowing about the spheroid and

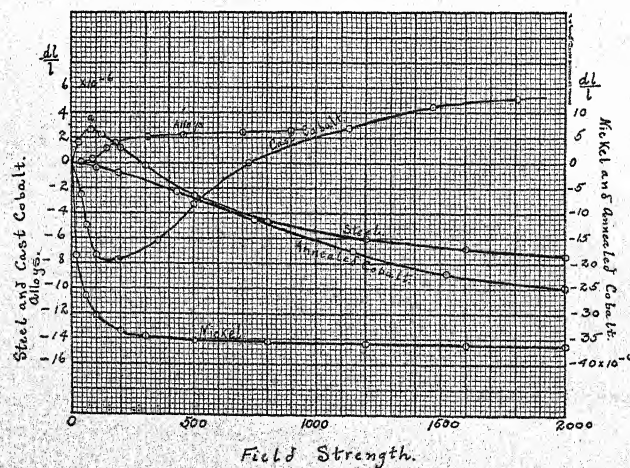


Fig. 8.

the initial orientation of the spheroid with respect to the imposed magnetic field, curves intermediate between these types just mentioned may be obtained but all may be classified in the four groups. Curves *z*, *e*, *d* and *b* are the types found in the change in length in iron, nickel, cast cobalt and the Heusler alloys respectively. This may be seen by comparing the curves in Fig. 8 with those in Fig. 5. This seems to me an important relation. The curves for nickel, cobalt and steel are from the works of Nagaoka and Honda¹ and the curve for the Heusler alloys from a paper by McLennan.²

APPLICATIONS OF RESULTS.

Suppose we consider a specimen of virgin iron as made up of a great many magnetic atoms like the one described. The equatorial planes may be considered as oriented in all possible directions but which may be represented by the six positions shown in initial condition, Fig. 9, where the arrows indicate the direction of the fields due to the revolving electrons. From the strong magnetism we find in iron and from the character of our curves we may infer that the number of electrons revolving about each nucleus is sufficient to make the axial force predominant for all field strengths. If such be the case all of the planetesimals will be turned with poles pointing in the directions indicated in final condition, Fig. 9, if a magnetic field, *H*, be applied.

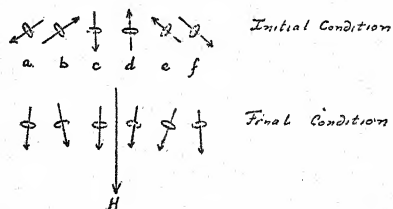


Fig. 9.

In this process *a* and *b*, during the first stages of rotation, will annul each other in their effects of changing length as well as *e* and *f*. For *c* and *d*, *c* will remain in about the same position it occupied in the initial position while *d* will make a turn of about 180° just as the model did in curve *z*, Fig. 5. There will be no others to counteract the effect of the atoms so oriented, thus their combined effect is to give a change in length represented by the curve for steel in Fig. 8. In some specimens of iron the resultant orientation of all of the magnetic atoms may not be fairly represented by the groupings shown in Fig. 9. In that case we will get some intermediate curve as Bidwell³ and others have done.

It will be noted that in the final condition, Fig. 9, all of the magnetic atoms are oriented with the fields of the revolving electrons in a definite

¹ Nagaoka and Honda, Phil. Mag., July, 1902, p. 45.

² McLennan, Phys. Rev., June, 1907, p. 449.

³ Bidwell, Proc. Roy. Soc., p. 228, Vol. 55, 1894.

direction. When coming into this position their mutual effects doubtless have much to do with setting them all more nearly parallel to each other as Ewing found in his model of a magnet.

Thus by means of the initial orientation of the magnetic atoms and the relative values of the axial and equatorial forces operative upon them it is possible to show how the variation of length with change in magnetic field strength occurs as it does in the various specimens of ferro-magnetic substances. Having shown how such a model may explain the Joule effect it becomes an easy matter to explain the other magneto-strictive effects as I pointed out in the paper referred to. This follows at once from the general relation that *elongation indicates that the fields of the revolving electrons of many of the magnetic atoms are being turned normal to the imposed field while contraction means they are assuming a position parallel.*

A striking relation between the Joule effect and another magnetic effect is in the case of the Kerr phenomenon. If plane polarized light is reflected normally from the pole of an iron magnet, along the lines of force, the plane of polarization will be rotated by reflection. In the case of iron the planes of revolution of the electrons will be normal to the lines of force and the revolving electrons will influence the rotation of the plane of polarization. In the Heusler alloys, however, we have elongation for all field strengths in the Joule effect. This means, from the general statement given above that the planes of revolution of the electrons are parallel to the lines of force and hence do not affect the rotation of the plane of polarization. Ingersoll¹ has found that the Kerr magnetic effect is not present in the Heusler alloys.

CONCLUSIONS.

Whether such a model as described is a true picture of the elementary magnet or not is a question which only a great amount of experiment will answer. One thing does seem clear: that whatever theory does finally coördinate the diverse magnetic phenomena which we know, that theory must have as its salient feature something which corresponds to the two forces of which I have spoken as the axial and equatorial forces.

In conclusion I wish to point out again that the important result which this theory leads to is that the orientation of the elementary magnet in ferromagnetic substances determines the character and magnitude of the various magnetic phenomena. Consequently since the Joule effect gives us the clue as to how the elementary magnets are turned, the Joule

¹ Ingersoll, Phil. Mag., p. 41, Jan., 1906.

effect becomes a means whereby we may predict how all the other magnetic phenomena will occur in the same specimen.

In a series of papers I wish to point out some of these striking relations which I have found between the Joule magneto-strictive and other magnetic effects.

PHYSICAL LABORATORY, OBERLIN COLLEGE,
OBERLIN, OHIO, Sept. 13, 1911.

AN IMPORTANT PRACTICAL PROBLEM IN
GYROSTATIC ACTION.

BY W. S. FRANKLIN.

THE gyrostatic mechanism of the Brennan monorail car has two distinct functions, namely, (*a*) to prevent the tipping of the car when it is for a short time out of balance, and (*b*) to right the car before the limit of action (*a*) is reached; indeed it is essential that this righting action be exaggerated so that the unbalanced condition of the car may be momentarily reversed.

The action (*a*) exists in every case in which it has been proposed to utilize the steadying action of the gyrost. This action depends upon precessional motion, and it is important to consider how much the steadied structure yields to a disturbing force while the precessional motion is being established.

For example, the gyrost. in a torpedo exerts a force on the steering gear of the torpedo when the torpedo turns momentarily from its straight course, and the pivoted frame which carries the gyrost. always yields to some extent in exerting this force.

Another example would be furnished by a gun platform supported on gimbals and steadied by gyrostats. Any disturbing force, such as friction at the gimbal bearings, would be counteracted by precessional motion of the gyrostats, but the entire structure would yield to some extent to the disturbing force during the establishment of the precessional motion.

The calculation of the amount of this yield and of the time during which it is produced is the object of this paper, and fortunately the general theory of rotatory motion can be greatly simplified under the conditions which would be encountered in practice, because the spin velocity of the gyrost. wheels would be very great in comparison with the angular velocity of precessional motion.

The simplest example of the action under consideration is presented by an ordinary toy gyrost. as shown in Fig. 1 when a temporary support of the gyrost. ring at *p* is removed so as to allow the pull of gravity and the upward push of the post at *P* to exert a torque *TT* upon the gyrost. ring.

Let *AB*, Fig. 1, be called the *y*-axis; and let a horizontal axis through

the top of the post P and at right angles to the plane of the gyrostator ring (assumed to be rotating about AB at uniform angular velocity Ω as explained later) be called the z -axis. The positive direction of the z -axis is inwards in Fig. 1.

The motion of the gyrostator after the torque TT begins to act may be considered in three parts as follows:

(a) A uniform precession about the y -axis at angular velocity Ω such that

$$\omega \Omega K = T \quad (1)$$

where ω is the angular velocity of spin of the gyrostator wheel, K is the moment of inertia of the wheel about its axis of spin, and T is the value of the torque TT , Fig. 1.

(b) An oscillatory movement about the y -axis, and

(c) An oscillatory movement about the z -axis.

Let y be the angular velocity at a given instant of the oscillatory movement about the y -axis, and let z be the angular velocity at the same instant of the oscillatory movement about the z -axis. Then at the instant that the torque T begins to act the gyrostator ring is stationary and therefore we have the initial conditions:

$$\left. \begin{aligned} t &= 0, \\ y &= -\Omega, \\ z &= 0. \end{aligned} \right\} \quad (2)$$

The two angular velocities y and z are precessional motions like Ω and the torques required to produce y and z are determined by equations similar to equation (1). Therefore the torque required to produce y is $\omega y K$ (acting about the z -axis), and the torque required to produce z is $-\omega z K$ (acting in a negative direction about the y -axis). Furthermore dy/dt is the angular acceleration of the ring and wheel about the y -axis, and dz/dt is the angular acceleration of the ring and wheel about the z -axis. And the torques required to produce these angular accelerations are $l \cdot (dy/dt)$ (acting about the y -axis), and $m \cdot (dz/dt)$ (acting about the z -axis); where l is the combined moment of inertia of the ring and wheel about the y -axis, and m is the combined moment of inertia of the ring and wheel about the z -axis. But there is no torque acting on the gyrostator except the torque T due to gravity which is wholly engaged, as it were, in producing the uniform precessional motion Ω . Therefore the sum of the two torques $l \cdot (dy/dt)$ and $-\omega z K$ is equal to zero, and the sum of the two torques $m \cdot (dz/dt)$ and $\omega y K$ is equal to zero. Therefore we have:

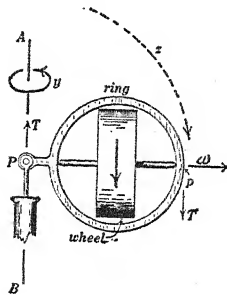


Fig. 1.

$$l \cdot \frac{dy}{dt} = \omega z K \quad (3)$$

and

$$m \cdot \frac{dz}{dt} = -\omega y K. \quad (4)$$

The values of l and m are sensibly constant because of the small angular displacements resulting from the oscillatory movements about the y and z axes. Therefore equations (3) and (4) give by differentiation and elimination:

$$\frac{d^2y}{dt^2} = -\frac{\omega^2 K^2}{lm} \cdot y \quad (5)$$

and

$$\frac{d^2z}{dt^2} = -\frac{\omega^2 K^2}{lm} \cdot z. \quad (6)$$

The general solutions of equations (5) and (6), written in the form most easily used, are:

$$y = C \cos (pt + \theta) \quad (7)$$

and

$$z = C' \sin (pt + \theta'), \quad (8)$$

where

$$p^2 = \frac{\omega^2 K^2}{lm}. \quad (9)$$

The constants of integration C , C' , θ and θ' are determined by the conditions imposed by equations (3) and (4) and by the initial conditions (2), and the expressions for y and z which satisfy all of the conditions are

$$y = -\Omega \cos pt \quad (10)$$

and

$$z = \sqrt{\frac{l}{m}} \cdot \Omega \sin pt. \quad (11)$$

Let Z be the maximum angular displacement of the gyrostat frame in the direction of the suddenly applied torque T , and let τ be the time which elapses before this maximum angular displacement is reached. It is evident from equation (11) that the oscillatory movement z is periodic and that its period is $2\pi/p$, and it is also evident that the time τ must be half a period. Therefore $\tau = \pi/p$, or, using the value of p from (9) we have

$$\tau = \frac{\pi \sqrt{lm}}{\omega K}. \quad (12)$$

The value of Z is obtained by integrating $z \cdot dt$ between the limits $t = 0$ and $t = \tau$, and therefore Z is equal to $2\Omega/\omega K$, or, using the value of Ω from equation (1), we have

$$Z = \frac{2lT}{\omega^2 K^2}. \quad (13)$$

It is interesting to note that the value of Z can be derived from the principle of conservation of momentum as follows: From equations (10) and (11) it is evident that, $y = +\Omega$ when the axis of the gyrostat has reached its lowest point Z radians below the horizontal. Therefore the total angular velocity of the gyrostat about the y -axis is 2Ω , and the angular momentum of the gyrostat about the y -axis is $2l\Omega$. On the other hand, the vertical component of the spin momentum ωK of the gyrostat wheel is equal to $Z\omega K$, the angle Z being very small. But there is no angular momentum about the y -axis at the start and there is no torque action about the y -axis. Consequently $2l\Omega + Z\omega K$ must be equal to zero, or ignoring sign we find $Z = 2l\Omega/\omega K$.

A possible method for steadying the motion of a platform aboard ship would be to mount the platform on gimbals, and place upon the platform two independent Brennan pairs of gyrostats, an A -pair for controlling motion about the A -axis of the gimbals, and a B -pair for controlling motion about the B -axis of the gimbals. The relation of the A -pair to the A -axis is shown in Fig. 2, and the B -pair would be similarly related to the B -axis.

Such a mechanism has two entirely independent¹ oscillatory movements. The oscillatory movement involving the A -pair of gyrostats, for example, would consist of an oscillatory movement of the entire system about the A -axis of the gimbals combined with an oscillatory movement of the A gyrostats about their vertical trunion-axes yy as shown in Fig. 2. These two connected oscillatory movements are in time quadrature with each other, and their angular amplitudes are in a fixed ratio. And the two connected oscillatory movements involving the B -pair of gyrostats are similarly related to the B -axis and the B gyrostats.

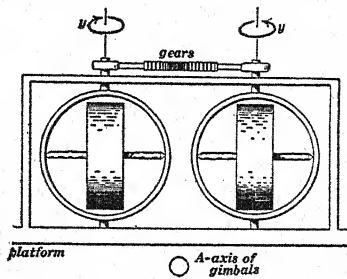


Fig. 2.

The equations of motion of the system in Fig. 2 are two pairs of equations very similar to equations (3) and (4), and the range Z and duration τ of the yield of the platform due, for example, to a friction torque exerted steadily on the platform through the gimbal bearings while the ship slowly rolling or pitching would be calculated by using equations similar to (12) and (13).

¹ It must be remembered that the y and z movements in Fig. 1 are mutually dependent.

The use of a perfectly controlled platform as a gun-mount on a warship would seem to be highly desirable, but perfect control is of course impossible, and any gyrostatic mechanism introduces high-frequency oscillations; and although these high-frequency oscillations are of small amplitude they might be much more troublesome to the gunner than the very wide range but slow oscillations of the ship itself.

One can appreciate the essential mechanics of the mechanism in Fig. 2 by reducing the pair of gyrostats to an equivalent spring, and loading the platform with the weights WW so as to leave its weight and moment of inertia unchanged. Thus Fig. 3 shows a mechanism which is exactly equivalent to Fig. 2 insofar as oscillatory movements of the platform are concerned. The spring in Fig. 3, however, can exert a steady torque

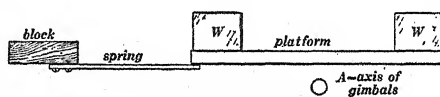


Fig. 3.

upon the platform for an indefinite time which cannot be done by the mechanism in Fig. 2. The block in Fig. 3 is to be thought of as traveling up and down with the gimbal axis so that the spring is bent only when the platform is tilted. If such a spring-controlled platform would be of important usefulness on board ship then the mechanism of Fig. 2 would be of important usefulness but of course very expensive.

ON THE MOBILITY OF IONS IN AIR AT HIGH PRESSURES.

BY A. J. DEMPSTER.

ALTHOUGH several theories bearing on the mobility of ions in gases have been given, none as yet can be said to be completely satisfactory. On this account it was thought that a determination of the mobility of ions in air at pressures much above those hitherto used might be of interest and perhaps afford further evidence on which to base theoretical considerations. The following paper contains an account of a series of such measurements.

I. MOBILITY MEASUREMENTS.

In determining the mobilities the method used was practically that devised by Frank and Pohl.¹ The air was ionized in the chamber *PG*, Fig. 1, by polonium, which was placed on the plate *P* facing the gauze *G*. As the pressures used were high the alpha rays from the polonium were all absorbed within the space *PG*. The plate *P* was kept at a constant potential with regard to *G*, and *G* was made alternately positive and negative by means of a battery of small storage cells and a commutator. At high pressures a difference of potential of 300 to 500 volts was required between *P* and *G* to get enough ions into the space *GS* to give measurable deflections. The commutator was run at a constant rate and the charges received on the electrode *E*, for different voltages on *G*, were measured with a quadrant electrometer. The voltage between *G* and *S* when the ions were travelling towards *E*, which was always less than when they were travelling in the opposite direction, was measured with a Kelvin electrostatic voltmeter. Readings were taken with different voltages applied between *G* and *S* and from a curve representing these the voltage was determined which was just required for the ions to reach *E* in the time interval in which they were moving. In some of the measurements two or three seconds elapsed between alternations of the potential of *G*. Under these conditions oscillations of the electrometer needle were set up by the induced charges, and so a second gauze *S* was introduced and by means of a battery of small dry cells in the cylinder kept at a constant potential of 17 volts above zero. This device, which had been used by Lattey² with success in his experiments almost entirely eliminated the swing of the needle due to induction.

¹ Verh. der. Deutsch. Phys. Ges., p. 69, 1907.

² Proc. Roy. Soc., July 28, 1910.

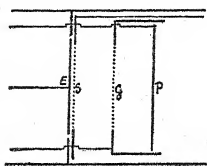


Fig. 1.

The gauzes were supported by ebonite posts and the whole apparatus as described was placed in a steel cylinder which was filled with air to the pressure desired by means of the compressor of a liquid air apparatus. The air used was drawn through lime into the compressor and by it forced into a high pressure cylinder where it was of necessity in contact with water. From this cylinder it was passed through a second cylinder containing caustic potash in order to be dried and thence into the ionization chamber. The pressures were read on ordinary commercial pressure gauges which were afterwards calibrated with a standard gauge.

The commutator was made to run as uniformly as possible by keeping constant the electromotive force given by a small dynamo, with permanent magnets, connected with the commutator. The electromotive force of this dynamo was opposed by a couple of standard cells and the resultant difference of potential was measured with a second electrometer. With this arrangement slight variations could be readily detected and corrected.

The distance that the ions actually travelled could not be determined accurately by direct measurement. The distance between the two gauze sheets was 1.55 cm. but as the meshes were considerable, being 1.65 mm. in width, the path of the ions could only be said to be approximately 1.55 mm.

In the course of the experiments the velocity of the positive ion was found to vary inversely as the pressure, and so the value of the velocity found by other methods was assumed and the distance the ions travelled calculated to be 1.66 cm. This distance was used therefore in calculating the mobility of the negative ion.

The following results were obtained where

P_1 = pressure in atmospheres (1 atmo. = 76 cms. of mercury),

k = mobility of the ion in cm. per sec. per volt per cm.,

pk = product of the mobility and the pressure.

The measurements cited in Table I. were made one after another in the order given and the two principal sources of irregularity in obtaining them arose in making the pressure readings and in determining the critical voltage from the curves which represented the readings obtained with the different fields applied between G and S . This uncertainty was greater in some measurements than in others and would largely account for the variations, shown in the table, of the product pk . These variations, however, are not large, and we may conclude therefore that the mobility of the positive ion varies inversely with the pressure up to as high as one hundred atmospheres.

TABLE I.
Mobility of the Positive Ion in Air.

Pressure.		k	pk
Cm. of Hg.	Atmos.		
7,726.16	101.66	.0139	1.42
7,726.16	101.66	.0138	1.41
6,410.80	83.30	.0154	1.28
4,496.16	59.16	.0227	1.34
2,971.60	39.10	.0344	1.35
1,989.68	26.18	.0517	1.36
1,086.30	14.29	.0945	1.35
576.23	7.58	.178	1.35
379.85	5.00	.262	1.31

A summary of the measurements made on the mobility of the negative ion in air is given in Table II.

The first nine readings were taken before the measurements with the positive ion were made and the last five afterwards. The lower value obtained for the product pk corresponding to the last five pressures cited can very probably be attributed to a change in the state of the air sup-

TABLE II.
Mobility of the Negative Ion in Air.

Pressure.		k	pk
Cm. of Hg.	Atmos.		
7,922.24	104.24	.0224	2.33
6,304.96	82.96	.0281	2.33
4,547.84	59.84	.0378	2.26
3,152.48	41.48	.0532	2.21
3,152.48	41.48	.0536	2.22
2,945.76	38.76	.0561	2.18
2,585.16	33.66	.0637	2.15
1,757.12	23.12	.0906	2.10
1,086.04	14.29	.143	2.04
2,264.80	29.80	.064	1.96
1,550.40	20.40	.0915	1.87
1,033.60	13.60	.130	1.76
558.14	7.34	.253	1.86
379.85	5.00	.330	1.65

plied by the liquid air machine, as the mobility of the negative ion in air at atmospheric pressure is known from Lattey's investigation to be greatly affected by changes in the amount of moisture present.

It is evident, however, from a survey of all the results given in Table II., that there is a gradual increase in the value of pk as the pressure rises, since the ratio of the mobility of the negative ion to that of the positive

ion at high pressures is about 1.7, while at the lowest pressures investigated this ratio was only about 1.26.

II. MOBILITY AND POTENTIAL DIFFERENCE IN ANTECHAMBER.

The above readings were all made with a potential difference of more than 300 volts between P and G . At low pressures measurable deflections could also be obtained with a much lower potential difference than this voltage, but it was noticed that the velocity appeared to depend on the strength of the field in which the ions were made, for example, at a pressure of 7.59 atmospheres the positive ion made in a field of 319 volts between P and G had an apparent velocity 1.47 times the velocity when made in a field of 10 volts between P and G . The following groups of measurements which illustrate this point were made with different potentials V between P and G , each group being made with the same sample of air.

Positive.

Pressure.		V	k	pk
Cm. of Hg.	Atmos.			
379.85	5.00	384	.262	1.31
		10	.186	.95
576.23	7.59	319	.178	1.35
		83	.167	1.27
		39	.155	1.18
		10	.134	1.01
		10	.121	.92
1,086.30	14.29	310	.0945	1.35
		83	.087	1.24

Negative.

Pressure.		V	k	pk
Cm. of Hg.	Atmos.			
379.85	5.00	485	.330	1.65
		9.5	.300	1.50
		9	.293	1.46
558.14	7.34	408	.253	1.86
		408	.250	1.84
		82	.241	1.77
		82	.246	1.73
		10	.223	1.64
1,033.60	13.60	755	.130	1.76
		10	.118	1.60
1,550.40	20.40	800	.092	1.87
		29.5	.0895	1.78

This effect may possibly in part be due to the lines of force, with higher potentials, extending further into the chamber GS , but the larger effect

with the positive ion appears to be due to an actual change in the value of k .

A few measurements were also made at atmospheric pressure by placing the polonium on G facing P . The same effect was noticed but the results obtained in this set of measurements were not so reliable as the contacts on the commutator were not so definite at the increased speed of running.

III. SUMMARY OF RESULTS.

I. The mobility of the positive ion made by the alpha rays from polonium in air has been shown to vary inversely with the pressure up to 100 atmospheres.

II. At high pressures the mobility of the negative ion made in air by the alpha rays from polonium does not appear to vary inversely with the pressure but seems to decrease less rapidly with the pressure than it should if it followed this law.

III. From a series of observations with Frank and Pohl's method it has been shown that the mobility of the negative ion made in air by the alpha rays from polonium is greater with high voltages applied to the antechamber of the ionization vessel than with low ones.

In conclusion I wish to express my thanks to Professor McLennan who suggested the problem and aided in every possible way throughout the progress of the experiment.

THE PHYSICAL LABORATORY,
UNIVERSITY OF TORONTO,
October 20, 1910.

ON THE PRODUCTION OF A HELIX OF RAYS FROM THE WEHNELT CATHODE.

BY CHAS. T. KNIPP.

THE hot lime cathode furnishes us with a convenient method of obtaining a compact beam of cathode rays direct from the source and without the use of diaphragms. This makes it possible to perform with very simple apparatus a number of beautiful and instructive experiments on the magnetic and electrostatic deflections of these rays. The apparatus herewith described represents one step in an endeavor to experimentally realize a compact helical beam of cathode rays and I present it as being of possible general interest, especially to those giving courses on recent advances in experimental physics.

The essential parts of the apparatus are shown in Figs. 1 and 2. The containing vessel *mn* is an ordinary one gallon blue cell glass jar with the top cut off, leaving the jar 10 or 12 cm. high. The lip is ground on emery cloth until quite plane. The jar is closed by a quarter inch plate *pp'* of plate glass. Within two centimeters of the bottom a hole *O* of

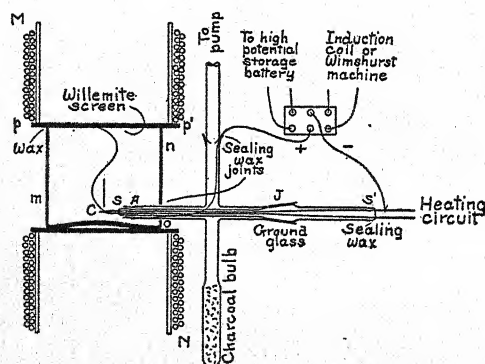


Fig. 1.

about two centimeters diameter is ground through the jar to receive the electrodes. A large solenoid *MN*, in two sections of several hundred turns each, of number 10 or 12 copper wire placed as shown in Fig. 1, furnishes a fairly uniform field with its lines parallel to the axis of the containing vessel *mn*. I find it convenient to wax on the plate *pp'* with beeswax and resin half and half. However, before waxing, this

plate should have a thin willemite screen spread over its lower surface. This may be very conveniently done by shaking the powdered willemite up in alcohol, allowing it to settle on the flat plate and siphoning off when the right thickness is reached.¹

Both the anode and the cathode are inserted through the tube *AJ* which in turn is waxed into the opening *O* in the jar. The anode consists of an aluminum collar supported at the inner end of this tube and allowed to project from it about one centimeter. Connection to the anode is made by a fine copper wire (No. 30) through the sealing wax joint connecting to the pump. A charcoal bulb should also be provided as shown. The construction and mounting of the cathode are very important, as it is necessary that the cathode beam emerging from the hot lime be at right angles to the leading-in wires which are carried by the glass tube *SS'*. The details of the construction of the platinum Wehnelt cathode are shown in Fig. 2. The leading-in wires are hammered flat at the cathode end, and bent into the form shown in *b*.

The platinum strip is cut as shown in *c*, the dotted lines showing where it is to be bent. The tongue of this strip should not be more than 1 mm. wide.

The small dot in *c* (its position is also indicated in *d*) represents the point where the lime should later on be deposited. In *d* is shown a side view of *c* after the platinum strip is bent twice at right angles. The strip thus prepared is mounted between the ends of the flattened leading-in wires and secured in position by the small metal clamp shown in *b*. Two small pieces of mica properly placed will secure insulation and at the same time cause the current to flow through the platinum. In *a* is given an end view of the mounted cathode, the vertical line showing the position and direction of the cathode beam.

For the lime I find it convenient to use, as they do at the Cavendish Laboratory, a touch of Bank of England sealing wax. This is easily applied by first warming the cathode, then increasing the current until the white lime disc appears. For the best results, this disc should be only about one half millimeter in diameter. The cathode made in this manner has distinct advantages over the platinum foil strip mounted in the usual way. In this the foil is not under tension and hence the tendency to burn out is very much lessened. The mounting is also much more compact and rigid. The platinum strip can be replaced by a new one in a comparatively few minutes. In the second place the cathode and leading-in wires, as a whole, are mounted free to turn at right angles to the direction of the cathode stream. To attain this the supporting tube *SS'*

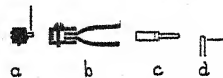


Fig. 2.

¹ J. J. Thomson, *Phil. Mag.*, Vol. 20, p. 753, Oct., 1910.

is ground into the glass system at J . The ends of this tube have each two small holes blown in to receive the heavy copper (No. 10) heating circuit leads. The end S' is closed with red sealing wax. This supporting tube should be centrally seated so as to retain its axial position on turning. This is not difficult, since this is a type of joint that is easily ground and made to hold.

The various connections for heating the lime cathode and producing the discharge are shown in Fig. 1. For a steady cathode beam a high potential storage battery is best. The discharge can be maintained easily by 300 volts connected through a water resistance. By means of a suitable switch, the discharge may be thrown from the induction coil or Wimshurst machine to the storage cells. It is quite important to have the coil or machine on the circuit as starting the discharge is hastened by first ionizing the gas. The density of the cathode beam may be varied through wide limits by varying the temperature of the Wehnelt cathode. A sufficient degree of exhaustion may be obtained by first pumping the vessel out by means of a Geissler or Geryk pump and then immersing the charcoal bulb in liquid air; or, in the absence of the latter, by using a Gaede pump. Of course, the best result—the sharpest outlined beam—is obtained by using both a Gaede pump and a charcoal-bulb-in-liquid-air.

Now, if the supporting tube SS' is turned in the ground joint J so that the plane of the Wehnelt cathode is at right angles to the axis of the containing vessel mn , that is, the cathode beam parallel to the lines of magnetic force, the beam will be unaffected except in this wise. An ion tends to move along the lines of magnetic force. By reason of the platinum surface not being strictly plane the beam when the magnetic field is off will tend to diverge. The phosphorescent spot on the screen will therefore be more or less spread out. But by turning on the field, in accordance with the statement above, the beam will become more compact and the spot on the willemite screen more clearly cut. If now the cathode is slowly turned in either direction the succeeding positions of our cathode beam will present a most beautiful spectacle. For a slight turning, the beam will take up the form of a long graceful spiral of low pitch. As the angle increases, the pitch will also increase—the spiral consisting now of many turns. If the magnetic field is truly uniform, the spiral will take up the form of a helix. As the angle between the plane of the cathode and its initial position approaches 90 degrees, the helix (theoretically) will degrade into a circle. What really takes place is: owing to the fact that the beam is slightly divergent, our circle will have a helix extending up and also down, the prominence of these

two helices depending, as suggested, upon the divergency of the beam, which in turn depends upon the convexity of the platinum surface and also on the degree of exhaustion of the containing vessel.

The diameter of the helix may be reduced at will, of course, by the application of a stronger magnetic field. In the apparatus in question, 10 amperes through the solenoid *MN* gave a field of about 50 gaussess. This strength of field formed a helix of some 2.5 cm. in diameter.

Another interesting point brought out by the apparatus is the definition on the willemite screen. On viewing the helix from the side (when the pitch is about one centimeter) it presents a rather diffused appearance. The individual turns are not clearly discernible, but fade off on either side and overlap. But when viewed from the top, the cross-section as pictured on the screen is a circle of extreme sharpness—a line oftentimes of less than one millimeter's width. In short, the helix assumes the form of a drawn-out shaving.

Other interesting experiments are the application of an electrostatic field, either from the sides for lateral displacement, or placed vertically for the extension or compression of the helix.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
November 24, 1911.

PROCEEDINGS
OF THE
AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE FIFTY-NINTH MEETING.

A REGULAR meeting of the Physical Society was held at the Kent Chemical Laboratory of the University of Chicago on Saturday, December 2, 1911. President W. F. Magie presided.

The following papers were presented:

Rays of Positive Electricity from the Wehnelt Cathode. CHAS. T. KNIPP.
Spontaneous Negative Potential of Alkali Metals. JAY W. WOODROW.

The Charges on Thermions Produced in Air and Hydrogen at Atmospheric Pressure. (By title.) J. C. POMEROY.

The Effect of Prolonged Illumination on the Photo-Electric Discharge in a High Vacuum. R. A. MILLIKAN and J. R. WRIGHT.

An Absolute Determination of the Minimum Ionizing Energy of an Electron, and the Application of the Theory of Ionization by Collision to Mixtures of Gases. EDWIN S. BISHOP.

Current Produced by Light in a Metallic Film. (By title.) P. A. ROSS.

The Production of Light by Cathode Rays. GORDON S. FULCHER.

The Electron Theory of Metallic Conduction. (By title.) O. W. RICHARDSON.

The Kerr Magnetic Rotation for Transverse Magnetic Fields. L. R. INGERSOLL.

The Spectrum of Titanium in a Partial Vacuum and the Proportionality of Displacement to Pressure at Moderate Pressures. WALTER S. ADAMS and HENRY G. GALE.

The Effect of Temperature upon the Hydrogen Spectrum as Produced by Alternating Current. HARVEY B. LEMON.

A Study of the Reversible Pendulum. (Second Paper.) (By title.) J. C. SHEDD and J. A. BIRCHBY.

An Analytical Investigation into the Cause of Instability in Certain Varieties of Light-sensitive Selenium. F. C. BROWN.

Experiments on the Loss of Heat by Hot Bodies through Gas Conduction and Convection. (By title.) CHAS. F. LORENZ.

The Oscillatory Discharge of a Leyden Jar. R. R. RAMSEY.

A Method of Measuring the Effective Capacity of High Tension Cables. G. W. STEWART and D. M. TERWILLIGER.

Electro-static Induction in Long Lines. CARL KINSLEY.

Application of the Potentiometer Method to the Determination of the Curve of Decay of the Counter E. M. F. of the Aluminium Rectifier. C. W. GREEN.

Measurement of Inductances and Capacities with the Thomson Double Bridge. (By title.) S. J. BARNETT.

An Oscillographic Method of Plotting Hysteresis Curves. N. H. WILLIAMS.

Effect of Frequency on the Capacity of a Condenser with Kerosene for the Dielectric: and the Limits of the Wheatstone Bridge in Measuring Small Capacities. S. HERBERT ANDERSON.

On the Value of the Sparking Potential below Minimum Sparking Potential. CARL ENGLAND.

A Determination of the Ratio of the Specific Heats for Air and Carbon Dioxide. H. W. MOODY.

The Pressure Coefficient and Specific Heat of Kerosene. (By title.) R. R. RAMSEY.

The Intensity Factors in the Binaural Localization of Sound. G. W. STEWART.

The Restored Positive Thermionic Current from Hot Platinum as Caused by Sodium as an Impurity. F. C. BROWN.

The Number of Ions Produced by an Alpha Particle from Polonium. T. S. TAYLOR.

On the Production of a Helix of Rays from a Wehnelt Cathode. CHAS. T. KNIPP.

Upon request, President Magie informally reported upon the present status of negotiations between the Society and the PHYSICAL REVIEW.

The meeting adjourned at 5 P.M.

G. W. STEWART,
Secretary pro tem.

MINUTES OF THE SIXTIETH MEETING.

A JOINT meeting of the Physical Society with Section B of the American Association for the Advancement of Science was held in Washington, D. C., December 27-30, 1911. The presiding officers were President W. F. Magie, of the Physical Society, and Vice-President R. A. Millikan, of Section B. At the session of December 30, both presiding officers being absent, E. B. Rosa was made temporary chairman. All sessions were held at the National Bureau of Standards.

During the morning session of Thursday, December 28, the presidential address, on "The Primary Concepts of Physics," was delivered by President W. F. Magie. During the afternoon session of the same day the vice-presidential address of E. B. Rosa, retiring chairman of Section B, was delivered, the subject being "The Work of the Electrical Division of the Bureau of Standards."

Following the presidential address the remainder of the morning session of December 28 was devoted to a symposium on "Ether Theories," led by A. A. Michelson. Messrs. E. W. Morley, A. G. Webster, W. S. Franklin, D. F. Comstock and G. N. Lewis took part in the discussion.

The Thursday sessions, which were in charge of the officers of Section B of the American Association for the Advancement of Science, also included addresses by H. A. Wilson on "The Structure of the Atom," and by S. W. Stratton on "Physical Work at the Bureau of Standards."

The Annual Business Meeting was held on December 29. In the absence of Mr. Ames the treasurer's report for the year ending December 1, 1911, was read by the secretary. The report is printed at the close of these minutes.

Tellers being duly appointed by the president the ballots cast for officers and members of the council were counted and the following were declared elected: *President*, W. F. Magie; *Vice-President*, B. O. Pierce; *Secretary*, Ernest Merritt; *Treasurer*, J. S. Ames; *Members of the Council*, H. A. Bumstead, Theodore Lyman.

On behalf of the council the president reported progress in the consideration of a plan for the publication and control of the *PHYSICAL REVIEW* by the Society.

The secretary announced that two amendments to the constitution had been proposed, as follows:

1. To amend Article IV. of the Constitution so as to read as follows:

The officers of the Society and all those who have at any time held the office of president of the Society, together with eight other members to be elected in the manner specified in Art. V., shall constitute a Council, which shall have general charge of the affairs of the Society.

2. To amend Article V. of the Constitution so as to make the first paragraph read as follows:

1. The officers of the Society and other members of the Council shall be elected by ballot at the Annual Meeting of each year. An official ballot shall be sent to each member at least one month prior to the Annual Meeting, and such ballots, if returned to the Secretary in envelopes bearing the names of the voters, shall be counted at the Annual Meeting. At least two months before the Annual Meeting the secretary shall send to each regular member of the Society a request for nominations, specifying what vacancies are to be filled. The names of all those who are nominated by not less than two members shall be printed on the official ballot. The official ballot shall also contain a name proposed by the Council for each vacancy, with blank spaces in which the voter may substitute other names. A majority of all votes cast in person or by mail shall be necessary to election. In case of failure to secure a majority for any office, the members present at the Annual Meeting shall choose by ballot between the two having the highest number of votes. The term of office shall be one year for officers and four years for other members of the Council, and until their successors shall be elected. No member shall be elected as President more than two years in succession. No elected member of the Council, having completed a term of four years, shall be reelected until at least one year shall have intervened.

It was voted to recommend to the next meeting of the Society, at which meeting, in accordance with Article VI. of the Constitution, the proposed amendments will come up for final action, that the first be adopted. A similar motion in the case of the second proposed amendment was laid on the table.

At the close of the meeting a vote of thanks to the scientific staff of the Bureau of Standards for the hospitality shown the Society was unanimously adopted.

The following papers were presented:

An Important Practical Problem in Gyrostatic Action. W. S. FRANKLIN.

A Relation between the Magnetic Hysteresis and the Tensile Strength of a Series of Iron-carbon Alloys. (Read by title.) C. W. WAGGONER.

Relation between the Joule Effect and the Permeability in the Same Specimens of Steel. S. R. WILLIAMS.

A Magnetic Test as a Means of Determining Flaws and Mechanical Strains in Iron and Steel. CHAS. W. BURROWS.

The Electrical Resistance and the Polarization E.M.F. of a Mixture of Clay, Feldspar, and Quartz. A. A. SOMERVILLE and O. E. BUCKLEY.

A Kinetic Theory of Gravitation; Some Explanatory Remarks on my Paper of Last Year. CHARLES F. BRUSH.

Some Diffraction Photographs. MASON E. HUFFORD.

Demonstration of Linear and Surface Thermopiles of Bismuth and Silver. W. W. COBLENTZ.

The Vertical Temperature Gradient of the Atmosphere. WM. R. BLAIR.

A Modified View of Electronic Conduction. WALTER P. WHITE.

The Application of Statistical Principles to Photoelectric Effects and Some Allied Phenomena. (Read by title.) O. W. RICHARDSON.

The Velocity-distribution Curves of Electrons Liberated by Different Sources of Ultra-violet Light, and the Bearing of these Curves on the Planck-Einstein Theory. R. A. MILLIKAN.

A Study of Crystal Rectifiers. R. H. GODDARD.

The Half-value of the Radioactive Deposit Collected in the Open Air. F. A. HARVEY.

Distribution of Current in Point-plane Discharge. ROBT. F. EARHART.

The Influence of Temperature on the Phenomena of Phosphorescence in Zinc Sulphide. H. E. IVES and M. LUCKIESH.

On the Free Vibrations of a Lecher System using a Lecher Oscillator, II. F. C. BLAKE and CHAS. SHEARD.

The Thomson Effect in, and the Thermal Conductivity of Tungsten, Tantalum, and Carbon at Glowing Temperatures. A. G. WORTHING.

The Effect of the Electrical Discharge on Solids and Liquids Suspended in Air. W. W. STRONG.

A Quantitative Measure of Development in Scientific Observation. OTTO STUHLMANN, JR.

Elastic Hysteresis in Metal Bars. A. G. WEBSTER and T. L. PORTER.

Evidence that the Velocity of Light is Independent of the Motion of the Source. D. F. COMSTOCK.

The Specific Heat of Wood. FREDERICK DUNLAP.

The Spectra of Iron and Titanium at Moderate Pressure. W. S. ADAMS and H. G. GALE.

On the Relation between Pressure Displacement and Wave-length. W. S. ADAMS and H. G. GALE.

The Spark Spectra of the Alkaline Earths in the Schumann Region. THEODORE LYMAN.

A Convenient Device for Obtaining a Steady High Potential for Electrometer Work. A. H. FORMAN. Introduced by J. S. SHEARER.

The Form of CO_2 , SO_2 , and NH_3 Crystals. H. E. BEHNKEN. Introduced by J. S. SHEARER.

A New Method of Photographing Sound Waves. A. L. FOLEY and W. H. SOUDER.

Another Instrument for Photographing Sound. A. G. WEBSTER.

The Influence of the Natural Periods of Concentrating Horn and Diaphragm upon Sound Wave Records, and the Quantitative Analyses of Tones from Several Musical Instruments. D. C. MILLER.

The Expansion of Water below 0°C . J. F. MOHLER.

Demonstration of the Resonance Spectrum of Iodine in Vacuo and in Helium. R. W. WOOD.

Slit Width Corrections in the Photometry of Black Body Spectra. E. P. HYDE.

The Effect of Temperature on the Absorbed Charge in Electric Condensers. ANTHONY ZELENY.

The Influence of Neighboring Conductors upon a Klemencic Receiver of Electric Waves. A. D. COLE.

The Absorption of Beta Rays by Gases. A. F. KOVARIK.

The Absorption of Gamma Rays of Radium by Air at Different Pressures. H. A. ERIKSON.

Poynting's Tangential Method for Showing the Existence of Radiation Pressure an Assumption Unwarranted by Experiment. R. A. WETZEL.

A New Type of Curve Drawing Instrument and Controller Mechanism. M. E. LEEDS.

The Silver Voltameter as a Precision Instrument. E. B. ROSA.

Recent Work with the Silver Voltameter. G. W. VINAL.

The Dielectric Constant, Specific Resistance, and Electrostatic Absorption of Crystals. H. L. CURTIS.

A New Type of Apparatus for Measuring Linear Expansion. ARTHUR W. GRAY.

Temperature Influence upon the Refraction of Quartz, Boro-silicate Crown Glass, and Dense Flint Glass, from 100°C . to -190°C . F. A. MOLBY.

The Thermo E.M.F. of the Nernst Filament. J. S. SHEARER.

The Transmission of the Active Deposit of Radium in an Electric Field. E. M. WELLISCH and H. L. BRONSON.

Diffraction Gratings with Controlled Groove and Anomalous Distribution of Intensity. Illustrated with Experiments. R. W. WOOD.

Further Investigations with the Radiant Emission from the Electric Spark. R. W. WOOD.

A New Type of Neutral Double Potentiometer. WALTER P. WHITE.

Note on the Ascensional Rate of the Free Balloons used for Meteorological Purposes. WM. R. BLAIR.

A New Form of Vacuum Pump. J. JOHNSTON.

A Simple Dynamical Example of the Genesis of an Integral Equation. A. G. WEBSTER.

A New Way to Determine g . A. G. WEBSTER.

On the Effect of Close Electrostatic Coupling on the Free Period of a Lecher System. F. C. BLAKE and CHAS. SHEARD.

Poynting's Theorem and the Equation of Electromagnetic Action. (Read by title.) W. S. FRANKLIN.

A Simple Slit for the Spectroscope. (Read by title.) J. P. NAYLOR.

The Applicability of the Planck Equation to the Radiation from Tantalum and Tungsten. (Read by title.) E. P. HYDE.

The Emission of Light by Hydrogen Canal Rays. G. S. FULCHER.

A Sensitive Vacuum Thermal Couple and Method for Producing High Vacua. A. H. PFUND.

On Magnetic Rays. (Read by title.) L. T. MORE and E. G. RIEMAN.

The Wave-lengths of Neon. I. G. PRIEST.

The Electric Discharge from Pointed Conductors. JOHN ZELENY.

Ellipticity and Rotation in Optically Active Solutions. L. B. OLMSTEAD.

The Language of Meteorology. C. F. TALMAN.

The Joule Thomson Effect in CO_2 . E. S. BURNETT.

On the Theory of the Hysteresis Loop of Iron. J. KUNZ.

The Photo-electric Effect in Phosphorescent Materials. G. A. BUTMAN.

An Absolute Determination of the Coefficient of Viscosity of Air. L. GILCHRIST.

Adjourned at 1:30 P. M., Saturday, December 30.

ERNEST MERRITT,
Secretary.

TREASURER'S ACCOUNT FOR THE YEAR 1911.

Disbursements for Year 1911.

Executive expenses:

President, mileage.....	\$ 16.80
Secretary, office expenses.....	100.00
mileage.....	56.84
Treasurer, office expenses.....	50.00

Stationery, printing and postage:

Orders of Secretary.....	189.66
Orders of Treasurer.....	46.00

Subscriptions to Journals:

PHYSICAL REVIEW, 574 members.....	1,722.00
Science Abstracts "A," 568 members.....	1,420.00
Science Abstracts "B," 34 members.....	68.00

Repayment of Error in check.....	2.00
Payment for Science Abstracts "B" for 1910.....	2.00

\$3,673.30

Receipts for Year 1911.

Annual dues, 554 members ¹	\$3,323.71
Payments for Science Abstracts "B".....	68.00
Entrance fees, 9 members.....	27.00
Interest and exchange.....	28.23
Error in check.....	2.00
	<hr/>
	\$3,448.94
Deficit for year.....	224.36
	<hr/>
	\$3,673.30

Summary.

In bank December 1, 1910.....	\$ 833.53
Received, back dues.....	8.00
Received, dues in advance.....	6.00
Received, old accounts.....	394.00
	<hr/>
	\$1,241.53
Less deficit.....	224.36
	<hr/>
Balance in bank December 1, 1911.....	\$1,017.17

Summary of membership: Honorary.....	6
Life.....	16
Regular and Associate.....	554
	<hr/>
Total.....	576

(Of the regular members, two did not wish the journals.)

JOSEPH S. AMES,
Treasurer.

I have examined the receipts and the vouchers for the expenses, as stated in the above account; and I have verified the bank balances, and found all the accounts correct as given in the report of the Treasurer.

J. A. ANDERSON.

THE EFFECT OF PROLONGED ILLUMINATION ON PHOTO-ELECTRIC DISCHARGE IN A HIGH VACUUM.²

BY R. A. MILLIKAN AND J. R. WRIGHT.

IN this paper attention will be confined to the effect of illumination upon potential-photocurrent curves, although it has been found that there are other factors which influence these curves much more profoundly. A consideration of these other factors will be presented in a subsequent article.

In work previously reported³ it has been shown not only that in a very high vacuum prolonged illumination with ultra violet light produces no fatigue effects such as characterize photo-electric phenomena in gases, but that marked increases both in the quantity and the velocity of electronic discharge result from such illumination. It was suggested that these effects were probably

¹ One member is in arrears \$0.29.

² Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

³ Millikan, *PHYS. REV.*, 30, p. 287, 1910. See also Millikan and Winchester, *PHYS. REV.*, 29, p. 85, 1909.

due to the removal by the light of some kind of a surface film, presumably of occluded gas. Since then v. Baeyer and Gehrts¹ and Hughes² have obtained results which they have interpreted in a similar way.

A much more complete study of the effect of illumination has now been made in the best attainable vacua and under conditions which preclude the possibility of a contamination of the surfaces by mercury vapor. The metals experimented upon were aluminium, platinum, copper, gold, zinc, and steel. The powerful source of light was a spark furnished by zinc electrodes fed by twelve Leyden jars which were excited by a twelve-inch Scheidel coil used as a transformer, on a sixty cycle 100-volt A. C. circuit. Before and after each

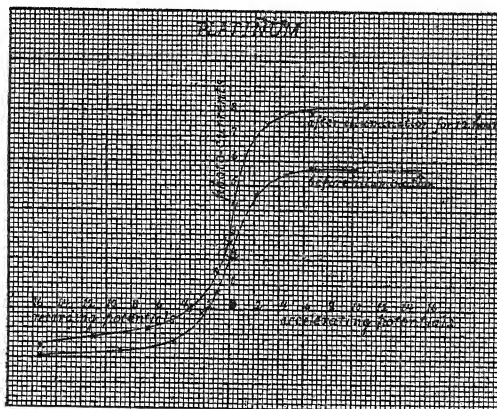


Fig. 1.

period of illumination a complete potential-discharge curve was taken, the potentials of the illuminated metals varying from -16 to $+16$ volts. This curve was taken with the aid of a small, constant spark-source. The photo-currents in all cases increased markedly with illumination throughout the full length of the curve. The figure shows some results obtained with platinum. These are typical of the curves obtained with all the metals. The apparatus was not well suited to the accurate determination of the positive potential necessary to prevent entirely the escape of electrons, since the effects due to reflected light are unusually large, as can be seen from the large negative ordinates corresponding to the higher retarding potentials. Nevertheless it can be seen from the platinum curve that this potential is in this case more than $+15$ volts since at this point the curve has not yet become parallel to the axis of potentials.

When the potential on the metals was -15 volts the photo-currents from the six metals before and after illuminations of from ten to seventeen hours' duration were as follows.

¹ v. Baeyer u. Gehrts, *Verh. d. Deut. Phys. Ges.*, 12, p. 877, 1910.

² A. W. Hughes, *Cambridge Phil. Soc. Proc.*, 16, p. 167, 1911.

Metal.	Photo-Current. ¹	
	Before.	After.
Al.....	1.88	7.72
Pt.....	5.42	8.14
Steel.....	3.09	5.50
Cu.....	1.72	2.77
Zn.....	.37	3.46
Au.....	.28	1.85

In all cases save that of copper a rest of from 24 to 48 hours caused a decrease of from 10 to 30 per cent. in the photo-currents, though there was in general no return to the initial values. Copper, however, showed a different behavior. Its initial photo-current at -15 volts was 1.72; nine hours of strong illumination brought it up to 2.77; after a rest of twelve hours it had risen to 3.41; and after a rest of 30 days it was 4.26.

Preceding experiments in a tube from which mercury vapor had not been excluded, while showing all of the increases with illumination shown above, did not show the subsequent changes with time, though these were carefully looked for, particularly in the case of copper.

In all of these experiments the tubes and method of observing were similar to those used previously by Millikan and Winchester.²

UNIVERSITY OF CHICAGO,

November 15, 1911.

THE PRODUCTION OF LIGHT BY CATHODE RAYS.¹

BY GORDON S. FULCHER.

IT seems clear now that light is emitted by an atom or molecule usually only as a result of the shock accompanying a sudden change of electrical condition, ionization or neutralization. In the case of any one gas, there are several groups of spectrum lines which may be emitted independently; for instance, the spark lines of nitrogen are quite different from the bands obtained under different conditions. The difficulty, however, is to determine which electrical state of the gas molecule is responsible for each group, since the phenomena occurring in the case of an ordinary discharge through a gas are so numerous. An attempt has been made to secure simpler conditions by causing cathode or canal rays to pass out of the region through which the discharge was going. A fine hole was made in the anode or cathode, and the fine luminous beam of rays issuing from it was used as a line source of light, taking the place of the slit of the spectrograph. Thus only the light actually produced in the direct path of the rays was analysed. Precautions were taken to obtain pure cathode rays, unmixed with retrograde canal rays, and the following results were obtained.

Cathode rays in air produce, as Lewis³ had concluded, chiefly the negative

¹ Phil. Mag., 14, p. 194, 1907.

² Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

³ P. Lewis, Astrophysical Journal, 17, 258, 1903.

bands of nitrogen. Only the faintest trace of the positive bands is visible (3,000 volts). In *hydrogen*, they produce both the compound line and series line spectrum, but the latter is relatively much weaker than in the canal ray spectrum. In *oxygen*, they produce chiefly the negative bands $\lambda\lambda$ 526, 560, 586, as Wüllner¹ reported, and also the spark lines and series triplets, though very faintly.

Slow *canal rays* in *air* (1,000 volts) produce both the positive and negative band spectrum of nitrogen. In *hydrogen*, they seem to cause the emission, primarily of only the series spectrum. In *oxygen*, the spectrum produced shows very strong main series lines, $\lambda\lambda$ 3,947 and 4,638, and fairly intense subordinate triplets. The variation in the relative intensity of the various groups of lines, produced in various ways, is shown in the table giving the average intensity of the lines of the various groups.

Oxygen.	Very Slow Canal Rays.	Canal Rays (2,000 v.).	Canal Rays (5,000 v.).	Cathode Rays (3,000 v.).
Negative bands	?	?	?	20
Spark lines	4	4	4	4
λ 4368	40	12	4	4
λ 3947	25	8	2.5	1
1st subseries	6	3	0.5	0.4
2nd subseries	2.5	1.3	0.3	0.2

The only transformations that can happen, due primarily to cathode rays, are ionization with and without dissociation. For velocities below a certain minimum, probably dissociation cannot be produced; when, therefore, slow cathode rays cause the emission of a single group of lines, that group may be confidently assigned to the positively charged molecule.

We may conclude, then, that the negative bands of nitrogen and the negative bands of oxygen are emitted by the molecules of the respective gases as a result of ionization by cathode rays. Similarly, the positive bands of nitrogen and the series of triplets of oxygen are probably emitted by the molecules ionized by collision with canal rays.

WISCONSIN, November 12, 1911.

RAY'S OF POSITIVE ELECTRICITY FROM THE WEHNELT CATHODE.²

BY CHAS. T. KNIPP.

THIS paper is a preliminary note and deals with the work done by the author at the Cavendish Laboratory on the measurement of e/m and v when the hot lime cathode is used. The photographic method recently devised by J. J. Thomson in his study of Positive Rays was employed. In order to get photographic records of the positive ray spectrum, it was necessary to accelerate the rays. The points brought out are:

¹ A. Wüllner, *Physicalische Zeitschrift*, 1, 132, 1899.

² Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911

1. The velocity of the carriers of positive electricity in the case of the hot lime cathode is not great enough to affect a photographic plate upon which they may strike.
2. These slow moving particles can be accelerated at will, the minimum potential difference required to give them a sufficient velocity to affect the plate being about 500 volts per centimeter.
3. There is secondary radiation present as well as primary—both straight lines and their continuations as parabolas are shown in a few of the photographs.
4. The value of e/m is the same for the various carriers (*e. g.*, approximately 1.00×10^4 for H , and $.50 \times 10^4$ for H_2) as that found by J. J. Thomson in the case of positive rays formed by the ordinary perforated cathode with high potential discharge.
5. The greater the acceleration the more clearly cut and well defined are the photographs, also the nearer the parabolas extend to the origin.
6. The later photographs show a negative counterpart to some of the curves.
7. The work thus far has been with residual air only.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
November 4, 1911.

CURRENT PRODUCED BY LIGHT IN A METALLIC FILM.¹

By P. A. ROSS.

POYNTING has shown experimentally that when light is incident at an oblique angle on an absorbing body there is a tangential force as well as a normal pressure. He has also shown that this tangential force exists at a surface where refraction takes place.

A totally reflecting body would have two equal and opposite tangential forces due to the incident and reflected beams. But since all bodies are imperfect reflectors there must be in all cases where the light is incident at an oblique angle a differential tangential force.

Since the electrons are apparently the cause of the phenomena of reflection, refraction, and absorption, it must be that this tangential force is exerted upon the electrons. In a nonconductor this would result in a slight displacement of the electrons but in a metal where they are free to move it should result in a drift of electrons along the metallic surface in the direction of the force, and an "eddy" back through the metal at a depth greater than the penetration of the light.

Then by taking a film of metal thin enough to be transparent we ought to be able to prevent a part of this "eddy" back and the light beam ought to drive the electrons before it and build up a negative potential at one end of the film.

The first experiment was made with a gold film deposited by a cathode discharge. The film was about one fifth light wave in thickness, one centimeter

¹ Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

wide, and four centimeters long. The film was deposited on plate glass. Copper wires were soldered to two screw pinch cocks and these were screwed down tightly at each end of the plate on tin foil cushions over the gold film. The resistance of the gold film and connections was 3.01 ohms. The film was then connected to a D'Arsonval galvanometer (resistance 91.7 ohms, constant 2.56×10^{-10} amp./cm.). The film was placed on the table of a spectrometer and a beam of sunlight from a heliostat concentrated on the film by means of a lens. The arrangement of the film and connections with respect to the beam of light was as symmetrical as possible in order to avoid thermal currents as much as possible. The beam illuminated the whole width but only about one centimeter of the length of the film. In no case did the light strike the connections.

The film was first placed normal to the beam. There was a slight wandering of the galvanometer (about .1 cm.) but it soon returned to its original null point. With the beam incident at 30° a constant deflection of .15 cm. was produced. With the light incident at 30° on the other side of the normal the galvanometer showed a constant deflection of .15 cm. on the other side of the null point. In each case the current corresponded to a flow of negative electricity in the direction of the tangential force due to the light. At 45° the deflection was .20 cm. At 60° the deflection was .28 cm., and at 70° it was .30 cm. In each case the galvanometer deflection could be reversed *instantly* by shifting the beam to the other side of the normal.

The same experiments were tried with ordinary gold leaf and with a copper film deposited by a cathode discharge. The results are given below together with the current and E.M.F. produced by the beam. It will be seen that the deflections are roughly proportional to the sine of the angle of incidence.

Film.	Resistance.	Angle of Incidence.	Deflection of Galv., Cm.	Current, Amperes.	E.M.F., Volts.
Gold (cathode)	3.01 ohms	0°	0	7.7×10^{-11}	7.3×10^{-9}
		30°	.15		
		45°	.20		
		60°	.28		
		70°	.30		
Gold (leaf)	2.85 ohms	0°	0	6.4×10^{-11}	6.06×10^{-9}
		30°	.13		
		45°	.18		
		60°	.23		
		70°	.25		
Copper (cathode)	2.6 ohms	0°	0	1×10^{-10}	9.4×10^{-9}
		30°	.20		
		45°	.28		
		60°	.35		
		70°	.40		

STANFORD UNIVERSITY.
October 2, 1911.

THE MEASUREMENT OF INDUCTANCES AND CAPACITIES WITH THE THOMSON DOUBLE BRIDGE.¹

BY S. J. BARNETT.

WHILE the Thomson double bridge has been used extensively for the measurement of resistances, especially small resistances, it does not appear to have been used hitherto for measurements of inductances or capacities. For these it is often at least as convenient as the ordinary bridge, and in some cases it offers important advantages.

1. For the comparison of two self-inductances L and N , the bridge is arranged as in Fig. 1, conventional symbols designating resistances, inductances, etc. When balances have been obtained for both steady and variable currents, $L/N = A/B (= a/b)$. The branches C and D may be either inductive or non-inductive. The bridge compares the effective inductance between the

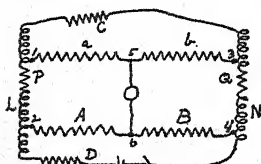


Fig. 1.

points 1 and 2 with that between the points 3 and 4. The inductance of the leads is eliminated. The inductance of the central part of a long solenoid traversed by a uniform current, which is simple to calculate, may be compared with any other inductance, provided that, as is usually the case, $A + B$ and $a + b$ are sufficiently large to insure that the current in all parts of the solenoid is practically the same.

2. To compare a self-inductance and a capacity, the branch 34 is made non-inductive, non-inductive variable resistances r and R are introduced between the galvanometer terminals and the points 5 and 6 respectively, and condensers with capacities s and S are inserted between the upper and lower galvanometer terminals and the points 3 and 4 respectively. A double balance will be obtained when $P/Q = A/B (= a/b)$, $S/s = r/R = a/A$, and $L = SQ[A + (A + B)/B \cdot R]$.

3. To compare the mutual inductance M of two coils M and L with the self-inductance L of one of them, L is inserted as in the figure, the branch 34 is made non-inductive, and M in series with an adjustable rheostat is inserted between the points 2 and 4. When the double balance has been obtained, $M/L = W/[P + Q + (a + b)C/(a + b + C)]$, W designating the total resistance through M between the points 2 and 4.

4. To determine a self-inductance by comparison with a resistance and a time, one of the coils L in the figure is shunted with a non-inductive resistance Z , and a balance is obtained with an alternating current supply of frequency n . Then

$$L^2 = [Q(P + Z)^2 - PZ(P + Z)B/A]/[4\pi^2 n^2 (ZB/A - Q)],$$

and

$$N^2 = [Q - ZPB/A(P + Z)](ZB/A - Q)/4\pi^2 n^2.$$

¹ Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

If the inductance under investigation is of the kind referred to in the last part of 1, it must obviously take the place of the unshunted coil N in the bridge.

5. To compare two capacities, the coils L and N are replaced by the two condensers with capacities S and S' respectively. When a balance has been obtained, $S/S' = B/A$.

Methods (1), (2), (4) and (5) are strictly zero methods. In method (3), however, only the time integral of the current in the branch 56 vanishes, in general, when the key in branch D is opened or closed. A corresponding zero method is easily developed, but the experimental adjustments are more troublesome.

In all the above methods the positions of the generator and detector may be interchanged without altering the balance. If the points 1, 3, and 5 are made to coincide, these methods (and many other double bridge methods which will immediately suggest themselves) reduce to well known Wheatstone bridge methods. The double bridge methods described here have all been tested in this laboratory, and the theoretical formulae have been verified experimentally, but the bridge has been used only in the ordinary form in which $A = a$.

THE PHYSICAL LABORATORY,
THE OHIO STATE UNIVERSITY.

A DETERMINATION OF THE RATIO OF THE SPECIFIC HEATS FOR AIR AND CARBON DIOXIDE.¹

By H. W. MOODY.

IN this work, use was made of the law of adiabatic expansion,

$$(p_1/p_2)^{\gamma-1} = \left(\frac{\theta_1}{\theta_2}\right)^{\gamma},$$

from which we have

$$\gamma = \frac{\log p_1/p_2}{\log p_1/p_2 - \log \theta_1/\theta_2}.$$

The method is, in essentials, similar to that used by Lummer and Pringsheim, save that a thermocouple was used in determining the temperature after expansion. A thermocouple of mil copper and constantan wire was employed as a part of a potentiometer system.

The results show a higher degree of concordance than has been noticed in other determinations by this method, where a bolometer strip or platinum thermometer was used for measuring θ_2 .

The corrected mean of a series of determinations gives for air, at 25° C. $\gamma = 1.401$, and for carbon dioxide, at the same temperature, $\gamma = 1.300$. In these, it is thought that the probable error does not exceed $\pm .0005$.

¹ Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

THE INTENSITY FACTORS IN THE BINAURAL LOCALIZATION OF SOUND.¹

BY G. W. STEWART.

THE "intensity theory" is usually regarded as the ratio of the sound intensities at the two ears.

Occasionally reference is made to the differences of intensity.

The "intensity theory" has played a prominent part in the various studies that have been made of the binaural localization of sound. From the writer's theoretical results of the acoustic shadow of a rigid sphere, with the sound located at a distance of 477 cm. and the wave length varied from 240 cm. to 30 cm., the intensities at the two ears are determined. The computations are made for different positions of the head rotating about an axis perpendicular to the aural axis, and the differences, ratios and sums of the intensities at the two ears ascertained. The sum of intensities is proposed as the chief intensity factor. If this be correct, then there must be a wave length about 60 cm. in length which is the most easily localized when placed in front of the hearer. Experiment in the open shows that this is the case. Neither the differences of the intensities nor the ratios can account for this fact. Further, the facts that have been ascertained by physicists and psychologists, especially threshold observations and distance perceptions in the horizontal plane, give a preponderance of evidence in favor of the sum of intensities as the principal intensity factor. This does not mean that the psychologist should abandon a consideration of the separate intensities or their differences and ratios, for all these must be given due weight. Other physical factors entering into the binaural localization of sound are not discussed.

A METHOD OF MEASURING THE EFFECTIVE CAPACITY OF HIGH TENSION CABLES.¹

BY G. W. STEWART AND D. M. TERWILLIGER.

THE already known method of utilizing the partial capacities (in the case of a polyphase cable) is utilized, but the partial capacities are computed from the observations of a quadrant electrometer before and after dividing its charge with the cable. The method therefore depends upon the calibration and capacity of the electrometer. The results show that the effective capacity of a very short piece of cable, less than 50 cm., can be measured with an accuracy of three per cent.

The manufacturers of the cable tested state that the ballistic galvanometer method shows a temperature variation of from 10 per cent. to 50 per cent. between 60° and 80° F., but the writers found no perceptible variation of the effective capacity between 23° C. and 50° C.

The method here proposed eliminates practically all the effect of absorption. The other advantage is the requirement for only a short piece of the cable.

¹ Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

THE ELECTRON THEORY OF METALLIC CONDUCTION.¹

BY O. W. RICHARDSON.

THE first part of this paper describes the results of an investigation undertaken in order to obtain information about the law of force which governs the collisions of electrons in metals. Lorentz has shown how to calculate the electric and thermal conductivities, without neglecting the complications which arise from the Maxwell distribution of velocity, on the assumption that the collisions occur with particles which can be treated as immovable elastic spheres. A very similar calculation can be carried out if the elastic spheres are replaced by immovable centers of force. One finds, in fact, for the electric conductivity σ ,

$$\sigma = \frac{4\pi}{3} k \frac{e^2}{m} \frac{\Gamma\left(\frac{2}{s-1} + 2\right)}{h \frac{2}{s-1} + 1} A, \quad (1)$$

for the thermal conductivity T ,

$$T = \frac{\pi k m}{3 \theta} \frac{\Gamma\left(\frac{2}{s-1} + 3\right)}{h \frac{2}{s-1} + 3} A, \quad (2)$$

and for the ratio of the two

$$T/\sigma = \frac{2s}{s-1} \frac{R^2}{e^2} \theta. \quad (3)$$

In these formulæ,

$$k^{-1} = 4\pi n \left(\frac{K}{m}\right)^{\frac{2}{s-1}} \int_0^\infty \sin^2 \theta \alpha d\alpha, \quad (4)$$

n is the number of centers in unit volume, the repulsive force exerted by a center on an electron at distance d from it is $+ K/d^s$, the integral in (4) has the meaning attributed to the same integral by Maxwell (Scientific Papers, Vol. 2, p. 36), R is the constant in the equation $pv = R\theta$ reckoned for a single molecule and $\Gamma(p+1) = \int_0^\infty e^{-x} x^p dx$. The other symbols have the meaning assigned to them by Lorentz (Theory of Electrons, p. 267).

Since experiment shows that for all the good conductors $T/\sigma = 3R^2\theta/e^2$, we conclude that $s = 3$, so that the collisions take place with centers which exert forces on the electrons varying as the inverse third power of the distance. A similar conclusion has been reached by Sir J. J. Thomson from a comparison of the electro-magnetic energy radiated by the electrons, during their collisions, with the experimental measures of the radiation from a black body (Phil. Mag., Vol. 14, p. 217, 1907; Vol. 20, p. 238, 1910). The strength (K) of the centers deduced by Thomson from the constants in the radiation formula is of the same order of magnitude as that which I have deduced from

¹ Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

the values of the electrical and thermal conductivities of the good conductors. The comparison indicates that the centers of force are parts of the atom rather than the whole of it.

The second part of the paper gives a new and apparently exact, thermodynamical calculation of the equilibrium pressure p of the electrons emitted by a conductor. The value found is

$$p = A_1 \theta^{\frac{\gamma}{\gamma-1}} e^{-\frac{\omega}{R\theta}} - \frac{e}{R} \int_0^\theta \frac{\sigma}{\theta} d\theta. \quad (5)$$

Here θ is the absolute temperature, R has the same meaning as in (3), γ is the ratio of the two specific heats of the electrons, σ is the specific heat of electricity in the material of the conductor (Thomson effect), ω is the internal latent heat of evaporation of one electron and A_1 is a quantity which is characteristic of the substance and independent of the temperature.

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J.

THE CHARGES ON THERMIONS PRODUCED IN AIR AND HYDROGEN AT ATMOSPHERIC PRESSURE.¹

By J. C. POMEROY.

THE value of Ne , where N is the number of molecules of any gas under standard conditions and e the charge on an ion, has been determined for these ions by Townsend's method (Roy. Soc. Proc. A., Vol. 81, p. 464). The results indicate that at low temperatures both the negative and the positive ions have the electronic value of Ne but as the temperature is raised an increasing proportion of the carriers have a value of Ne equal to twice the electronic value. At a white heat, apparently, all the ions, whether positive or negative, carry a charge equal to twice the electronic value.

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J.

ON THE RELATION BETWEEN PRESSURE SHIFT AND WAVE-LENGTH.¹

By HENRY G. GALE AND WALTER S. ADAMS.

WORKING in the laboratory of Mount Wilson Solar Observatory we have found that the lines of the iron spectrum may be divided into four groups on the basis of their behavior under pressure. The "flame" lines form group *a* and are characterized by small shifts. The remaining lines of comparatively small shift form group *b*, a large and probably complex group. Criteria are at present lacking for separating it into smaller groups. Group *c* consists of two clusters of lines, one in the violet and one in the blue-green. The displacements are distinctly larger than for group *b*. Group *d* consists of

¹ Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

the lines which show very great displacements, unsymmetrically broadened toward the red, sometimes extending for 5 Ångströms from the maximum. The precision of measurement is necessarily low for such lines.

The various groups divide naturally into clusters. The group, number of lines in a cluster, the average wave-length, and the average displacement in Ångström units produced by a pressure of 8 atmospheres are shown in the following table. The last two columns give the observed minus the computed values of the shifts, assuming first that the shift is proportional to the square of the wave-length and finally that it is proportional to the cube of the wave-length.

Group.	No. of Lines.	Mean λ .	Mean Δ .	Residuals (Obs.-Comp.).	
				$\Delta = \left(\frac{\lambda}{\lambda_0}\right)^2 \kappa$	$\Delta = \left(\frac{\lambda}{\lambda_0}\right)^3 \kappa$
<i>a</i>	17	3813	0.0105	-0.0032	+ 0.0002
	5	4409	.0158	- .0025	- .0001
	19	5398	.0292	+ .0017	- .0001
<i>b</i>	27	3791	.0164	- .0072	+ .0007
	29	4287	.0219	- .0083	- .0008
	27	6292	.0719	+ .0069	+ .0001
<i>c</i>	11	4395	.0547	- .0058	- .0020
	10	4902	.0803	+ .0051	+ .0016
<i>d</i>	4	4249	.09	- .01	+ .02
	15	5498	.14	- .02	- .01
	5	6339	0.25	+0.04	+ 0.01

The average relative displacements for the four classes, reduced to the same wave-length, are

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
1	1.5	3.4	6.6

The results show conclusively that in the spectrum of iron, within the limits of observational error, the shift is proportional to the cube of the wave-length.

In the case of titanium we have been unable to disentangle the various groups which probably exist. The enhanced lines, however, show conspicuously large shifts, often fully 50 per cent. larger than the neighboring arc lines. There are, however, exceptions, 4338.084 and 4443.976 showing smaller displacements than the neighboring arc lines. Except in the ultra-violet the enhanced lines are all unreversed in the arc at a total pressure of 9 atmospheres. They are broad, diffuse and difficult to measure. There are a few lines also which give displacements of the order of 0.2 Ångström and are undoubtedly of the same nature as the *d* group of iron. Omitting these lines and the enhanced lines and taking the simple mean displacement at various parts of the spectrum we have the following result.

Region.	Number of Lines.	Mean λ .	Mean Δ .	Residuals (Obs.-Comp.).	
				$\Delta = \left(\frac{\lambda}{\lambda_0}\right)^2 \kappa$	$\Delta = \left(\frac{\lambda}{\lambda_0}\right)^3 \kappa$
3700-4700	115	4288	0.0250	+0.0016	0.0064
4700-5700	100	5142	.0344	+ .0008	.0023
5700-6850	38	6127	0.0440	-0.0038	0.0102

The last two columns indicate that in the case of titanium the pressure shift is proportional to the square, rather than the cube of the wave-length.

We have also made a few measurements on some of the series lines of calcium at the same pressure of 9 atmospheres (total). The second subordinate series contains a triplet in the violet beginning at 3949 and another in the red at 6,102. The red lines are of fairly good quality for measurement, but the violet lines are extremely poor. We have, however, succeeded in obtaining a few measures on two of them. The results follow:

λ	Δ	Number of Plates.
3957.177	0.081	3
3973.864	.085	3
6102.937	.139	2
6122.434	.136	2
6162.390	0.137	3

averaging

Mean λ	Mean Δ	Obs.-Comp.
3965	0.083	— .003
6129	0.137	+ .004

The relation between displacement and wave-length is clearly very nearly a linear one, and in the last column we have given the residuals on such an hypothesis. Although these lines do not permit a high degree of precision in measurement, it is hardly possible that the errors are so large as to admit of a law containing the second power of the wave-length, much less the third.

Although the results for titanium and calcium are not so conclusive as those for iron on the law relating displacement to wave-length, there is perhaps sufficient evidence to warrant a reference to the chemical relationship of these three elements. Calcium, titanium and iron appear in the second, fourth, and eighth groups of the Mendeléjeff table, with atomic weights of 40, 48 and 56, and atomic volumes of 25, 13 and 7, respectively. It is perhaps conceivable that the variation of pressure displacement with wave-length should be different for different elements, being a function of the atomic weight. It is interesting to note in this connection that Duffield has suggested a third power law for gold and silver, which are in the iron group, and that Rossi, in connection with his recent work on vanadium, which is in the fifth group, concludes that "the displacement seems roughly proportional to the square or a higher power of the wave-length."

We are greatly indebted to Miss Lasby for the measurement of many plates, and for continued interest throughout the work.

THE PHYSICAL REVIEW.

REACTION EFFECTS PRODUCED BY THE DISCHARGE OF ELECTRICITY FROM POINTS IN GASES AND THE BEARING OF THESE EFFECTS ON THE THEORY OF THE SMALL ION.¹

BY EDWARD J. MOORE.

ARRHENIUS² published in 1897 a series of observations on the reaction effects produced when electricity discharges from points in gases. He found that the ratio of the reaction to the current was very nearly constant and that the force of reaction was approximately inversely proportional to the pressure when the current was constant.

His results are discussed by J. J. Thomson³ from the standpoint of the electron theory and the following equation developed for the force of reaction on the point:

$$F = \frac{i}{K} \int dz, \quad (1)$$

where i is the current, z the direction of the normal from the point to the plane and K the velocity of the ion in unit field. This equation assumes that ions of one sign only are present in the region between point and plane, that these ions are formed in the immediate neighborhood of the point, and travel without change in character to the plane opposite. It seems improbable, as Thomson himself points out, that both these conditions are strictly fulfilled in practice. This investigation was undertaken in order to see to what extent the above relations would hold under the experimental condition that must be imposed. It was hoped that even if the method did not prove to be capable of yielding satis-

¹ A preliminary report of this paper was presented at the Minneapolis meeting of the Physical Society, December, 1910.

² Wied. Ann., Vol. 63, p. 305, 1897.

³ Conduction of Electricity through Gases, 2d ed., p. 507.

factory results for the absolute values of the mobilities it would, nevertheless, yield consistent relative values and in that way, serve to throw light on the interesting and much discussed question of the character of the gaseous ion.

THE APPARATUS AND THE METHOD.

Two forms of the apparatus were used. The arrangement was very simple. In the first form two fine sewing needles were fastened to a wire and formed into a couple (see Fig. 1). This couple was suspended by a phosphor bronze ribbon in such a way that electricity could be made to discharge from the points to the planes P and P' placed opposite. These planes were five cm. in diameter and were connected to earth through a

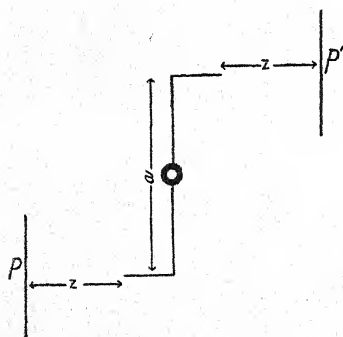


Fig. 1.

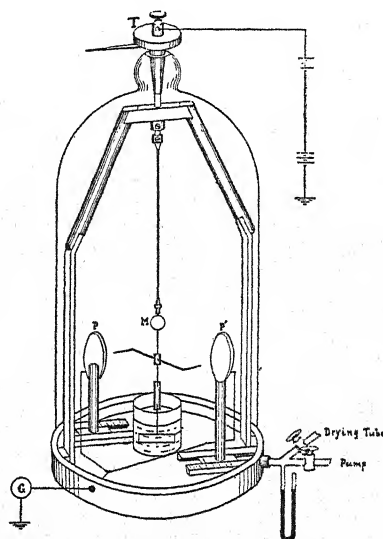


Fig. 2.

galvanometer G , Fig. 2. The distance z between point and plane could be varied between 0 and 5.5 cm. The position of the couple could be read by means of the mirror M . The force of reaction for a given value of z could be measured by the torsion of the suspension, and at the same time the current flowing from the points could be measured by the galvanometer. The apparatus was placed under a large bell jar and arranged so as to admit of varying the pressure. The inside of the bell jar was covered with tin-foil and connected to earth. Electrical connection with the points was effected through a stop cock T in the top of the bell jar. This stop cock served also as a torsion head for measuring the reaction.

The second form differed from the first in having, as it were, two

couples (see Fig. 3) instead of one. The planes were replaced by a cylindrical brass ring 18 cm. in diameter and 5 cm. wide. The ring was connected to earth through the galvanometer and observations were made in precisely the same way as with the single couple. This form of the apparatus was well adapted for measuring the mobility with varying pressures, and yielded curves identical in form with those obtained with the single couple. An aluminum vane, connected to the couple by a piece of ebonite and dipped into a vessel of oil, served to damp the couple.

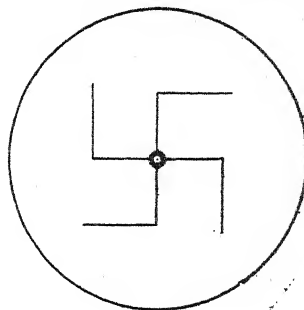


Fig. 3.

The galvanometer was of the ordinary D'Arsonval type. One centimeter deflection at a distance of 2 meters corresponded to 1.9×10^{-8} amperes.

A constant source of potential was obtained from small storage batteries. 10,000 volts were available. Without this constant source this investigation could not have been successfully carried out, especially at the lower pressures, where a variation of a few volts produced a large variation in the current.

The equation of the couple is

$$Fa = T_0\theta, \quad (2)$$

where F is the reaction on the point due to the ions in the gas streaming from it, a is the couple arm, T_0 the torsion constant and θ the angle of twist in radians. Substituting this in equation (1) we have

$$\theta = \frac{ai}{T_0K} \int dz. \quad (3)$$

Observations were made in the following way. The zero position of the couple was first observed with a telescope and scale. A potential was then applied to the points and the torsion head turned to keep the couple in the zero position. In this way the potential was varied until the desired current was obtained. The values of the current and the torsion were then noted. Owing to the fact that the current, the reaction, the distance between the points and the planes and the potential all varied simultaneously, it was not easy to duplicate conditions in consecutive readings. However, the results of these observations seldom differed among themselves by more than two or three per cent.

If a charge q is placed at a distance d from an infinite plane AB maintained at zero potential the density of the induced charge on

the surface of AB is inversely proportional to the cube of the distance, i. e., $\sigma = \alpha/r^3$.

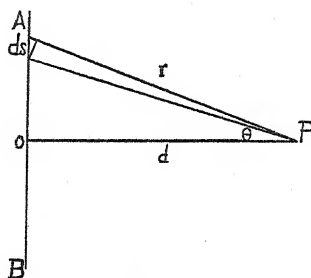


Fig. A.

Correction for electrostatic attraction between the point and plane.

The total charge on the plane will be

$$\int \sigma ds \quad (a)$$

but

$$ds = r^2 \frac{dw}{\cos \theta}.$$

From (a) we get

$$\int_0^{2\pi} \frac{\alpha dw}{d} = \frac{2\pi\alpha}{d} = q,$$

$$\therefore \alpha = \frac{qd}{2\pi} \quad \text{and} \quad \sigma = \frac{qd}{2\pi r^3}.$$

Now the pressure per sq. cm. on the plane is

$$2\pi\sigma^2 = \frac{q^2 d^2}{2\pi r^6}.$$

The total pressure

$$F = \int p ds = \frac{q^2 d^2}{2\pi} \int \frac{ds}{r^6},$$

$$\frac{q^2}{2\pi} \int_0^{\pi/2} \frac{\cos^3 \theta \sin \theta d\theta}{d^2} = \frac{1}{4} \frac{q^2}{d^2}.$$

Since $q = CV$, the above equation becomes

$$F = \frac{1}{4} \frac{C^2}{d^2} V^2, \quad (4)$$

which gives the relation of the attraction between point and plane in terms of the applied potential and the distance. All the observations on the reaction produced on the point by a discharge from it had to be corrected for this force, which of course was of opposite sign to the force

of reaction. The method of making this correction was to observe the attraction between point and plane, using equation (2), as the potential was increased up to the point where a current began to flow. These attractive forces F were plotted with F as ordinates and PD as abscissas and the equation of the experimental curve determined in the following way. Assuming that the equation of this curve was of the form of equation (4), viz.,

$$v^n = PF,$$

then

$$n \log V = \log P + \log F,$$

replacing $\log V$, $\log P$ and $\log F$ by $V'P'$ and F' we have

$$F' - nV' + P' = 0.$$

Since F' and V' are numbers obtained from the experimental curve they can be plotted. The result should be a straight line. From this

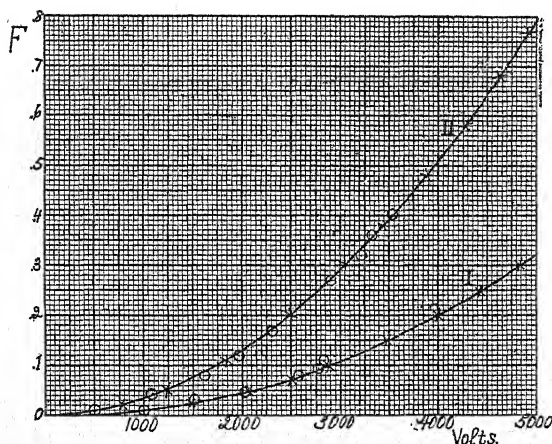


Fig. 4.

line n and P can be found. Fig. 4 shows typical curves of this kind obtained for different couples. The equation of the first is

$$V^{2.060} = 1.3 \times 10^8 F,$$

the second,

$$V^{1.971} = 2.47 \times 10^7 F.$$

Circles indicate observed points and crosses points calculated from the equations of the curves. The equations of a large number of these curves were determined and the values of the exponent of V were all within the limits of those given in the above equations. By extrapolating on these curves the values for the correction at the higher potentials were obtained.

RESULTS AT ATMOSPHERIC PRESSURE.

The curves in Figs. 5 and 6 were found by first determining the equation for the attraction at the different values of z and then measuring the reaction at these points. Fig. 5 shows the results obtained when i is constant and z varied. Fig. 6, the results when z is constant and i

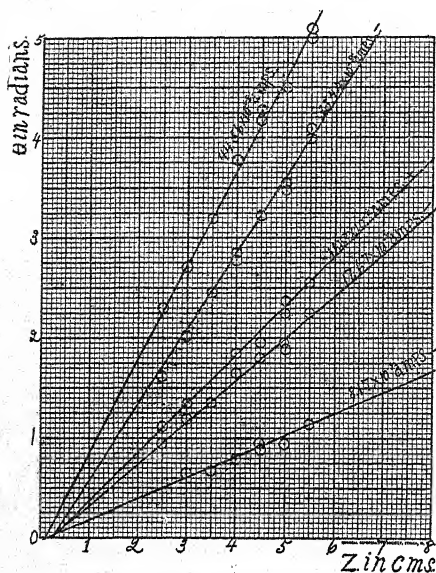


Fig. 5.

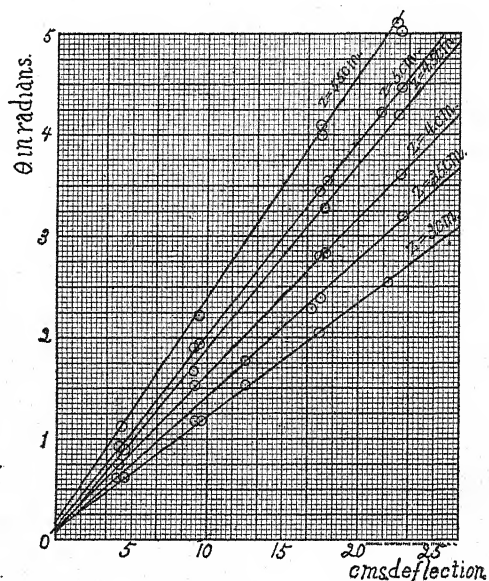


Fig. 6.

varied. The experimental points are not mean values but single observations. When one considers the number of variables that must be controlled in making an observation these points lie fairly close to the straight line which equation (3) would require.

According to equation (3), therefore, the slopes of the lines in Fig. 5 should equal ai/T_0K . Since we know all these quantities K can be calculated at once. However, when this was tried it was found that the value of K increased with the current. The slopes of the lines in Fig. 5 were plotted with the current and the straight line of Fig. 7 was obtained. The slope of this line is a/KT_0 . This gives a constant value for K and indicates that for this experiment equation (1) should be written

$$F = \left(\frac{i}{K} + C \right) \int dz, \quad (5)$$

where C is a small constant.

K computed in this way came out 9.72 cm. for the negative ions and

6.43 cm. for the positive ions. Chattock¹ by measuring the wind pressure produced by electricity discharging from a point obtained $K_- = 1.80$ cm. and $K_+ = 1.32$ cm. Zeleny's² values were $K_- = 1.87$ cm. and $K_+ = 1.36$ cm. Franck³ found K_- to vary from 8.8 to 12.26 cm. and K_+ from 2.5 to 3.77 cm. when he measured his velocities over short distances close to the point in strong fields using Zeleny's method. However, when he measured the velocities over greater distances using

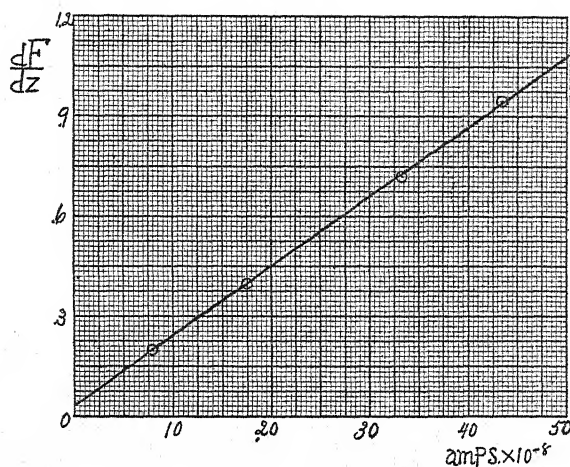


Fig. 7.

a modification of Rutherford's method, producing his ions from point discharge, he obtained $K_- = 1.79$ cm. and $K_+ 1.34$ cm., values which are in good agreement with those obtained by other observers.

If we consider the ratio, K_-/K_+ , Zeleny's⁴ value is 1.37, Chattock's⁵ 1.36, Franck's⁶ 1.35. This work gives 1.51 or taking the inverse ratio of the reactions from the data given in the second part of this paper $K_-/K_+ = 1.49$. The average of a number of observations made using the form of the apparatus of Fig. 3 gives 1.485. If we take Kovarik's⁷ value for the velocity of the negative ion and the average of the above values for the positive we get for this ratio 1.507. Thus it appears that the ratios of the mobilities obtained from the reaction effects are in fair accord with those given by other methods, while the absolute values are very much too high. The following considerations may in

¹ Chattock, Phil. Mag., Series 5, 48, 1899.

² Zeleny, Phil. Trans. A., Vol. 195, p. 193, 1900.

³ Franck, Ann. d. Phys., 4, Vol. 21, p. 972, 1906.

⁴ Loc. cit.

⁵ Loc. cit.

⁶ Loc. cit.

⁷ Kovarik, PHYS. REV., Vol. 30, p. 415, 1910.

part account for these discrepancies. Franck's work indicates that the velocity is very much higher near the point than it is farther out. Tyndall¹ concludes that whatever clustering there is, in air at atmospheric pressure, occurs immediately after the formation of the electron, *i. e.*, the electron does not travel an appreciable distance before it becomes attached. Now, if the ionization in the case of the negative begins farther from the point than in the case of the positive, and this is a most reasonable assumption, the method used by Franck would give a large difference in the apparent velocities. The mobility as calculated from equation (5) depends upon dF/dz . If this is cut down in any way the apparent value of K is increased. If we assume that the gas is ionized for some distance from the point and that the region of ionization extends farther out as z is increased, which is reasonable, as the field increases with z , the ions of opposite sign traveling back to the point would tend to reduce the value of dF/dz . If we assume also, that there is back discharge, *i. e.*, ions of opposite sign traveling from the planes to the points, and assume further that this back discharge increases as z increases, we have another very important factor in cutting down the value of dF/dz . Many observers have found back discharge, and Chattock and Tyndall² point out that in their experiments it increased with z . Zeleny³ finds the back discharge greater for positive ions traveling from point to plane than for negative. If this is a correct view of the mechanism of point discharge large values of K would be expected from this method.

Another possible effect upon the reaction might be a forward drag upon the points due to the air currents set up in the gas by the moving ions. Such a drag, if it existed, would be directly proportional to the area of the points. Points whose area varied in the ratio 1, 2, 3 and 4 were used. The results showed that if such an effect existed it was negligibly small. This also tends to confirm the view that ionization begins at some distance from the point.

THE VARIATION OF MOBILITY WITH PRESSURE.

A number of experimenters have worked on this problem and have reached conclusions very much at variance with one another. Two methods have been used for this work. Rutherford⁴ made the first observations in 1898, using the alternating field method. This method lends itself to measurements on the negative ion only. Rutherford

¹ Tyndall, *Phil. Mag.*, S. 6, Vol. 21, May, 1911.

² Chattock and Tyndall, *Phil. Mag.*, S. 6, Vol. 19, p. 455, 1910.

³ Zeleny, *Phys. Rev.*, Vol. 33, p. 70, 1911.

⁴ Rutherford, *Proc. of Camb. Phil. Soc.*, IX., p. 401, 1898.

concluded, as a result of his work, that the mobility of a negative ion was inversely proportional to the pressure. He used pressure as low as 34 mm.

Langevin¹ devised a method which enabled him to measure the mobility of both positive and negative ions with varying pressure. His observations extended over a range of pressures from 142 cm. down to 7.5 cm. of Hg. His results indicate a marked increase in the mobility of the negative ion beginning in the neighborhood of 20 cm. of Hg., the positive showing only a tendency to increase at 10 cm. of Hg. These observations were made in air, as were also Rutherford's.

Wellisch² in the Cavendish Laboratory, using Langevin's method, measured the mobility of both positive and negative ions in a large number of gases and vapors. His values for the mobilities at normal pressure are in good agreement with the results obtained by other observers. He found, however, that the mobilities of both positive and negative ions were inversely proportional to the pressures. The range extended from atmospheric pressure to 1 cm. of Hg.

Kovarik³ used a slight modification of Rutherford's alternating field method. He worked with air and CO₂ and his range of pressures extended from 760 mm. to 8.8 mm. The method limited him to observations on the negative ion and his results show a very rapid increase in the mobility at about 10 cm. pressure. This rapid increase in the velocity of the negative ion at the lower pressures has always been interpreted to mean that the negative ion at ordinary pressure is an electron with a cluster of one or more molecules attached to it. At the lower pressures according to the results of Langevin and Kovarik it simplified its character by freeing itself from some or all of the attendant molecules, thus being able to move through the gas at a very much higher velocity, whereas the results of Rutherford and Wellisch seem to indicate no such simplification. Langevin's observations on the positive indicate a tendency toward simplification below 10 cm. while Wellisch finds that it appears to persist in its original form at pressures as low as 1 cm.

In view of the lack of complete agreement in the results obtained by the methods heretofore employed for measuring the mobility with varying pressure further experiments along this line seem desirable. Notwithstanding the unsuitability of the present method to the problem of a correct determination of the absolute value of K , the above observations would seem to indicate that it would probably yield consistent values

¹ Langevin, *Ann. de Chemie et de Physique*, t. XXVIII., p. 289, 1903.

² Wellisch, *Phil. Trans., A.*, Vol. 209, pp. 249-279, 1909.

³ *Loc. cit.*

for the variation of the mobility with varying pressure and if so, would be admirably adapted to throw light upon the disputed point regarding the change in the character of the ion at low pressure, since fairly accurate observations on the reaction could be obtained at lower pressures and in stronger fields than those at which it had been found possible thus far to measure mobilities. The results reached below tend to confirm this inference. Also observations made at 40 cm. pressure, when reduced to 76 cm. on the assumption that $PK = \text{constant}$, agreed well with values of K obtained at atmospheric pressure.

The arrangement of the apparatus was similar to that shown in Fig. 2. The pressure was measured with a mercury manometer which could easily be read to one tenth millimeter. The error in reading the pressure was well within the limits of the other errors of the experiment. The distance between the points and the planes was kept constant at 5.3 cm. Since it had been shown that for a given value of z , K is constant when i is constant, the observations were made with approximately constant currents and the value of K was obtained as follows from (1) and (2):

$$\frac{T_0\theta}{a} = F = \frac{i}{k} \int dz, \quad \therefore K = \frac{i}{\theta} \times \text{constant}.$$

Assuming the law $K_0P_0 = KP$, we get

$$K_0 = \frac{Pi}{P_0\theta} \times \text{constant}.$$

The values of K_0 obtained in this way were plotted as abscissas with P as ordinates. If the mobility is inversely proportional to the pressure this should give a straight line.

Observations were made in air, hydrogen and CO_2 . The pressure was varied from 760 mm. down to 5 mm. of Hg. The gases used were first thoroughly dried by passing them over calcium chloride and phosphorus pentoxide. Commercial CO_2 was used. The hydrogen was obtained by decomposing zinc with HCl in a Kipp generator. In order to remove oxygen and other impurities it was passed through pyrogalic acid and KOH , then through the drying tubes and finally admitted to the observation apparatus. The usual precautions were taken to free the apparatus from traces of air when filling it with the gas to be used.

RESULTS AT REDUCED PRESSURES.

Figs. 8, 9 and 10 show the type of variation obtained. The negative curves in air and CO_2 begin to show an increase in the mobility of the ion, over that which the law $PK = \text{constant}$ would require, at about 20 cm. pressure. At 10 cm. there is a decided bend in the curve and as the

pressure is still farther reduced the mobility increases enormously. The curves for hydrogen show the same characteristics except that the increase begins at a higher pressure and the decided bend occurs at about

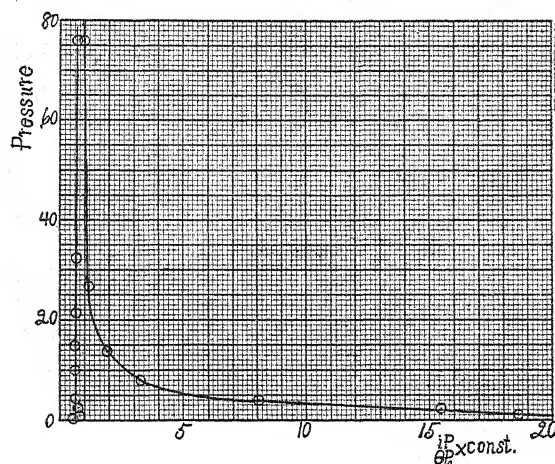


Fig. 8.

Air.

20 cm. Very high velocities are reached by the time the pressure is reduced to 8 cm. The hydrogen curve indicates a tendency for the

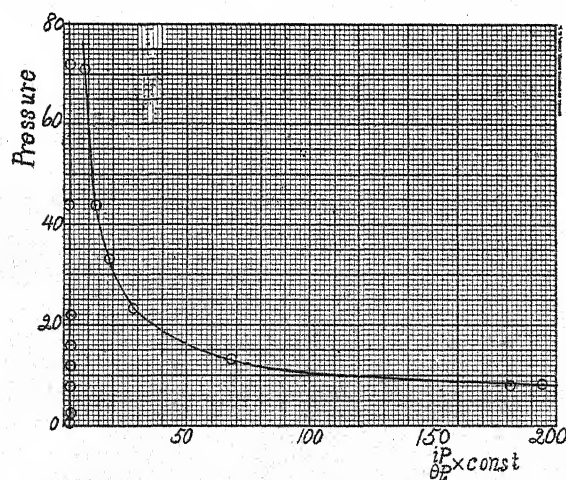


Fig. 9.

Hydrogen.

mobility to increase from the beginning. This may be due to the back discharge¹ which is so prominent in hydrogen.

¹ Chattock and Tyndall, *Phil. Mag.*, Series 6, Vol. 19, p. 454, 1910; Tyndall, *Phil. Mag.*, S. 6, Vol. 21, p. 585, 1911.

Fig. 11 shows the kind of accuracy that can be obtained from this method of measurement. The observations for the points marked with

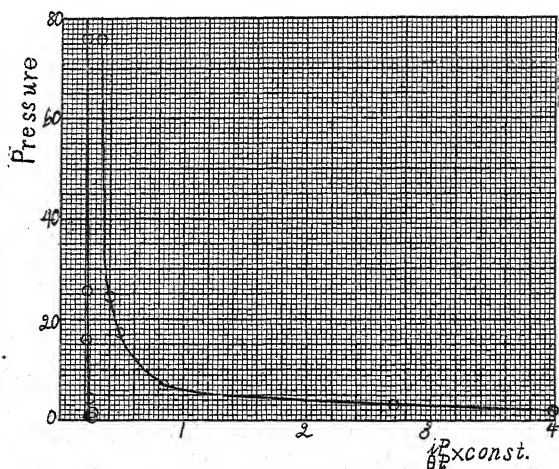


Fig. 10.

CO₂.

crosses were made three or four days after those for the points marked with circles. In the meantime the apparatus was repaired and refilled with CO₂.

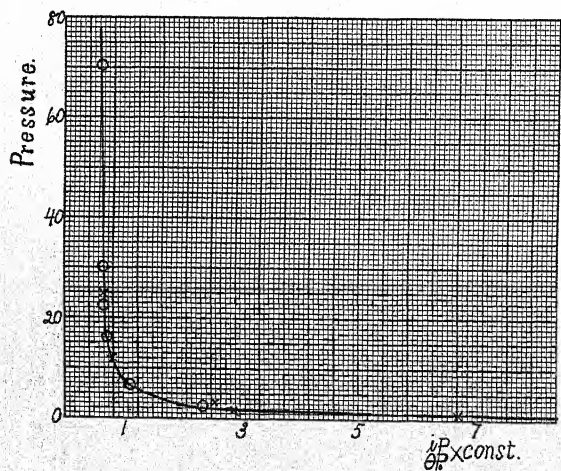


Fig. 11.

CO₂.

Tables I., II. and III. will serve to indicate the character of the data obtained by these observations.

TABLE I.

AIR.
Positive.

i Amperes.	P.D. Volts.	θ Observed.	θ Corrected.	P Cms.	$\frac{iP}{\theta F_0} \times \text{Const.}$
12.7×10^{-7}	4,560	5.883	6.139	32.53	.749
12.0×10^{-7}	3,700	3.839	4.007	21.17	.716
12.3×10^{-7}	3,300	2.858	2.997	14.95	.684
12.3×10^{-7}	2,870	1.857	1.958	10.09	.709
12.6×10^{-7}	2,500	1.337	1.387	7.19	.729
12.3×10^{-7}	1,970	.658	.705	4.01	.779
12.2×10^{-7}	1,500	.303	.331	2.01	.797
12.2×10^{-7}	1,170	.125	.142	.85	.820
12.5×10^{-7}	1,070	.088	.092	.52	.794

Negative.

10.5×10^{-7}	4,000	4.533	4.729	40.70	1.019
10.15×10^{-7}	3,100	2.525	2.643	27.00	1.164
10.7×10^{-7}	2,190	.867	.926	14.18	1.855
11.25×10^{-7}	1,490	.274	.301	8.01	3.34
11.8×10^{-7}	1,000	.0573	.0698	4.26	8.06
10.7×10^{-7}	725	.0112	.0167	2.17	15.56
10.0×10^{-7}	720	.00101	.0065	1.12	18.75

TABLE II.

HYDROGEN.
Negative.

i Amperes.	P.D. Volts.	θ Observed.	θ Corrected.	P Cms.	$\frac{iP}{\theta F_0} \times \text{Const.}$
21.6×10^{-7}	2,940	1.975	2.081	71.28	8.29
22.6×10^{-7}	1,960	.736	.883	44.72	12.39
22.6×10^{-7}	1,580	.4084	.439	33.43	19.22
22.2×10^{-7}	1,180	.195	.213	23.88	28.11
23.8×10^{-7}	890	.0426	.0524	13.51	68.70
22.6×10^{-7}	800	.0382	.0462	8.27	181.3

Positive.

12.05×10^{-7}	3,160	2.500	2.622	71.70	2.30 ¹
12.4×10^{-7}	2,801	1.705	1.801	31.42	2.28
12.2×10^{-7}	2,430	1.186	1.259	22.26	2.41
12.2×10^{-7}	2,140	.8238	.899	16.54	2.42
11.8×10^{-7}	1,880	.500	.543	12.08	2.51
12.2×10^{-7}	1,560	.359	.389	8.10	2.91
12.4×10^{-7}	1,250	.188	.208	4.63	2.82
12.7×10^{-7}	980	.0875	.0995	2.18	3.15
12.5×10^{-7}	815	.0419	.0489	.89	3.02
12.9×10^{-7}	823	.0274	.0345	.51	2.63
					2.14

¹ Calculated.

TABLE III.
CARBON DIOXIDE.
Negative.

i Amperes.	P.D. Volts.	θ Observed.	θ Corrected.	P Cms.	$\frac{iP}{\theta F_0} \times \text{Const.}$
7.9×10^{-7}	—	—	16.0	70.4	.387
7.9×10^{-7}	3,800	5.035	5.21	24.48	.407
7.9×10^{-7}	3,280	3.167	3.29	17.69	.473
8.2×10^{-7}	2,000	.7365	.785	7.15	.830
8.6×10^{-7}	1,230	.1012	.1202	3.43	2.74
6.8×10^{-7}	800	.0101	.0109	.95	4.00

Positive.

4.3×10^{-7}	4,760	5.21	5.44	25.38	.225
5.6×10^{-7}	3,500	2.959	3.009	10.8	.226
5.5×10^{-7}	2,380	1.033	1.102	4.40	.246
4.0×10^{-7}	1,750	.2304	.267	1.53	.258
5.6×10^{-7}	1,550	.063	.092	.41	.268

From these results it seems clear that whether the negative ion is a charged molecule (Wellisch's view) or a cluster of molecules (Langevin's view) it simplifies its character in the neighborhood of 10 cm. pressure for air and CO_2 and 20 cm. for hydrogen; on the other hand, whatever the character of the positive ion it remains unchanged at the lowest pressures observed.

Since this work was completed (September, 1910), and presented to the American Physical Society (December, 1910), a paper has appeared by Todd¹ which gives the results which he has obtained by so modifying Rutherford's method as to make it possible to use it in measuring the mobility of positive ions at reduced pressure.

Todd's results are in complete agreement with those found above, in that the law $PK = \text{constant}$ holds for positive ions at pressures as low as 1 mm.

CONCLUSIONS.

1. The relation between the force of reaction and the distance between point and plane in point discharge has been investigated and found to be linear within the range of currents and distances used. The relation between the force and the current for different values of the distance was also found to be linear. The experimental relation is expressed by the equation

$$F = \left(\frac{i}{K} + C \right) \int dz.$$

¹ Camb. Phil. Soc. Proc., 16, p. 21, February, 1911.

2. The absolute values of the velocities of the ions in unit field were found to be very much higher than those obtained by other methods. The ratio of the velocities, however, was only slightly higher. A possible explanation of the high velocities is suggested.

3. The variation of the mobility with pressure has been measured from 760 mm. to 5 mm. pressure in air, hydrogen and CO_2 . The mobility of the positive was found to be inversely proportional to the pressure in all cases down to 5 mm. The negative ion showed a rapid increase in mobility, beyond that required by the law $PK = \text{constant}$, beginning at about 20 cm. Below 10 cm. the mobility became very large. In the case of hydrogen these effects occur at somewhat higher pressures.

Before concluding the author wishes to thank Professor A. A. Michelson and the members of the Ryerson staff for their interest and encouragement, especially Professor R. A. Millikan, at whose suggestion this investigation was undertaken, and who has offered many helpful suggestions during its progress.

RYERSON LABORATORY,
UNIVERSITY OF CHICAGO.
August, 1911.

THE MAGNETIC ROTATION AND ELLIPTICITY PRODUCED
BY MIRRORS OF MASSIVE METALS.

BY PAUL D. FOOTE.

KERR,¹ Righi² and others have shown that plane polarized light, incident at any angle upon a mirror of a magnetic substance placed in a magnetic field, experiences certain modifications upon reflection. In general the reflected vibration takes the form of an ellipse the axes of which are inclined at an angle to the plane of the incident vibration. The magnitudes of this ellipticity and rotation vary with the intensity of magnetization, the angle which it makes with the normal to the mirror, the wave-length of the light, the angle of incidence, and the material of which the mirror is constructed. The more simple case of normal magnetization and normal incidence of the light is of especial interest because the effect is not then complicated by the presence of phenomena arising from ordinary metallic reflection.

Rotation.—DuBois³ has observed a direct proportionality between the rotation (R) and the intensity of magnetization (I) normal to the reflecting surface; that is,

$$R = K \cdot I,$$

where K is known as the "Kerr constant" for the given substance and wave-length. For iron, cobalt, and nickel, he found K to be negative (the rotation being contrary to the direction of the magnetizing current) while in the case of magnetite K was positive. The dispersion of the rotation was also investigated—iron showing a linear increase of the Kerr constant with increasing wave-length; nickel presenting a minimum at $580\mu\mu$; cobalt a slight minimum in the blue-green; and magnetite a distinct maximum at $590\mu\mu$.

Ingersoll⁴ working in the infra-red has extended DuBois' curves to a wave-length of 4μ and found that beyond a wave-length of 1μ K decreases for the magnetic metals and magnetite. The rotatory dispersion curves obtained by him resemble typical dispersion curves in the region of an

¹ J. Kerr, *Phil. Mag.* (5), 3, p. 339, 1877; (5), 5, p. 161, 1878.

² A. Righi, *Ann. de Chim. et Phys.* (6), 4, p. 433, 1885; (6), 9, p. 132, 1886.

³ H. E. J. G. DuBois, *Wied. Ann.*, 39, p. 25, 1890.

⁴ L. R. Ingersoll, *Phil. Mag.*, p. 41, Jan., 1906.

absorption band. In the case of magnetite and nickel the rotation changes sign in the neighborhood of 1μ . Heusler's alloy was found to give, if any effect at all, at most one tenth that of iron or steel. Measurements¹ made upon iron mirrors deposited by a cathode discharge gave curves similar to those from massive steel.

Loria² has added several substances to the list of those which exhibit the Kerr effect. The rotation in the case of magnetite he found to change sign at 464μ . With invar it increased with increasing wave-length up to 600μ where a slight negative maximum occurred. Cupriferrite showed a positive rotation up to 587μ where it passed through the zero and finally reached a slight maximum negative rotation at about 650μ . The magnitude of all these rotations was small—the maximum for the saturated cupriferrite being approximately but one fifteenth of the maximum for saturated iron as observed by DuBois.

Tokmatschew³ predicted a rotation in saturated Heusler's alloy of about $14'$ for 450μ , but neither Loria working in this region of the spectrum, nor Ingersoll in the infra-red, discovered any effect.

Ellipticity.—Comparatively little work has been done on the ellipticity which generally is present with rotation. The first measurements of this, at normal incidence and normal magnetization, were made by Zeeman,⁴ who investigated the so-called Sissingh constant for iron and cobalt at a wave-length of 589μ .

Skinner and Tool⁵ have also investigated, incidentally, both rotation and ellipticity in the case of iron, cobalt and nickel cathodic films used as mirrors and found them when present at all to be of the same sign as for the massive metals, that is, in a direction opposite to the magnetizing current. In their measurements the ratio of the axes of the reflected elliptic vibration was of an order 3×10^{-3} and approximately the same for all wave-lengths. Practically no ellipticity could be observed in light reflected from films of cobalt either electrolytic or cathodic, nor were cathodic films of nickel "active" even in transmitted light.

The present work is a report of an investigation of primarily the ellipticity, but along with it also the rotation, produced in plane polarized light incident normally upon mirrors of massive iron, cobalt, nickel, steel, magnetite, invar, and Heusler's alloy, all magnetized normally to the re-

¹ L. R. Ingersoll, Phil. Mag., p. 74, July, 1909.

² Stanislaw Loria, Koninklijke Ak. v. Wet. te Ams., May, 1910.

³ S. Tokmatschew, Journ. de russ. phys.-chem. Ges., 42 (Phys. T.), 1910. Beibl., N. 13, p. 718, Art. 51, states that this rotation was *observed*, while Loria claims it to be *predicted* from theoretical considerations. The author was unable to obtain the original article.

⁴ Dr. P. Zeeman, Leiden Comm. No. 15, 1895.

⁵ C. A. Skinner and A. Q. Tool, Phil. Mag., p. 833, Dec., 1908.

flecting surface. In the case of iron, nickel invar and magnetite the refractive indices and absorption coefficients of the same surfaces were also measured. These were obtained by Mr. J. T. Tate¹ of this laboratory, using the method of measurement described by Tool.² In order to preclude the possibility of any time-change in the surfaces affecting the results his observations were made as soon as those on the rotation and ellipticity were completed.

METHODS OF MEASUREMENT.

Rotation.—Figure 1 shows diagrammatically the manner in which the apparatus was arranged. For the measurement of rotation, bright sunlight was concentrated upon the slit of a Fuess spectral system and passing successively through a lens L_2 , a total reflecting prism, and polarizer, was reflected from the tested mirror M , and brought to a focus

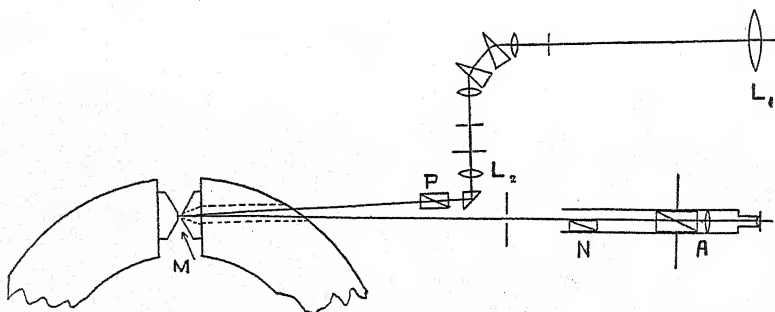


Fig. 1.

upon the Lippich half nicol N set at a small angle to the analyzer A . The angle between the incident and reflected ray was about 2° —well within the limit of 15° for which, as has been shown by Righi, the rotation is practically the same as for normal incidence. The spectral system was so adjusted that it gave a field of light ranging in wave-length over 20μ in the red and 15μ in the violet. Inasmuch as there were no rapid variations of the observed effects with wave-length this gave a sufficiently homogeneous light.

Ellipticity.—For the measurements of ellipticity, a Brace halfshade and compensator analyzing system was used. In the place of the Lippich N , was substituted the fixed halfshade on which a telescope was focused. Between this and the analyzing nicol, was mounted the compensator. To compensate for the rotation of the plane of polarization, before attempting to measure the ellipticity, the polarizer was provided with a

¹ The author wishes to acknowledge his indebtedness to Mr. Tate.

² PHYS. REV., page 1, July, 1910.

means for being slightly rotated so that in all cases the axes of the elliptic vibration could be set at 45° to the azimuth of the fixed halfshade—this being the condition of maximum sensibility. The order of the compensator which was carefully measured by Mr. Tate¹ was 6.89° for a wave-length of $550\mu\mu$. The order for other wave-lengths was calculated from the data obtained by Tool with similar mica compensators. The order of the halfshade was about 3° . A slight modification of the general formula by Professor Tuckerman² of this laboratory for such an arrangement of the analyzing system is

$$E = \tan N_1[\sin 2(\varphi - \varphi_0) + \sin 2\varphi_0],$$

in which E = ratio of minor to major axis of the ellipse; $2N_1$ = order of the compensator; φ_0 = angle through which the compensator is rotated to change from a match on plane polarized light to the nearest position of no effect on this light; and φ = the angular displacement of the compensator from its position of match on plane polarized light to a match on light of ellipticity E .

All measurements of both ellipticity and rotation were obtained by reversing the magnetizing current and thus doubling the optical effect.

The direction of the vibration in the ellipse was determined by comparing the compensator with a quarter wave plate, of which the "fast" and "slow" azimuths had been determined by means of the interferometer.

Accuracy of the Optical Measurements.—The probable error of the observations varied with the conditions of the sunlight and the number of individual settings made. In the case of rotation it usually amounted to $\pm .005^\circ$. Similarly the error in ellipticity was approximately $\pm .05 \times 10^{-3}$. When the observed magnitudes were very small, a larger number of readings, frequently from 20 to 30, were taken and the probable error of the mean thereby considerably reduced. Added to the above error in the ellipticity, due to the compensator setting, would be any error in the calibration of the compensator plate. A liberal allowance for this is one per cent.

Mirror Polish.—The polishing of the mirrors especially in the case of the softer metals such as cobalt, nickel, invar, or Heusler's alloy was very troublesome. However a satisfactory surface free from scratches could be obtained by the use of soft "silver rouge" on a bed of asphaltum. In a number of cases mirrors were polished with rouge or tin oxide upon a bed of pitch. These latter were plane and gave an excellent image but

¹ Using a combination of Stokes's method and the halfshade principle, Tool, loc. cit.

² The Transmission of Light through Doubly Refracting Plates, with Applications to Elliptic Analyzing Systems. L. B. Tuckerman, Jr., University of Nebraska Studies, April, 1909.

were slightly scratched. Those polished on asphaltum were also fairly plane, but did not present the bright clear appearance of the others. Regarding the effect of methods of polishing upon the magneto-optical constants, one appears warranted in concluding that, as with the ordinary optical constants, the form of the curve is practically unchanged though the magnitude of the constants may differ somewhat. Great care was always necessary lest oils, benzol or any other cleaning agent, should dry upon the mirror surface, as the deposit from these substances can affect the constants to a marked degree.

Measurement of Field Strength.—The measurement of the strength of the magnetic field was made by means of the rotation produced by a slab of glass for which the Verdet constant had been carefully obtained. One very important consideration seems to have been neglected by previous observers in this line of work. This is the variation of the field strength with the distance from the mirror surface. A thin cover glass was mounted at different distances from the pole piece by means of brass rings of various thicknesses. The magnetic rotation thus obtained when steel and magnetite mirrors were used is plotted in Fig. 2 against distances

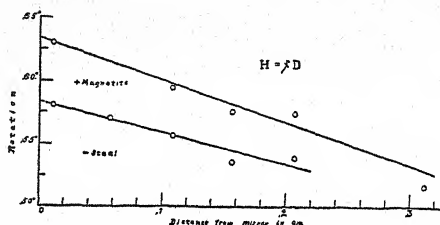


Fig. 2.

to the center of this cover glass. The curves for the other mirrors were similar. The variation with distance is marked but the relation seems to be approximately linear. When therefore a standard calibration slab is used the field measured is that at the center of the slab rather than at the surface of the mirror. The true value at the mirror was, in the present work, obtained by extrapolation. The largest correction necessary was about six per cent.

EXPERIMENTAL RESULTS.

The measurements of rotation for all the mirrors are incorporated in Table I., and those of ellipticity in Table II. In Table III. are given the ordinary optical constants of several, in which as commonly used, ν_0 is the refractive index for normal incidence and κ_0 the extinction coefficient—being the ratio of the amplitude at any point to that at a distance

farther ahead (in the direction of propagation) equal to one wave-length in the substance.

IRON.

The curves representing the variation with wave-length of both rotation and ellipticity for the various iron mirrors are given in Fig. 3.

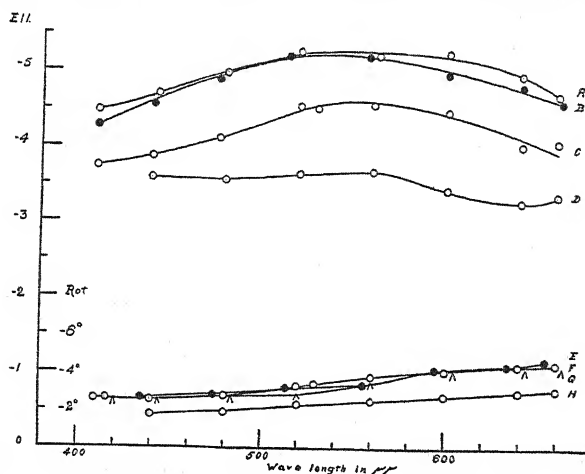


Fig. 3.

Iron.

- | | |
|---|--|
| A, Mirror 7, $E \times 10^3$, 21,000 C.G.S., | E, ●, Mirror 4, $R \times 10$, 20,400 C.G.S., |
| B, Mirror 4, $E \times 10^3$, 20,400 C.G.S., | F, ○, Mirror 8, $R \times 10$, 22,900 C.G.S., |
| C, Mirror 8, $E \times 10^3$, 22,900 C.G.S., | G, △, Mirror 7, $R \times 10$, 21,000 C.G.S., |
| D, Mirror 1, $E \times 10^3$, 13,600 C.G.S., | H, Mirror 1, $R \times 10$, 13,500 C.G.S. |

Commercial Swedish Iron Mirror 1.—Polished with rouge on a bed of pitch. Field strength, 13,600 C.G.S. The rotation is negative throughout the spectrum, the magnitude increasing with increasing wave-length. The ellipticity is also negative with a slight maximum value at 560μ . The rotation is about twice the magnitude of that obtained by Skinner and Tool, with cathodic films of iron in a slightly weaker field (13,200), while the ellipticity is about 70 per cent. greater than their values. The effects with this mirror were not maxima since the magnetic saturation was not reached.

Commercial Swedish Iron Mirror 4.—Polished two years before using—surface bright and clear. Field strength, 20,400. The rotatory dispersion and ellipticity curves of this are similar in form to those obtained for Mirror 1 but due to the stronger field are somewhat greater in magnitude.

Pure Wrought Iron Mirror 7.—This iron was obtained from Kahlbaum, somewhat flawed, but admitted a satisfactory polish with pitch and

rouge. Field strength, 21,000. The rotation by this mirror increased with increasing wave-length, but not exactly linearly. The ellipticity curve follows closely that obtained for Mirror 4.

Pure Wrought Iron Mirror 8.—From same sample as 7. Field strength, 22,900. The rotation increased with wave-length, changing from a slight convexity to the axis of abscissas below $520\mu\mu$ to a slight concavity above. The ellipticity curve resembles in form that of the preceding mirrors, but its magnitude is less than that of either Mirror 4 or 7.

Fig. 6 gives a graph of the variation of the rotation produced by this mirror with field strength. It has been shown by DuBois that the abscissa of the point of intersection of the straight line, $R = KH$, and the asymptote, $R = \text{constant}$, has the value $4\pi I_m$, where I_m is the saturated intensity of magnetization. Accordingly for pure iron (Mirror 8) we find from the figure the intersection at about 19,100, hence $I_m = 1,520$. Fig. 6 also shows for the same mirror a plot of the ellipticity as a function of field strength. The curve is very similar to that of rotation.

If the values of the intensity of magnetization, obtained from the rotation curve by the aid of the equation $R = KI$, be plotted against ellipticity (Fig. 8), we see at once a direct proportionality exists, so that we may write for the ellipticity a similar equation

$$E = FI,$$

where F is the proportionality constant of the mirror for any given wave-length. That the same relation exists for transmitted light has already been shown by Skinner and Tool (l. c.). The ordinary optical constants of this mirror are given in Table III.

COBALT.

The cobalt was obtained from Kahlbaum in rolled sheets 2 mm. thick. The curves from the various samples are plotted in Fig. 4. The rotation for all is negative, with a very slight minimum in the yellow. The ellipticity is also negative, the magnitude decreasing with increasing wave-length. In the blue all tend toward a maximum ellipticity and would probably show decreasing values in the ultra-violet. The general form of both sets of curves for the four mirrors are in agreement, but the magnitudes are somewhat different. Skinner and Tool obtained approximately the same rotation with cathodic films of cobalt but were unable to observe any ellipticity.

Fig. 6 shows the variation of rotation and ellipticity with the magnetic field. Both curves resemble typical I - H curves, and if the intensity of

magnetization, obtained from the rotation values, be plotted against ellipticity, the result is a straight line, as was the case with iron (Fig. 8). I_m is found from the above relation to be approximately 1,390 C.G.S.

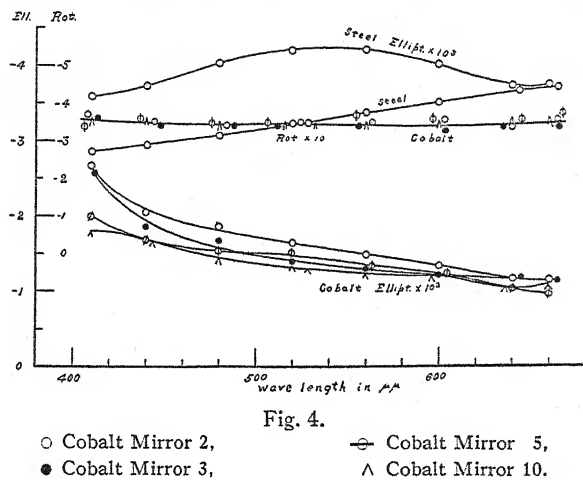


Fig. 4.

○ Cobalt Mirror 2,
● Cobalt Mirror 3,

△ Cobalt Mirror 5,
△ Cobalt Mirror 10.

NICKEL.

The nickel was obtained from Kahlbaum also in the form of sheets 2 mm. thick.

Mirror 6.—This mirror was polished by buffing. The surface was

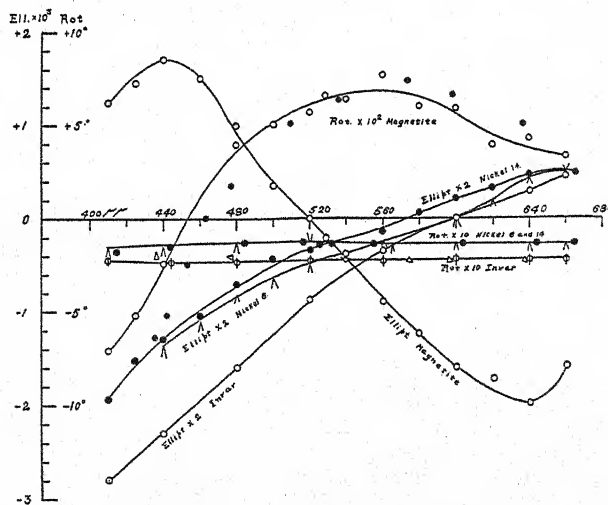


Fig. 5.

somewhat pitted and not exactly plane. Field strength, 17,800. The curves from it are found in Fig. 5. The rotation is negative with a slight minimum in the green. The ellipticity presents an interesting feature

in that it reverses sign in passing through the spectrum. In the violet, green, and yellow, it is negative, the magnitude decreasing gradually with increasing wave-length; at $600\mu\mu$ it passes through the zero (*i. e.*, the light remains plane polarized upon reflection) and from this point to $660\mu\mu$ continues positive with gradually increasing values.

Mirror 14.—Polished with rouge and tin oxide on a bed of pitch. The surface was plane, bright and clear, but showed a few fine scratches. Curves for this also in Fig. 5; field strength, 20,500. The rotation is negative and practically of the same magnitude as that obtained with Mirror 6. A very slight minimum probably exists at $520\mu\mu$. The ellipticity reverses sign as in the case of the preceding mirror crossing the axis at $572\mu\mu$ instead of at 600. This disagreement in the two may be due to the difference in the method of polishing or possibly to errors of observations, since the compensator rotation was very small, although to balance this the values for both curves were made the mean of from 20 to 30 observations. Fig. 7 presents a graph of the rotation as a function of the field strength. I_m is found to be 600 C.G.S.—less than half that of iron or cobalt. The ordinary optical constants of this mirror are given in Table III.

STEEL.

One hardened steel mirror was studied. This was obtained from a Lissajous figure apparatus and possessed a large bright clear surface. The curves are found in Fig. 4—field strength, 19,200. They are similar in form and of approximately the magnitude of those from iron. Figure 6 shows the variation of rotation and ellipticity with the field. Both

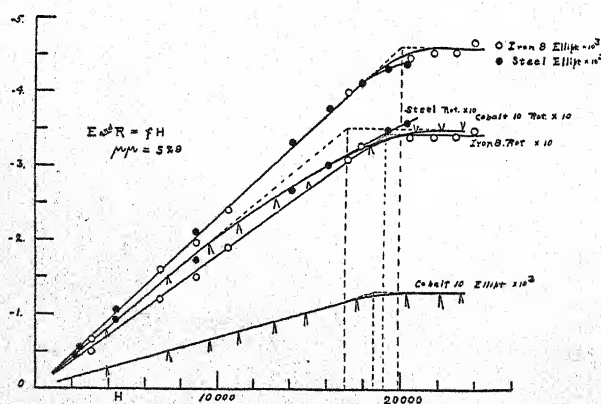


Fig. 6.

curves have the general I - H form, but, although the induction was brought up to 20,000, it is seen that the asymptotic sections of the curves are not quite reached. As was to be expected, the maximum intensity

of magnetization is accordingly higher than that for any other substance tested.

INVAR.

The invar, containing 36 per cent. nickel and 64 per cent. iron, was obtained in the form of bars $3 \times 12 \times 250$ mm. from the Haen chemical works, Seelze, Germany. An excellent polish was secured by means of silver rouge and asphaltum. The curves from this are given in Fig. 5. The rotation is negative and of a magnitude lying between that of nickel and iron, the curve bearing a closer similarity to the former. A slight maximum is reached at 480μ but the general form is approximately a straight line of zero slope. The values given by Loria are represented in the plot by triangles. The agreement in values is fairly satisfactory. The ellipticity shows strikingly the effect of the presence of nickel. Precisely as with that of Nickel Mirror 6 the value is negative in the blue, gradually decreasing with increasing wave-length to the point 600μ where it passes through the zero and becomes positive, with gradually increasing magnitude. The negative values from the alloy, however, are larger and the positive values smaller than those from the nickel, indicating the influence of the iron present. The rotation and ellipticity as functions of the field, shown in Fig. 7, present no new char-

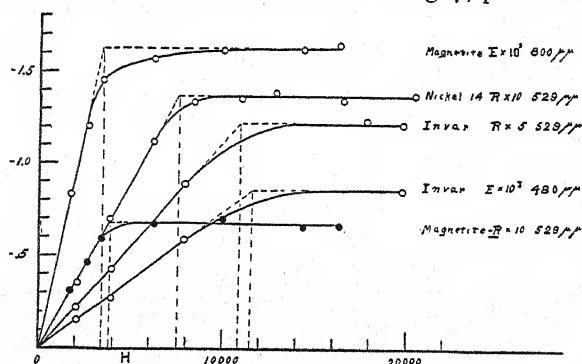


Fig. 7.

acteristics. The maximum intensity of magnetization obtained from a mean of the values given by each curve is 900 C.G.S.—much larger value than that found by Loria (530 C.G.S.). Fig. 8 shows the proportionality between ellipticity and intensity of magnetization. The ordinary optical constants for this invar mirror are found in Table III. They differ very little from those obtained from iron, scarcely showing any effect of the presence of nickel.

Magnetite.—(Ferroferrite, $\text{FeO} \cdot \text{Fe}_2\text{O}_3$.) Octahedral crystal from Pfischthal, Tyrol. A section 2 mm. thick was cut parallel to one of the faces

and polished with rouge and pitch. The surface was brilliant, plane and free from scratches. The rotatory dispersion is shown in Fig. 5. In the violet the rotation is negative, the magnitude rapidly increasing with decreasing wave-length. It is possible that a maximum value exists in the early part of the ultra-violet. The curve passes through the zero point at about $454\mu\mu$ and then continues positive throughout the visible spectrum until a maximum value is reached at $550\mu\mu$. It is interesting to note that Ingersoll¹ working in the infra-red found the curve to again pass through the zero at about 1μ and to continue negative in magnitude as far as investigated (4μ). The curve obtained by Loria (indicated by full dots) is of the same general form but appears to be shifted a small distance toward the red. DuBois² in 1890 using the crystal of magnetite which was later tested by Loria, obtained a curve shifted still farther toward the red.

The curve of ellipticity as a function of the wave-length is also of an interesting form. In the violet it is positive, a decided maximum existing at $440\mu\mu$. From here the magnitude decreases with increasing wave-length until the point $520\mu\mu$ is reached where the ellipticity is zero. As the wave-length still further increases, the magnitude of the negative ellipticity increases almost linearly up to the point $640\mu\mu$ where a sharp negative maximum exists.

In Fig. 7 are plotted the results of the measurements of both rotation and ellipticity with various field strengths. The saturated intensity of magnetization (I_m) is accordingly found to be 287 C.G.S. This value is considerably smaller than has heretofore been obtained with magnetite

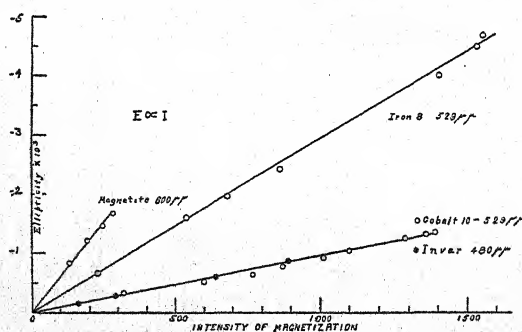


Fig. 8.

crystals. DuBois found the value 350, Loria 358, and Quittner³ by an entirely different method about 480. However magnetite, as has been

¹ Ingersoll, loc. cit.

² DuBois, loc. cit.

³ V. Quittner, Dissertation, Zurich, 1908.

shown by the latter investigator, is a substance which exhibits a great diversity of structure, the density and other properties varying with different samples of apparently the same crystalline form. It is possible, as has been suggested by Loria, that such phenomena have some influence upon the optical behavior of the material.

Fig. 8 shows the ellipticity as a function of the intensity of magnetization. In Table III. are found the ordinary optical constants of this mirror. As found by Skinner and Tool (l. c.) for cathodic films of iron, there appears to be nothing in these optical constants which suggests an explanation for the reversal of sign of the magneto-optical constants.

HEUSLER'S ALLOY.

This alloy was also secured from the Haen chemical works. It contained: Mn, 26 per cent.; Al, 13 per cent.; Cu, 61 per cent. The interior of the sample was full of bubbles, but a satisfactory slab 2 mm. thick was cut from the surface and made into several small mirrors which were polished with rouge on asphaltum. Practically no rotation could be detected in the violet, but a slight negative rotation was measured in the red. At 605μ this amounted to $-.03^\circ$ and at 640 to $-.05^\circ$. The ellipticity was negative but of an order too small to justify measurements throughout the spectrum. At 440μ a number of observations gave the magnitude as -3×10^{-4} . The effect was distinctly smaller in the red.

TABLE I.

Rotation (Degrees).

Mirror.	Field (C. G. S.).	410μ	440μ	480μ	520μ	560μ	600μ	640μ	660μ
Iron 1.....	13,600	—	— .17	— .19	— .23	— .25	— .28	— .30	— .31
Iron 4.....	20,400	—	— .26	— .28	— .32	— .34	— .42	— .44	— .47
Iron 7.....	21,000	— .25	— .26	— .28	— .29	— .36	— .42	— .43	— .44
Iron 8.....	22,900	— .250	— .253	— .277	— .329	— .379	— .413	— .443	— .449
Cobalt 2...	20,000	— .37	— .35	— .34	— .35	— .35	— .36	— .34	— .36
Cobalt 3...	20,000	— .36	— .34	— .34	— .34	— .34	— .33	— .34	— .34
Cobalt 5...	20,000	— .34	— .36	— .35	— .35	— .37	— .36	— .36	— .38
Cobalt 10..	22,100	— .362	— .363	— .352	— .353	— .354	— .358	— .367	— .368
Nickel 6...	17,800	— .15	— .15	— .13	— .13	— .15	— .13	— .14	— .14
Nickel 14..	20,500	— .181	— .147	— .129	— .129	— .138	— .144	— .137	— .140
Steel.....	19,200	— .270	— .285	— .313	— .348	— .378	— .405	— .438	— .445
Invar.....	19,800	— .219	— .234	— .235	— .231	— .230	— .222	— .227	— .227
Magnetite.	16,400	— .071	— .024	+ .040	+ .057	+ .077	+ .059	+ .043	+ .033

TABLE II.

Ellipticity $\times 10^3$.

Mirror.	Field (C. G. S.).	410 μ	440 μ	480 μ	520 μ	560 μ	600 μ	640 μ	660 μ
Iron 1.....	13,600	—	—3.60	—3.58	—3.66	—3.70	—3.47	—3.31	—3.40
Iron 4.....	20,400	—4.29	—4.58	—4.95	—5.27	—5.25	—5.02	—4.86	—4.65
Iron 7.....	21,000	—4.49	—4.67	—5.01	—5.30	—5.25	—5.30	—5.01	—4.75
Iron 8.....	22,900	—3.73	—3.88	—	—4.56	—4.60	—4.50	—4.06	—4.12
Cobalt 2...	20,000	—2.67	—2.04	—1.85	—1.63	—1.49	—1.33	—1.17	—1.15
Cobalt 3...	20,000	—2.61	—1.85	—1.67	—1.41	—1.31	—1.22	—1.20	—1.13
Cobalt 5...	20,000	—2.00	—1.68	—1.54	—1.52	—1.34	—1.27	—1.04	— .97
Cobalt 10..	22,100	—1.79	—1.68	—1.43	—1.33	—1.23	—1.20	—1.07	—1.08
Nickel 6...	17,800	—	— .68	— .41	— .25	— .12	—0.00	+ .21	+ .25
Nickel 14..	20,500	— .97	— .65	— .35	— .17	— .07	+ .11	+ .23	+ .24
Steel.....	19,200	—3.59	—3.73	—4.04	—4.21	—4.23	—4.03	—3.76	—3.77
Invar.....	19,800	—1.40	—1.15	— .80	— .44	— .17	—0.00	+ .14	+ .23
Magnetite..	14,400	+1.24	+1.71	+1.00	.00	— .91	—1.61	—2.00	—1.60

TABLE III.

μ	Iron Mirror 8.		Nickel Mirror 14.		Invar.		Magnetite.	
	ν_0	κ_0	ν_0	κ_0	ν_0	0	ν_0	0
460	2.032	1.445	1.927	1.556	2.12	1.44	2.394	.314
520	2.210	1.395	1.942	1.732	2.28	1.37	2.402	.297
560	2.340	1.335	2.005	1.788	2.39	1.37	2.425	.282
600	2.437	1.305	2.082	1.820	2.50	1.37	2.451	.266
660	2.548	1.270	2.215	1.837	2.66	1.36	2.468	.241

SUMMARY.

1. The magnetic rotation and ellipticity by reflection at normal incidence have been measured throughout the visible spectrum for massive mirrors of iron, cobalt, nickel, steel, invar, magnetite, and Heusler's alloy.

2. The rotation is in reasonable agreement with the work of preceding investigators.

3. The ellipticity, the investigation of which was the main object of the present work, and which has not been studied extensively by other observers, was found to be consistent with what might be expected from the work which has been done on transmission. For the majority of substances tested it was negative throughout; but for others it changed sign within in the visible spectrum.

4. The ordinary optical constants obtained for iron, nickel, invar and magnetite showed no evident connection existing between these and the magneto-optical constants.

5. In a number of cases, careful attempts to reproduce the mirror polish failed to give results which agreed in magnitude though the curves were usually of the same general form. In others practically identical results were obtained, although the mirrors were subjected to different methods of polish. That the polish may affect the magnitude and not the form of the dispersion curves has been quite generally observed by investigators of the ordinary optical constants of mirrors.

6. As with transmitted light the ellipticity of the reflected light in every case was found proportional to the intensity of magnetization for all wave-lengths.

In conclusion the writer wishes to thank Professors C. A. Skinner and L. B. Tuckerman for the aid he has received in this experiment.

THE BRACE LABORATORY OF PHYSICS,
THE UNIVERSITY OF NEBRASKA,
July, 1911.

A STUDY OF THE REVERSIBLE PENDULUM.¹

PART II. EXPERIMENTAL VERIFICATIONS.

BY JOHN C. SHEDD, J. A. and W. N. BIRCHBY.

FOR each axis of suspension of a physical pendulum there are, in general, three other parallel axes about which the pendulum swings with the same period. As shown in Fig. 1, these axes are symmetrically located in pairs with respect to the centroid of the pendulum. If z' and z'' be the distances of the respective pairs from the centroid, they are connected by the well-known relation

$$z'z'' = a^2,$$

in which a is the radius of gyration of the whole pendulum about a line passing through its centroid and parallel to the above axes.

If now the position of the centroid be varied (and with it the value of a) by altering the distribution of mass of the pendulum, the variation of z' is expressed by equation (9), as a linear function of d . The variation of z'' is more complex and may be derived as follows: If c be the distance of k from the centroid of the pendulum then

$$a^2(1 + R) = \alpha^2 + (c - b)^2 + R\beta^2 + R(c - d)^2.$$

Also, since

$$R = \frac{(c - b)}{(d - c)},$$

then

$$(d - c) = \frac{(d - b)}{(1 + R)} \quad \text{and} \quad (c - b) = \frac{R(d - b)}{(1 + R)}.$$

Substituting these values in the above expression the value of a^2 is found to be

$$a^2 = \frac{(\alpha^2 + \beta^2 R)}{(R + 1)} + \frac{R(b - d)^2}{(R + 1)^2}, \quad (44)$$

¹ In Part I., *PHYS. REV.*, Vol. XXV., pp. 274-293, the general equations of the pendulum were derived and discussed under three divisions. These equations are as follows:

$$\frac{gl^2}{\pi^2}(b + Rd) = Rd^2 + b^2 + \alpha^2 + R\beta^2, \quad (10)$$

$$\frac{gl^2}{\pi^2}[L(1 + R) - (b + Rd)] = R(L - d)^2 + (L - b)^2 + \alpha^2 + R\beta^2. \quad (11)$$

By substituting this value of α^2 and the value of z' given in equation (9) into the expression $z'z'' = \alpha^2$ the value of z'' is found to be:

$$z_1'' = \frac{(\alpha^2 + R\beta^2)}{(b + Rd)} + \frac{R(b - d)^2}{(R + 1)(b + Rd)}, \quad (45)$$

an expression in which z'' and d are the only variables.

Equation (9) appears graphically as a straight line; equation (45) as a hyperbola with a vertical asymptote a distance b/R to the left of the Y axis.

If now a new axis of suspension be chosen, at a distance L , a new set of four parallel axes having a common period will be determined, and the corresponding z values will be given by the equations

$$z_2' = \frac{(L - b) + R(L - d)}{R + 1}, \quad (46)$$

$$z_2'' = \frac{\alpha^2 + R\beta^2}{(L - b) + R(L - d)} + \frac{R}{R + 1} \cdot \frac{(b - d)^2}{(L - b) + R(L - d)}. \quad (47)$$

z_2' and z_2'' are here taken positive when the centroid is to the left of k_2 . The graphs of equations (9), (45), (46) and (47) are shown in Fig. 9, for a pendulum whose constants are as follows:

$$R = 1, \quad L = 60, \quad b = 40, \quad \alpha^2 + R\beta^2 = 600.$$

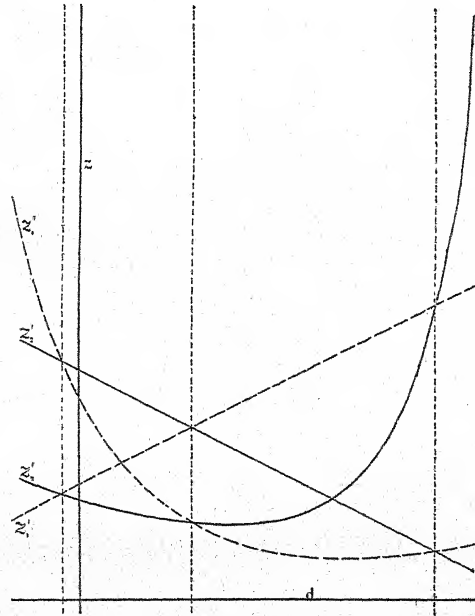


Fig. 9.

Of the ten intersections of the four graphs—two of the intersections being outside of the limits of the figure—six are paired for three values of d , and each of the six is between a z_1 and a z_2 curve. This means that whenever one of the four k_1 axes coincides with a k_2 axis, each of the three other k_1 axes coincides with one of the three remaining k_2 axes and thus the period is the same for all. Thus for the three values of d , the periods from k_1 and k_2 are equal. Further, for any value of d , the sum of the ordinates of the two straight lines is equal to L , as appears from equations (9) and (45); therefore the sum of the ordinates of the points of intersections corresponding to the first and last values of d is likewise equal to L , or $z_1' + z_1'' = z_2' + z_2'' = L$, and the pendulum is in Kater adjustment. From similar considerations it may be shown that this cannot, in general, be true of the middle value of d . It is true however if the two points of intersection coincide, and consequently,

$$z_1' = z_1'' = z_2' = z_2'' = a.$$

Thus there are in general three values of d for which the pendulum swings with equal periods from k_1 and k_2 , for two of which it is in Kater adjustment, and for one of which, in general, it is not. This last is the condition designated as the "third intersection" in the discussion of equations (10) and (11) when t and d are taken as the variables.

DIVISION I., t AND d TAKEN AS THE VARIABLES.

Equations (10) and (11), as previously shown, are cubics belonging to the 53d species named in Newton's classification. The general form is shown by the full line curve in Fig. 10A, where values of d are plotted as abscissæ and t as ordinates. As t must be positive, only that portion of the graph which lies above the X axis is realizable. Rewriting equation (10) in the form

$$t^2 = \frac{\pi^2}{g} \left(d - \frac{b}{R} \right) + \frac{b^2/R + b^2 + \alpha^2 + R\beta^2}{b + Rd}$$

it is seen that for large values of d the curve indefinitely approximates the parabola

$$\frac{gt^2}{\pi^2} = d - \frac{b}{R}.$$

This parabola is shown in the dotted curve of Fig. 10A, where its asymptotic character is clearly shown. Also, as d approaches the value $-b/R$, t increases without limit in both directions, giving

$$d + \frac{b}{R} = 0$$

as a vertical asymptote. It will be convenient to refer to the *vertical* and *parabolic* arms of this curve, meaning thereby those parts having the vertical and parabolic asymptotes respectively.

As already shown three only of the nine intersections of equations

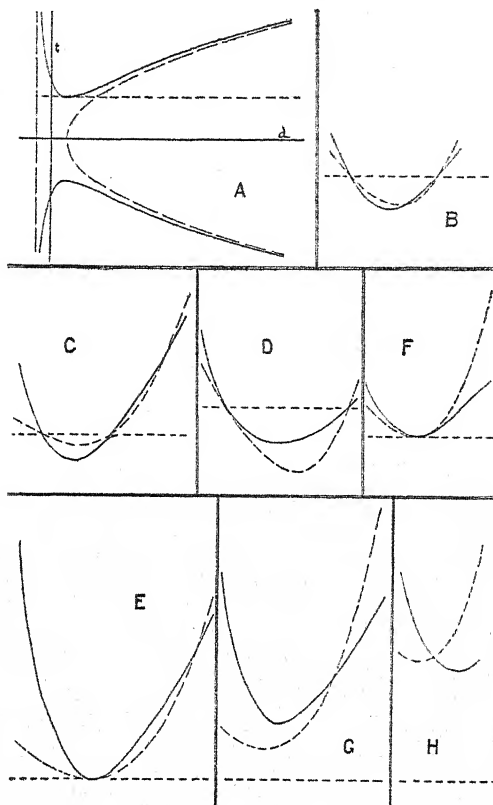


Fig. 10.

(10) and (11) belong to the present discussion. Their coördinates are given by equation (27). Two of the three intersections fall on the line

$$t = \pi \sqrt{\frac{L}{g}}$$

(the dotted horizontal line in the graphs of Fig. 10), and indicate Kater adjustment: while the third always falls on the line

$$d = \frac{L}{2} + \frac{(L - 2b)}{2R}.$$

The first two are disposed symmetrically about the line $d = L/2$. While these are sometimes imaginary, the third intersection is always real. It

may be on either side of, between, or coincident with the other two. These conditions are illustrated in Fig. 10, curves *B* to *H*.

As a check on this part of the theory an experiment was performed with a reversible pendulum arranged to give $d_2 > d_1$. The constants of the pendulum had the following values:

$$\alpha^2 = 610.4, \quad \beta^2 = 3.14, \quad R = 0.43, \quad b = 10.4, \quad L = 50.$$

Introducing these constants into equations (10) and (11) the following working equations are had:

$$\left. \begin{aligned} t_1^2 &= \frac{0.01006(d^2 + 1673.7)}{(d + 24.178)}, \\ t_2^2 &= \frac{0.01006[(50 - d)^2 + 5067.9]}{(142.063 - d)}. \end{aligned} \right\} \quad (48)$$

The results obtained both by experiment and calculation are given in the following table.

d	t_1 Experimental.	t_1 Calculated.	t_2 Experimental.	t_2 Calculated.
5	0.767	0.765	0.719	0.722
10	0.720	0.723	0.712	0.713
15	0.697	0.698	0.705	0.706
20	0.687	0.687	0.701	0.701
25	0.685	0.686	0.698	0.699
30	0.690	0.691	0.700	0.701
35	0.702	0.702	0.704	0.705
40	0.716	0.716	0.714	0.714
45	0.733	0.733	0.726	0.727
54	0.769	0.769	0.763	0.762
57	0.782	0.781	0.778	0.778
60	0.794	0.794	0.797	0.796
65	0.816	0.816	0.833	0.831

Fig. 10c is drawn from these data.

By the use of equations (48) these curves may readily be extrapolated to show the characteristics of the typical curve *A* in Fig. 10. This is done in Fig. 11, where d is extended to ≈ 330 . For purposes of comparison the ratio between the scales of ordinates and abscissæ is kept the same in Fig. 11 as in Fig. 10c.

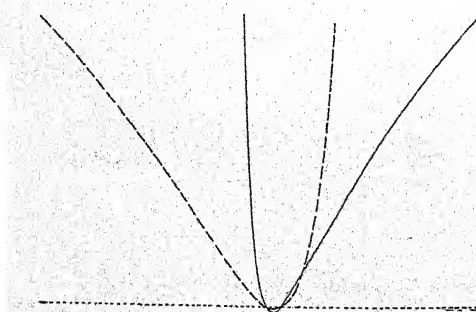


Fig. 11.

In discussing the possible arrangements of the three intersections of equations (10) and (11) it is convenient to consider their minimum points as the constants of the pendulum are varied. The coördinates (designated as δ_1 , τ_1 and δ_2 , τ_2) are as follows:

$$\left. \begin{aligned} \delta_1 &= -\frac{b}{R} + \left[\frac{b^2}{R^2} + \frac{b^2 + \alpha^2}{R} + \beta^2 \right]^{\frac{1}{2}}, \\ \tau_1 &= \pi \sqrt{\frac{2}{g}} \left\{ -\frac{b}{R} + \left[\frac{b^2}{R^2} + \frac{b^2 + \alpha^2}{R} + \beta^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}}, \\ \delta_2 &= L + \frac{L-b}{R} - \left[\frac{(L-b)^2}{R^2} + \frac{(L-b)^2 + \alpha^2}{R} + \beta^2 \right]^{\frac{1}{2}}, \\ \tau_2 &= \pi \sqrt{\frac{2}{g}} \left\{ -\frac{L-b}{R} + \left[\frac{(L-b)^2}{R^2} + \frac{(L-b)^2 + \alpha^2}{R} + \beta^2 \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}}. \end{aligned} \right\} \quad (49)$$

From these equations the loci of the minimum points may be derived, and are as follows:

$$\frac{g\tau_1^2}{\pi^2} = 2\delta_1, \quad (50)$$

$$\frac{g\tau_2^2}{\pi^2} = 2(L - \delta_2). \quad (51)$$

It will at once be noticed that the first of these equations is independent of all of the constants of the pendulum while the second one is dependent only upon L , and in such a way as merely to displace the curve along the axis of abscissas. Both curves are parabolae, are of the same size, but are turned in opposite directions. The vertex of the first one is at $(0, 0)$, while that of the second one is at $(L, 0)$: the X -axis is the axis of both curves. The somewhat remarkable conclusion is reached that every reversible pendulum, no matter how constructed, in which t and d are taken as variables, has the minimum-points of its period-curves upon two parabolae of fixed size, the distance apart of the vertices being the only variable quantity.

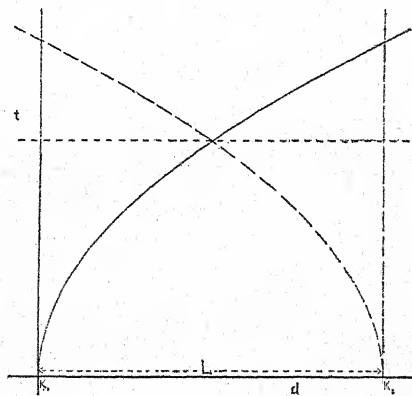


Fig. 12.

A study of the graph of these curves (Fig. 12) leads at once to several interesting conclusions. Thus the curves always intersect at the point $(L/2, \pi\sqrt{L/g})$, i. e., on the g -line and halfway between the knife-edges:

again, when the minimum points are above the g -line the k_1 minimum is to the right of the k_2 minimum; when they are below the reverse is the case. Also both minimum points must be either above or both below the g -line. This follows from the fact that the graphs of equations (10) and (11) cross the g -line at the same two points (Kater intersections), and if these intersections are real for one curve they must be for the other also. It may also be shown, by introducing the condition $\tau_1 = \tau_2$ into equations (50) and (51), that the ordinates of the minimum-points cannot be equal unless either the centroid of the fixed part is midway between the knife-edges, or the two minimum points coincide on the g -line. An interesting theorem may be noted concerning the relative positions of the minimum-points and the third intersection as follows: If the abscissa d_3 of the third intersection is to lie between those of the minimum points, *i. e.*, $\delta_1 \geq d_3 \geq \delta_2$ then the quantity $L/2 \cdot (R + 1)/R$ must be either greater or less than both of the quantities $[b^2/R^2 + (b^2 + \alpha^2)/R + \beta^2]^{\frac{1}{2}}$ and $[(L - b)^2/R^2 + [(L - b)^2 + \alpha^2]/R + \beta^2]^{\frac{1}{2}}$ according as the minimum points are below or above the g -line. For the abscissa of the third intersection to lie to the right or the left of the minimum points, *i. e.*,

$$d_3 \geq \begin{cases} \delta_1 \\ \delta_2 \end{cases}$$

the quantity $L/2 \cdot (R + 1)/R$ must be intermediate in value between the above two radicals.

It is now possible to state the relations necessary between the arbitrary constants of the pendulum, in order to produce the conditions illustrated by graphs $B-H$ in Fig. 10.

Curve B.—Third intersection between the first and second: for this it is sufficient that the third intersection be below the g -line, or that $t_3 < \pi\sqrt{L/g}$. This reduces to the condition

$$L^2 > (L - b)^2 \cdot R + \frac{4(\alpha^2 + R\beta^2)}{(R + 1)}. \quad (52)$$

Curve C.—The third intersection to the right or left of both of the others, *i. e.*, $d_3 > d_1$, or $d_3 < d_2$ (d_1 is always greater than d_2).

One of these conditions will always hold when,

$$L^2 > \frac{4b(L - b)}{1 - R} - \frac{4R(\alpha^2 + R\beta^2)}{1 - R^2}. \quad (53)$$

The third intersection will be to the right when in equation (53) $L/2 > b$, and it will be to the left when $L/2 < b$; for, since the third intersection requires that the centroid of the whole pendulum be midway between the knife-edges, it is evident that the centroids of the fixed and movable parts must be on opposite sides of this mid-way point.

Curve D.—The third intersection coincident with either the first or the second intersection. In this case,

$$b = \frac{L}{2} \pm \frac{1}{2} \left[\frac{RL^2(R+1) - 4R(\alpha^2 + R\beta^2)}{R+1} \right]^{\frac{1}{2}}, \quad (54)$$

the negative sign applying when $d_3 = d_1$, and the positive sign when $d_3 = d_2$.

Curve E.—The first and second intersections to coincide with each other but not with the third: or

$$d_1 = d_2 \quad \text{and} \quad \frac{L}{2} \leq d_3.$$

This requires $b \neq L$, and

$$b = \frac{L}{2} \pm \left[\frac{L^2}{4}(R+1) - (\alpha^2 + R\beta^2) \right]^{\frac{1}{2}}. \quad (55)$$

For $d_3 > L/2$ the positive sign in (55) applies, and for $d_3 < L/2$ the negative sign.

Curve F.—All three intersections to coincide. In this case

$$d_1 = d_2 = d_3 = \frac{L}{2}.$$

This requires $b = L/2$, and condition (55), which gives

$$\frac{L}{2} = \left[\frac{\alpha^2 + R\beta^2}{R+1} \right]^{\frac{1}{2}}. \quad (56)$$

Curve G.—The first and second intersections imaginary and the third to be between a vertical and a parabolic arm. The necessary condition is

$$\left[\frac{b^2}{R^2} + \frac{b^2 + \alpha^2}{R} + \beta^2 \right]^{\frac{1}{2}} \leq \frac{L}{2} \cdot \frac{R+1}{R} \leq \left[\frac{(L-b)^2}{R^2} + \frac{(L-b)^2 + \alpha^2}{R} + \beta^2 \right]^{\frac{1}{2}}, \quad (57)$$

the upper signs holding when the intersection falls to the right and the lower when it falls to the left.

Curve H.—The first and second intersections imaginary, and the third to be between vertical arms. The necessary condition is

$$\left. \begin{aligned} \frac{L}{2} \cdot \frac{R+1}{R} &< \left[\frac{b^2}{R^2} + \frac{b^2 + \alpha^2}{R} + \beta^2 \right]^{\frac{1}{2}}, \\ \frac{L}{2} \cdot \frac{R+1}{R} &< \left[\frac{(L-b)^2}{R^2} + \frac{(L-b)^2 + \alpha^2}{R} + \beta^2 \right]^{\frac{1}{2}}. \end{aligned} \right\} \quad (58)$$

If $b = L/2$, then $d_3 = L/2$, or the movable mass will be midway between the knife-edges when the third intersection is obtained, while $b > L/2$ throws it to the left and $b < L/2$ to the right.

The constants of the pendulum used in computing the graphs of Fig. 10 were as follows:

Curve.	α^2	β^2	R	b	L
<i>B</i>	610.4	3.14	0.430	21	45
<i>C</i>	610.4	3.14	0.430	10.4	50
<i>D</i>	610.4	3.14	0.430	33.3	50
<i>E</i>	610.4	3.14	0.430	8.2	50
<i>F</i>	610.4	3.14	0.430	20.68	41.35
<i>G</i>	610.4	3.14	0.430	10.4	40
<i>H</i>	610.4	3.14	0.430	16.0	30

A study of this table brings out several interesting points in connection with the curves of Fig. 10. Thus it will be noticed that curves *C*, *D*, and *E* are produced with the same constants except for variations in b : while curves *C* and *G* differ only in L , so that the k_1 curve is the same in both while the k_2 curve is displaced along the Y -axis.

A laboratory experiment often performed with the compound pendulum is to vary the position of the axis of suspension leaving all else constant. This is accomplished by suspending the pendulum bar on a fixed knife-edge passing through one of a series of holes cut in the bar to receive it, thus changing the axis without altering the distribution of mass (vide Nichols' or Duff's laboratory manuals). Thus L and b (or L and $L - b$) are varied and with them the period. As there is no movable mass d , R and β become zero. The variables are t and b and the case may be considered as a special one under the present discussion. It will be noticed that L occurs in equation (11) only in the expressions $L - b$

and $L - d$. This difference remains invariable as k_1 is moved, giving no variation in the period from the fixed knife-edge. Equation (10) may then be rewritten in the form

$$\frac{gt^2}{\pi^2} = \frac{\alpha^2 + b^2}{b} = l, \quad (58)$$

where l is the equivalent length of the pendulum. The application of the graphical method to this case while rather simple, brings out several interesting points. Let values of b be plotted as abscissas and values of l as ordinates. A hyperbola results, as shown in Fig. 13, having as

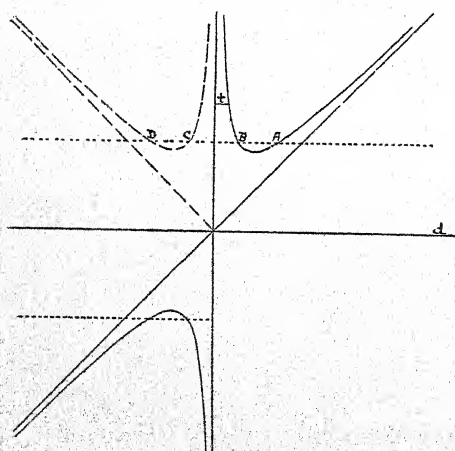


Fig. 13.

l as ordinates. A hyperbola results, as shown in Fig. 13, having as

asymptotes the lines $b = 0$ and $b = l$. As the pendulum reverses when the axis of suspension passes over the centroid, taken as the origin in the graph, it is convenient to "turn up" the third quadrant into the second, as shown by the dotted lines on the curve. It will now be seen that a minimum occurs when $b = \alpha$, and in that case $l = 2\alpha$, showing that the minimum period for any pendulum is given by the condition

$$t = \pi \sqrt{\frac{2\alpha}{g}}. \quad (59)$$

The graph for the fixed axis of suspension is given by the two straight lines $l = \pm l'$, the sign depending upon which side of the centroid the fixed axis is taken. Further if $l' > 2\alpha$, there are four intersections between the straight lines and the hyperbola, indicating four positions of the movable axis, A, B, C, D , which give the same equivalent length, l' . One of the points will coincide with the position of the fixed axis. As already shown, A and C , and also B and D , are l' cm. apart, and correspond to the Kater adjustment. If $l' < 2\alpha$ then the line $l = l'$ is tangent to the hyperbola and the four positions A, B, C, D reduce to two, the minimum points, 2α cm. apart. If $l' < 2\alpha$ there are no intersections and Kater adjustment is impossible.

DIVISION II., t AND β TAKEN AS VARIABLES.

As a method of determining the value of g this method has advantages as it is possible by taking four periods to determine g immediately. It is not necessary that the pendulum be brought into Kater adjustment, or even capable of such adjustment.

In this method the movable mass is made up of two masses m, m , which are slid along the pendulum rod as shown in Fig. 5, in such a way as always to be equidistant from a given point, d cm. from k_1 . Thus β may be varied and consequently t , without the variation of any other of the quantities entering into equations (10) or (11). In the graphical treatment, using equations (30) and (33), t^2 is plotted as y and q^2 as x , $2q$ being the distance between the movable masses m, m . As shown in Fig. 6 the graphs reduce to two straight lines, which intersect on the line

$$y = t^2 = \pi^2 \frac{L}{g},$$

and this intersection point may be used in the determination of g . As the moment of inertia of the pendulum is a minimum when $q = 0$, in this condition the equivalent length must be no greater than L for Kater adjustment to be possible. This leads to the condition,

$$L > [(\beta_0^2 + d^2)R + b^2 + \alpha^2] \div (b + Rd). \quad (60)$$

It is interesting to note that with this form of pendulum the condition signified by the third intersection, in Division I., is possible only when the centroid of the whole pendulum has first been adjusted midway between the knife-edges. If this be done the intersection secured will be the "third" and the Kater adjustment will be impossible, unless the pendulum is in the condition indicated by the coincidence of all three intersections as discussed in Division I. This will occur when β for the point of intersection has the value

$$\beta^2 = \frac{1}{R} \left[\left(\frac{(R+1)L^2}{4} - \alpha^2 \right) - \left(b - \frac{L}{2} \right)^2 - R \left(d - \frac{L}{2} \right)^2 \right]. \quad (61)$$

If $b = L/2$, then $d = L/2$, and this reduces to condition *F*, Division I., i. e.,

$$\beta^2 = \frac{1}{4R} [(R+1)L^2 - 4\alpha^2].$$

The following experiment was performed as a check on the theory: A pendulum was arranged with the following constants:

$$L = 60, \quad R = 1.51, \quad d = 0, \quad b = 15, \quad \beta_0^2 = 3.14, \quad \alpha^2 = 628.5.$$

β_0^2 was computed from the dimensions and mass of m , and α^2 from determinations of the periods from the two knife-edges, employing the fixed part alone, and using the equations,

$$\alpha^2 = b \left(\frac{gl^2}{\pi^2} - b \right) = (L - b) \left[\frac{gl^2}{\pi^2} - (L - b) \right].$$

As these constants fulfil the conditions of equation (60) the intersection will fall in the first quadrant.

By substituting the above constants into equations (30) and (33) the following working equations are obtained:

$$\left. \begin{aligned} t_1^2 &= 0.001013q^2 + 0.5759, \\ t_2^2 &= 0.000112q^2 + 0.6005. \end{aligned} \right\} \quad (62)$$

The following data were secured, and the graph is shown in Fig. 14*a*.

q^2	Experimental Values.		Calculated from Equation (62).	
	t_1^2	t_2^2	t_1^2	t_2^2
25	0.6005	0.6037	0.6012	0.6033
400	0.9811	0.6472	0.9811	0.6453

Determining analytically the value of t_2 for the intersection gives $t_2 = 0.60403''$ and solving for g gives the value $g = 980.61$.

In a second experiment to illustrate the case of extrapolation, when the Kater adjustment becomes impossible the following constants were used:

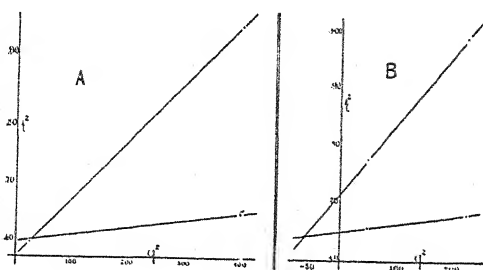


Fig. 14.

$$L = 64, \quad R = 2.28, \quad d = -2.5, \quad b = 25, \quad \beta_0^2 = 3.14, \quad \alpha^2 = 743.9.$$

The working equations are

$$\left. \begin{aligned} t_1^2 &= 0.00119q^2 + 0.7252, \\ t_2^2 &= 0.0001204q^2 + 0.65175. \end{aligned} \right\} \quad (63)$$

The graphs are shown in Fig. 14b and the experimental and calculated values below:

q^2	Experimental Values.		Calculated from Equation (63).	
	t_1^2	t_2^2	t_1^2	t_2^2
49	0.7779	0.6585	0.7835	0.6577
225	0.9940	0.6831	0.9930	0.6788

Determining g by substitution as above gives $g = 982.3$, a result well within the limits of accuracy, as the experiment was conducted.

DIVISION III., t AND R TAKEN AS VARIABLES.

In the third division of the discussion of equations (10) and (11) it was found necessary to modify the pendulum so as to make the value of β^2 very large in certain cases. Fig. 15 shows the form finally used. The

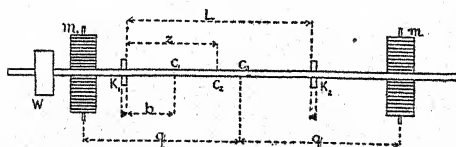


Fig. 15.

masses m and m , constituting the variable mass, are made up of brass discs which may be threaded onto appropriate supports. Discs of two sizes were used, one 7.67 cm. and the other 10.17 cm. in diameter. By

varying the number of discs, the value of R may be varied without changing any other of the constants of the pendulum. One of the disc supports remained fixed at one end of the pendulum while the other could be moved to any desired point, or left off altogether. When both were in use, an equal number of discs was placed on each, thus keeping the centroid of the variable mass half way between the supports.

If β_0 be the radius of gyration of one of the cylinders about its own axis, and $2q$ the distance between cylinders then the square of the radius of gyration of the variable mass is $\beta^2 = \beta_0^2 + q^2$. If but one disc support be in use then $\beta^2 = \beta_0^2$. In the pendulum employed β_0^2 could be given three values, in the neighborhood of 10, the value of q could be changed through a wide range, its maximum value being about 44 cm., giving $\beta^2 = 1,940 \pm$. A chronograph was available for recording the periods.

The results of four typical experiments are given, illustrating the four curves of Fig. 7. On page 291, Vol. XXV., a table is given showing the combinations of the constants of the pendulum necessary to produce the four cases. The following table gives the values selected in order to fulfil these conditions:

	L	b	d	a^2	β^2
I.	60	50.75	13.25	961.90	7.403
II.	60	17.75	24.85	600.22	1,933.8
III.	60	27.35	20.00	829.00	7.403
IV.	56	56.30	14.00	829.00	11.57

Below are given the data obtained, and in parenthesis immediately after, the corresponding value computed by means of the accompanying theoretical equations.

I.

$$t_1^2 = \frac{0.138895(R + 19.3462)}{(R + 3.8302)},$$

$$t_2^2 = \frac{0.4721(R + 0.4773)}{(R + 0.19786)}.$$

R	t_1	t_2
0.0	0.8376 (0.8374)	1.0677 (1.0673)
0.067	0.8317 (0.8316)	0.986 (0.984)
0.135	0.8262 (0.8258)	0.932 (0.935)
0.202	0.8205 (0.8203)	0.8955 (0.8956)
0.268	0.8154 (0.8151)	0.8686 (0.8690)
0.335	0.8103 (0.8106)	0.8480 (0.8480)
0.402	0.8051 (0.8048)	0.8316 (0.8319)
0.468	0.8012 (0.7999)	0.8185 (0.8186)
0.533	0.7954 (0.7954)	0.8063 (0.8077)
0.598	0.7910 (0.7907)	0.7986 (0.7974)

R	t_1	t_2
0.663	0.7865 (0.7863)	0.7906 (0.7904)
0.729	0.7824 (0.7812)	0.7838 (0.7838)
0.793	0.7780 (0.7778)	0.7780 (0.7779)
0.861	0.7739 (0.7735)	0.7728 (0.7725)
0.927	0.7698 (0.7694)	0.7681 (0.7677)
0.993	0.7659 (0.7653)	0.7639 (0.7634)

II.

$$t_1^2 = \frac{1.03285(R + 0.35819)}{(R + 0.71268)},$$

$$t_2^2 = \frac{0.907066(R + 0.75368)}{(R + 1.20313)}.$$

R	t_1	t_2
0.0	0.7205 (0.7205)	0.7538 (0.7538)
0.358	0.8309 (0.8310)	0.8041 (0.8037)
0.716	0.8802 (0.8812)	0.8344 (0.8335)
1.075	0.9102 (0.9100)	0.8541 (0.8533)
1.433	0.9292 (0.9286)	0.8686 (0.8674)
1.792	0.9432 (0.9417)	0.8793 (0.8781)
2.151	0.9527 (0.9513)	0.8876 (0.8863)
2.509	0.9609 (0.9588)	0.8941 (0.8929)
2.867	0.9671 (0.9647)	0.8997 (0.8983)
3.224	0.9724 (0.9695)	0.9042 (0.9017)
3.580	0.9767 (0.9734)	0.9072 (0.9066)

III.

$$t_1^2 = \frac{0.204966(3.8722 + R)}{(1.368 - R)},$$

$$t_2^2 = \frac{0.80611(0.2958 + R)}{(0.408 + R)}.$$

R	t_1	t_2
0.0	0.7618 (0.7617)	0.7644 (0.7644)
0.065	0.7870 (0.7870)	0.7843 (0.7841)
0.130	0.8128 (0.8140)	0.7965 (0.7988)
0.196	0.8433 (0.8436)	0.8105 (0.8102)
0.261	0.8748 (0.8742)	0.8199 (0.8191)
0.326	0.9091 (0.9087)	0.8285 (0.8264)
0.392	0.9464 (0.9463)	0.8333 (0.8325)
0.458	0.9876 (0.9875)	0.8386 (0.8377)
0.524	1.0337 (1.0333)	0.8431 (0.8421)
0.591	1.0852 (1.0851)	0.8469 (0.8459)
0.658	1.1439 (1.1436)	0.8503 (0.8493)
0.724	1.2106 (1.2095)	0.8531 (0.8522)
0.792	1.2884 (1.2883)	0.8560 (0.8548)
0.859	1.3806 (1.3803)	0.8586 (0.8572)
0.926	1.4929 (1.4917)	0.8606 (0.8593)

IV.

$$t_1^2 = \frac{0.14899(R + 19.579)}{(R + 4.0214)},$$

$$t_2^2 = \frac{0.42524(R + 0.5008)}{(R - 0.007)}.$$

R	t_1	t_2
0.0	0.8518 (0.8517)	c above k
0.262	0.8307 (0.8307)	1.1259 (1.1278)
0.525	0.8117 (0.8117)	0.9173 (0.9176)
0.788	0.7943 (0.7943)	0.8379 (0.8377)
1.05	0.7785 (0.7785)	0.7972 (0.7951)
1.31	0.7640 (0.7640)	0.7690 (0.7687)
1.57	0.7507 (0.7508)	0.7510 (0.7503)
1.83	0.7384 (0.7383)	0.7367 (0.7370)
2.09	0.7268 (0.7268)	0.7282 (0.7270)
2.35	0.7157 (0.7161)	0.7197 (0.7190)

The condition indicated by the "third" intersection in Division I. is realizable in Division III. whenever it is possible by varying R , to throw the centroid of the whole pendulum midway between the knife-edges. This value of R , from equation (9), is given by the expression

$$R = -\frac{(L - 2b)}{(L - 2a)}. \quad (64)$$

To make R positive in this equation, it is further necessary that $b > L/2 > d$, or that $b < L/2 < d$. The verification of theory by experiment in the above tables is all that could be asked for. Figure 16 shows the graphs corresponding to the above four cases.

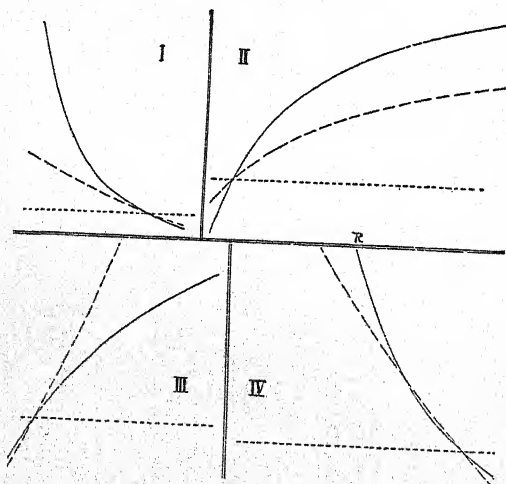


Fig. 16.

In a third paper it is proposed to discuss Best Values and to give a short critique of Kater's classic experiments.

September 1, 1911.

PROCEEDINGS
OF THE
AMERICAN PHYSICAL SOCIETY.

THE PRIMARY CONCEPTS OF PHYSICS.¹

BY WILLIAM FRANCIS MAGIE.

THE subject of the present address is one that does not often appear on a scientific programme. Physicists are so busy in enlarging the structure of knowledge that few of them concern themselves with the consideration of the fundamental concepts of the science. Yet it is plainly true that if those fundamental or primary concepts are not clearly apprehended, or if there is doubt as to what they are, the whole structure of the science rests on an insecure basis. I propose to examine certain questions concerning these primary concepts, about which there has been and is much unsettled opinion. The discussion necessarily rests upon my own beliefs about them. In the nature of the case each man can speak positively about them for himself only. It would be very improper to dogmatize, and I shall accordingly have to crave your pardon for a frequent expression of my own opinion, believing it less objectionable to be egotistic than to be dogmatic.

The first question which I shall consider is that raised by the advocates of the dynamical definition of force, as to the order in which the concepts of force and mass come in thought when one is constructing the science of mechanics, or in other words, whether force or mass is the primary concept. It will be of service in the discussion if we consider briefly the way in which some of the great builders of the science of mechanics used these concepts.

There is no need of presenting the views of Archimedes or of Stevinus, whose work was exclusively in statics and who used the concept of force given us by our muscular or motor sense, and measured forces by weights. The views of Galileo, however, are interesting as showing how far one can go in dynamics without using the concept of mass.

Galileo examined the problem of the motion of a body acted on by a constant force. The only constant force of which he could dispose was the weight of a body, or a component of its weight, and he accordingly was limited in his studies to the examination of the laws of falling bodies. Owing to the relation of proportionality between the weight of a body

¹ Presidential address delivered before the American Physical Society at Washington, D. C., December 28, 1911.

and its mass, this limitation in a way simplified the problem, while at the same time it made it more difficult to develop a complete doctrine of force and motion. By the famous experiment at the Leaning Tower Galileo satisfied himself that he could study any falling body as a type, and that the conclusions which he would reach from that study would apply to all. His attention was therefore directed almost wholly to the consideration of the motion of the falling body, while the question of the relation between the motion and the weight of the body was disregarded. The result of this was that he developed the laws of linear motion with constant acceleration, and numerous consequences of those laws, chiefly relating to motion down inclined planes, with really wonderful completeness, and was led in the course of his thought to a full appreciation and statement of the principle of inertia, while yet he did not, in this part of his work, attain to any useful conception of the relation of force to mass. He makes it clear that the conception of force which is sufficient for his purposes is that with which he was familiar from his study of statics. He says, in speaking about the "tendency" of a body to fall down inclined planes of the same height, that "It is clear that the tendency of a body to fall is as great as the resistance or the least force which suffices to prevent its falling and to keep the body at rest." In fact Galileo thought of the weight of a body, with which he was familiar from common experience, as a force which moved the body, and assuming that the weight was unchanged during motion his experiments demonstrated what kind of motion such a constant force will set up and maintain.

In the very interesting discussion which Galileo gives of the forces exerted by the collision of one body against another, he approaches nearer than in other parts of his discourse to an appreciation of mass as a characteristic of a moving body. He speaks in one place of the falling body being a composite of weight and velocity, and his discussion of the impulse applied by such a falling body to another on which it falls shows that he was very near the concept of momentum; but there is no real precision in his statements.

We now turn to Newton to get the full doctrine of the relations of force and motion. It will be clear to anyone who examines the introductory parts of the *Principia*, that Newton did not undertake in that book to present a systematic treatise on dynamics. He merely blocks out a rough set of definitions and postulates, in a very uncritical way, which are sufficient to enable him to go on as promptly as possible to the real task which was before him. A striking instance of this uncritical attitude of mind is found in Definition I., in which he says "Quantity of matter is its measure derived from its density and volume jointly." This quantity of matter thus defined he names mass. Since we can only define density in terms of the concept of mass it is surely uncritical to define mass in terms of density. In fact Newton on a later page uses the true definition when he says that bodies are of the same density, if their *vires inertiae* (that is, their masses) are proportional to their volumes.

The same sort of uncritical treatment appears in his presentation

of the various types of force. He mentions first the *vis insita*, which he defines as the power of resisting, by which a body persists in its state of rest or of uniform motion. He says it differs in no respect, except in the way of conceiving of it, from the inertia of a mass.

Then comes *vis impressa*, the action (*actio*) exerted on a body to change its state of rest or of uniform motion. This is force in our ordinary sense. Newton says that it arises from a blow, from pressures or from centripetal force.

Vis centripeta is the force by which bodies are drawn or impelled from all directions towards any point as a center, or tend toward it in any way whatever. The force of gravitation and magnetic force are examples of this centripetal force. So also is the force by which a sling draws a stone in it toward the hand, which force Newton explains as arising from the stretching of the cord of the sling.

Newton then goes on to define the various measures or modes of giving quantitative expressions for centripetal forces. He first describes the *vis centripetæ quantitas absoluta* as the measure of it as greater or less by comparison with the efficiency of the cause which transmits it from the center through the surrounding region. Thus the magnetic force is greater in one magnet than in another, either because of the greater mass of the one, or of the intensity of its power.

The *vis centripetæ quantitas acceleratrix* is the measure of it as proportional to the velocity, which it generates in a given time. Thus the power (*virtus*) of a magnet is greater at lesser distances, and less at greater distances; gravitating force is greater in valleys, less on mountain peaks, and less still at greater distances from the earth. At equal distances, he says, this gravitating *vis acceleratrix* is the same everywhere, because all falling bodies are equally accelerated.

The *vis centripetæ quantitas motrix* is the measure of it as proportional to the momentum which it generates in a given time. This quantity is the center-seeking or tendency to the center of the whole body, and (as Newton says, with an evident appreciation that he is limiting the generality of his conception) is the weight of the body. It is always known by the force opposite to it, and equal to it, by which the fall of the body can be prevented.

Newton calls these quantities of force of the various sorts described by the shorter terms motive, accelerative, and absolute forces, that is, he substitutes the general terms for the measured quantities of the forces which can be conceived only in those general terms. With this understanding he states that the *vis acceleratrix* is to the *vis motrix* as velocity is to momentum; for the quantity of motion (*momentum*) arises from the velocity and quantity of matter, and the *vis motrix* arises from the *vis acceleratrix* and the quantity of matter. For the sum of the actions of the *vis acceleratrix* upon the several particles of a body is the *vis motrix* of the whole body. Newton relates the *vis motrix* to a body as a striving of the whole body towards the center, made up of the strivings of all its parts; the *vis acceleratrix* to the position of the body, as a certain efficiency, diffused from the center through all places

around it, for moving bodies which are in those places. The *vis acceleratrix* as thus described reminds us of the conception of the strength of a field of force.

This analysis of the concept of force surely does not promote a clear apprehension of it. The only one of the distinctions which have been made which seems to be worth retaining is that between the *vis impressa* or action, and the *vis motrix impressa*, the one being force in its general or conceptual sense, the other the same force when given a measure or value. This distinction was clearly in Newton's mind and appears in the enunciation of the Laws of Motion. In the First Law the departure of a body from its state of rest or of uniform motion is ascribed to the *vis impressa*; that is, to force in general, without any specification as to its measure or even any declaration that it can be measured. In the Second Law the change of momentum is said to be proportional to the *vis motrix impressa*; that is, to force that is measured so that a proportionality to something else can be predicated of it. As has already been stated, Newton declared of this *vis motrix* in the special case of gravitation that it is known, or measured, by the force opposite to it and equal to it, by which the fall of the body, or, in the general case, the motion of the body, can be prevented. In the Third Law the force is called *actio*. This is the alternative word used in the definition of the *vis impressa*, as an equivalent for force in its general sense. The word in this sense is consistently used in the enunciation of the Third Law, in which forces are not considered as measured, but merely as compared by the condition of equality. From the examples of action and reaction which Newton gives (the finger pressed against a stone; the horse drawing a stone by a rope and drawn back equally toward the stone, because of the stretching of the rope and its exertion of equal forces at its two ends) it is plain that Newton conceived of forces in the way which is familiar to all of us, as the pushes and pulls which can be perceived by our motor sense, and as the causes of motions. He goes on to say that by these *actions* there are caused equal changes, not of velocity, but of momentum, so that the changes of velocity are inversely as the bodies (*corporibus*). In this way, without measuring forces, there is introduced the method of comparing masses.

It is difficult to perceive in these many definitions and declarations exactly what Newton's conception was of force, of the unit in which it is measured and of its relation to mass. After careful consideration of all that I can find in the *Principia* bearing on the question I am convinced that Newton viewed the concept of force as a primary one, or one directly given by intuition, and that he thought of the motions of bodies caused by these forces as connected quantitatively with them by the experimental relation embodied in the Second Law. Since Newton does not use a system of units, and states most of his laws and theorems in terms of proportions, the priority of force to mass, in the order of their apprehension, is not clearly presented.

In the matter of measuring a force he clearly asserts that a *vis motrix* is measured by the force which will counteract it and keep the body to

which it is applied at rest, and the force thus used can hardly be other than a force measured statically; but his frequent insistence on the measure of *vis motrix* by the momentum which it causes shows that he had a conception also of the dynamical measure of force. He further supplies the measurement of mass as a fundamental quantity which is needed to establish the dynamical measure of force by calling attention to the possibility of comparing masses by means of the velocities given them when acted upon by equal forces.

Lagrange in the *Mécanique Analytique* gives the most explicit expression to the definition of force in general which is the bug-bear of so many thinkers, and which yet, after all, is the real expression of our belief about force, when he says: "We understand by force the cause, whatever it may be, which impresses or tends to impress a motion on a body to which we suppose it applied." He goes on to say: "It should be measured by the quantity of motion impressed or ready to be impressed. In the condition of equilibrium, the force produces no actual effect; it produces only a simple tendency to motion; but it should be measured by the effect which it would produce if it were not restrained from acting." Lagrange repeats this definition of the measure of force in the introduction to his *Dynamics*, when he says that the product of the mass and the accelerating force (Newton's *vis acceleratrix*) or the acceleration, expresses the motive force (Newton's *vis motrix*). I cannot find that Lagrange gives any definition of mass. From a statement in his treatment of centers of gravity it would seem that he considered the mass to be determined by its weight. He seems to endeavor to measure force in the purely dynamical way, without going into the matter as fully as he should for a complete elucidation of it.

Thomson and Tait say flatly that force is a direct object of sense, and define it as any cause which tends to alter a body's natural state of rest, or of uniform motion in a straight line. They assert that the measure of force is the quantity of motion which it produces per unit of time. They give no other definition of mass than the one given by Newton.

From the account which has been given of the views held or expressed by some of the great leaders of thought in matters of dynamics it is clear that very indefinite notions existed in their minds with respect not only to the proper definition of force, but even with respect to the proper measure of force, which is fundamental and necessary in the development of dynamics. The acute and valuable criticism by Mach of this fundamental notion is so colored in its expression by Mach's favorite principle of economy that it is not altogether satisfactory, and I accordingly shall attempt to present what seems to me the proper order of thought on this matter. Similar statements have been many times made, but there is still no general consent in the minds of physicists as to the statement which should be acceptable to everyone.

There is no doubt that the dynamical *measure* of force is the correct one to use in building up a system of units. The point of difference on which dispute arises is the order of precedence of the two concepts force and mass in the establishment of this definition. It is not uncommon to

have force defined as the product of mass by acceleration, or of mass by the acceleration which the mass would have if it were free to move. In this definition mass is the primary concept. Now, as I view the question, force is the primary concept, a direct object of sense, and we know it to be a cause of motion, or of the distortion of a body to which it is applied and which counteracts it when the distortion has reached a certain limit. In particular we know it as counteracting, or as being counteracted by, the weight of a body. This conception of force is adequate for the development of statics, in which we treat the principles of statics as statements of laws which are derived from experiment and confirmed by the proof that they are mutually consistent. Galileo's experiments on falling bodies are then the direct experimental proof in a limited case of the proportionality between the force which acts on a body, measured at any one place by a weight, and the acceleration imparted to the body. Newton's Second Law is a statement of Galileo's discovery, with this addition, that the acceleration imparted by a force is not the same for all bodies, but depends upon a certain characteristic of the body. This characteristic, the mass of the body, first calls for recognition at this point. In the view I have taken the mass is the factor of proportion between the force which acts on a body and the acceleration which it imparts to the body. Since we can measure forces by comparison with a standard force, we can also measure masses by the aid of properly instituted experiments. Whether we measure masses in this way or not, and it turns out to be not a satisfactory way to do it, we at least get from this relation between force, of which we have a concept, and motion, of which we have a concept, an adequate working concept of mass. Force is the primary concept and mass is a derived concept.

Now owing to the permanency of masses of matter it is convenient to construct our system of units with a mass as one of the fundamental units. We are able to do this and to compare one mass with another chosen as standard, without going through the operation of measuring forces, by utilizing the principle embodied in Newton's Third Law. This law asserts that bodies which interact, that is, which exert forces on each other, exert equal forces, and thus if the bodies are free to move, their accelerations will be inversely as their masses. By observation of the accelerations of two mutually interacting bodies we may thus compare their masses, and so construct a set or scale of masses, and use these masses and their accelerations to measure forces. Thus while the concept of force is primary in the order of thought, we may make the unit of mass fundamental in the development of a system of units.

The point upon which I wish to insist is that both reason and the history of mechanics show that the foundation of the science is the purely intuitional concept of force which is shared by every intelligent being, and that this intuitional concept is not only accurate so far as it goes but adequate to serve as the foundation of a great science. No use of the concept of force in the theories of physics has ever violated in any particular this original and intuitional concept of it. Even the brilliant endeavor of Hertz to found all the principles of dynamics upon the three

concepts of time, space and mass, cannot escape the criticism that the concept of mass is meaningless to us unless it is given to us by our experience of the inertia of matter when we exert force upon it. Once that concept is attained it may be used, as Hertz so beautifully used it, in the logical upbuilding of a system of dynamics. Perhaps my contention will be made clearer if we consider briefly the question whether it would be possible for us to construct our present system of dynamics if we were disembodied spirits, gifted with the means of observing spaces, times, and colors, but without the sensation of force. We could see colored volumes, sometimes moving with constant velocity, sometimes with varying velocity, and we could ascribe the changing velocity to the action of a force. We further could connect the force with the moving volume by setting it equal to the acceleration multiplied by some factor which we might name the mass. This equation would contain two unknown and unmeasured quantities, and would mean nothing unless we could go further. Now the advocates of the purely dynamical definition of the concept of force say that we can go further, by observing the mutual accelerations of two bodies and using these to obtain the ratio of their masses. If this can be done the matter is settled. But could it be done by the disembodied spirit? In our use of the mutual accelerations of two bodies to get their masses we must explicitly state that the bodies are arranged so as to interact (that is, to exert force on each other), and unless that condition is established the mutual accelerations of two bodies, however often repeated, can tell us nothing about their masses. A man at a station might observe two trains leaving the station in opposite directions with the same accelerations every day for ten years, and yet he could not compare their masses by any such observations. Eyes and mind only will not do it. To get the measure of mass we must start with the intuitional knowledge of force, and use it in the experiments by which we first define and then measure mass.

I now come to a much more difficult part of my subject, the consideration of the other primary concepts of space and time. Not many years ago we should have been willing to pass them over with a mere mention, admitting the impossibility of giving a definition or even an intelligible description of either of them, admitting the impossibility of determining an absolute or fixed point in space, or an absolute instant of time, but still asserting that we knew something about them both of which we were sure. At present we are driven by the development of the principle of relativity to examine anew the foundations of our thought in respect to these two primary concepts.

I suppose that the old ideas about space and time that have been of service to physicists since the beginning of the science are summed up as well as anywhere in Newton's words:

"Absolute and real time, the time of the mathematician, flows on equably, having no relation in itself or its nature to any external object. It is also called duration. Relative, apparent time, the time of common life, is an external measure of any duration cognized by the senses, by means of motion. It is commonly used in place of real time."

"Absolute space, having no relation in its nature to any external object, always remains alike everywhere and immovable. Relative space is the measure of this space, or any movable dimension, recognized by our senses as limited by its situation with respect to bodies. This is commonly thought of as equivalent to absolute space."

These definitions have been often justly criticized for the emphasis laid on the unfruitful ideas of absolute time and space. Perhaps the criticism has fallen rather upon Newton's subsequent expansion of his thought on these ideas. But do they not contain in the first place the conceptions of time and space which have been uniquely useful up to this time in physics, and in the second place, do they not contain what each one of us really thinks about time and space when he makes an honest examination of his knowledge? The essential feature of both these descriptions for our present purpose is Newton's declaration, both as to time and space, considered as species and not as magnitudes, that they are in themselves and in their nature without relation to any external object. It is this statement which is contradicted by some of the enunciations of the principle of relativity.

It is not necessary for me to give an account of the genesis of the principle of relativity. It may fairly be said to be based on the necessity of explaining the negative result of the famous experiment of Michelson and Morley, and on the convenience of being able to apply Maxwell's equations of the electromagnetic field without change of form to a system referred to moving axes. It is not needed to explain many of the remarkable results obtained by Fizeau, by Mascart, and by Brace, in the field of experimental optics, which to a first inspection seem to show that the earth and the medium around it through which light passes are relatively at rest, but which a closer study by Lorentz and others shows may be compatible with a reasonable theory of the structure of matter and the hypothesis that the luminiferous medium is at rest. It is also not needed to explain the dependence of the path of an electron in a field of crossed electric and magnetic forces upon its velocity, as exhibited in the beautiful experiments of Kauffmann and of Bucherer, for other theories in which the principle is not used lead to expressions for the path, which, for the present at least, are in as good accord with observation as those which are deduced by the aid of the principle of relativity.

There are two ways of presenting the principle of relativity. In the first way the principle is stated as a direct inductive conclusion from the experiment of Michelson and Morley, and asserts that so far as a conclusion can be drawn from that experiment and the others which have been tried to test the matter, there is no way by which the relative motion of the earth and the luminiferous medium can be determined from observations made on the passage of light when the source of light and the observer are moving with the earth. As thus presented the principle holds out as the object of future study the construction of a suitable theory of the structure of matter and of the luminiferous medium to account for this fundamental experiment as well as for all other known truths in the domains of light and electricity. If this theory is expressed

in terms of the Lorentz transformation, and thus shows a dependence of the measure of time and the measure of length upon the velocity of the system in which the observer is placed, it will further be the object of inquiry to construct a theory of the relations between the material of the system and the luminiferous medium which will account for the change in the units of length and in the motions of bodies by which the unit of time is determined. When I say to account for, I mean to describe in terms of force, time, and space, as we conceive those notions in our everyday experience, and as we use them in our ordinary physical work, so that the description when apprehended will be reduced to the lowest terms in which our thought about the universe can be expressed. Such a description is, as I view it, a real explanation, and surely it is not yet time to say that such an explanation is impossible.

The other way of presenting the principle of relativity consists in laying down as a fundamental postulate a general proposition expressing the hopelessness of any attempt to settle the question raised by the experiment of Michelson and Morley by any theory of the structure of the universe. This postulate sometimes assumes a formidable aspect, and involves more than the mere postulate of relativity. Thus Laue says: "The principle of relativity asserts that from the totality of natural phenomena we may, with continually increasing approximation, determine a system of reference, x, y, z, t , in which the laws of nature hold in a definite and mathematically simple form. This system of reference is by no means uniquely determined by the phenomena. There is rather a triple infinity of equally admissible systems, which move relatively to one another with uniform velocities."

The feature of this enunciation of the principle to which I referred as an addition to the principle is the expressed condition that in the system of reference the laws of nature hold in a definite and mathematically simple form. There is no warrant in the past history of physics for the adoption of such a postulate as that. Surely the history of the discovery of the so-called secondary laws of physics, such as Boyle's law, the laws of friction, the laws of polarization and of absorption of light, the laws of magnetization, and many others, will bear out the statement that in very many cases the first enunciation of the law is in a definite and mathematically simple form, and that further knowledge shows that this form is only a first approximation to the truth. Even in the case of such laws as the law of gravitation, or of electrical attraction and repulsion, from which we have not yet detected any deviation, does anyone dare to say that they are universally true for all bodies and at all distances? Can we even feel sure that Maxwell's electromagnetic equations hold true with absolute exactness? They need supplementing when they are applied to material bodies. Can we be sure that they hold without modification, in rapidly moving bodies, or at extremely minute distances in free space? Or, from another point of view, admitting that the object of physical study is to reduce the description of natural phenomena to a set of simple laws, have we a right to assume that, in our analysis of the structure of matter and of the luminiferous medium, we have as yet

reached the ultimate model in which such simple laws will be operative? The answer to this question must be a negative one. Yet it is surely true that if it were not for this demand of simplicity, immediately attainable and at present expressed in the electromagnetic equations, the chief incentive to the development of the theory of relativity would be wanting.

But this is not the heart of the matter. With the principle of relativity as a basal postulate, not expressing our present inability but rather the hopelessness of any attempt to obtain ability, a complete description has been given of the phenomena now known to physicists, at least in the domains of mechanics, light and electricity. The difficulty which I find in accepting the principle, with the universality that is predicated of it, is that it does so much more than this.

The theories of J. J. Thomson and of Lorentz made physicists familiar with the notion of electrical mass, exhibited by the variability of the mass of a moving charged body, or by the apparent variable inertia of a moving charge expressed as a function of its velocity, and further with the notion that as the velocity of the charge approaches the velocity of light, the magnitude of the electrical mass approaches infinity, so that the velocity of an electrical charge, of an electron, and therefore presumably of matter, if it is entirely electrical in its structure, can never surpass the velocity of light. In these theories this remarkable conclusion was explained by the interaction between the moving charge and the ether. In the theory of relativity the same conclusion is reached as the consequence of a purely kinematical theorem, giving the rule for the addition of velocities, and not only does it hold for real moving charges, but for any action whatever which is conceivably transmitted through space. In particular this finite velocity of transmission must be ascribed to gravitational action. Now the Newtonian theory of gravitation assumes a practically infinite velocity of transmission of gravitational action, and astronomical observations have never given any warrant for the belief that its velocity of transmission is even of the order of magnitude of the velocity of light. The attempt has been made to reconcile the theory of relativity with the observed motions of the planets by the adoption of an arbitrarily chosen term in the formula for the force on a planet to represent what is equivalent to a counteracting force to annul the tangential acceleration which would arise from the finite rate of transmission of gravitational force. This is manifestly an artifice and not an explanation. If the principle of relativity is of universal application, it should not need the introduction of such an artifice to help it out in the solution of one of the classical problems of physics.

Furthermore, the principle of relativity in this metaphysical form professes to be able to abandon the hypothesis of an ether. All the necessary descriptions of the crucial experiments in optics and electricity by which the theories of the universe are now being tested can be given without the use of that hypothesis. Indeed the principle asserts our inability even to determine any one frame of reference that can be distinguished from another, or, what means the same thing, to detect any

relative motion of the earth and the ether, and so to ascribe to the ether any sort of motion; from which it is concluded that the philosophical course is to abandon the concept of the ether altogether. This question will be amply and ably discussed this morning, but I may venture to say that in my opinion the abandonment of the hypothesis of an ether at the present time is a great and serious retrograde step in the development of speculative physics. The principle of relativity accounts for the negative result of the experiment of Michelson and Morley, but without an ether how do we account for the interference phenomena which made that experiment possible? There are only two ways yet thought of to account for the passage of light through space. Are the supporters of the theory of relativity going to return to the corpuscles of Newton? Are they willing to explain the colors of thin plates by invoking "the fits of easy reflection and of easy transmission"? Are they satisfied to say about diffraction that the corpuscles near an obstacle "move backwards and forwards with a motion like that of an eel"? How are they going to explain the plain facts of optics? Presumably they are postponing this necessary business until the consequences of the principle of relativity have been worked out. Perhaps there is some other conceivable mode of connection between bodies, by means of which periodic disturbances can be transmitted. We may imagine a sort of tentacular ether stretching like strings from electron to electron, serving as physical lines of force, and transmitting waves as a vibrating string does. Such a luminiferous medium would not meet the postulate of simplicity, but it conceivably might work. But whatever the properties of the medium may be, there is choice only between corpuscles and a medium, and I submit that it is incumbent upon the advocates of the new views to propose and develop an explanation of the transmission of light and of the phenomena which have been interpreted for so long as demonstrating its periodicity. Otherwise they are asking us to abandon what has furnished a sound basis for the interpretation of phenomena and for constructive work in order to preserve the universality of a metaphysical postulate.

The electromagnetic equations, too, the retention of which in their present simple form is the *sine qua non* of the promoters of the principle of relativity, were not only developed by the conscious use of the hypothesis of a medium in which the electric and magnetic forces exist, but can be interpreted intelligibly only in terms of some such medium. The abandonment of this hypothesis reminds one of Baron Munchausen's feat performed while he was making his escape from prison. Since your historical reading may not have extended to the autobiography of this famous man, I may be permitted to relate that the Baron was letting himself down from the windows of a high town by a rope, and when he reached the end of it he found that he still had a long distance to go. The last part of the descent was particularly difficult, so to get rope enough he ingeniously spliced on an additional piece, which he obtained by cutting off the part above him.

The principle of relativity in its metaphysical form ignores the ac-

celerations of bodies. It is true that the experimental results to which the principle has been applied with such success are such that the study of acceleration in terms of the theory of relativity has not become necessary. But is it not reasonable to suppose that when suitable experiments have been invented and tried to test the effect of the acceleration of a system on the progress of light in it, it may be found that an effect can be detected? Some effect may be detected, for example, due to the rotation of a body. I have never been able to perceive any sound objection to Newton's assertion that we have evidence of absolute rotation by the observation of centrifugal force, and if a fixed direction of an axis and an absolute velocity of rotation can be determined in a mechanical system when accelerations are taken into consideration, why should the principle of relativity be treated as having universal validity?

But, after all, these questions raised by the development of the principle of relativity are of secondary importance. The central question is whether or not this principle can ever furnish a satisfactory explanation of natural phenomena. The formulas derived from it are evidently merely descriptive. This may be said with truth about all the formulas in which the general theories of physics have been embodied. Kirchhoff designates as the task of the science of mechanics, the description of the motions which occur in nature completely and in the simplest possible way. This assertion that the task of the theoretical physicist is done when he has reduced the phenomena with which he is dealing to a set of formulas, or, as we may say, when he has constructed an ideal model which will produce the phenomena, is one to which we would all assent in general. At the same time most of us would reserve the right to criticize each model thus presented, and to give to one or the other a preference based on considerations which are not necessarily limited to the simplicity of the model or to the completeness with which it reproduces the phenomena. Surely an additional test of the value of the model will be the intelligibility of the elements of which it is composed.

This last test has been generally met in the models which have been proposed as descriptions of natural phenomena. We can understand from what we see and feel what is meant by the motions of elastic spheres, and the model which uses them to represent the behavior of a gas is not only competent to reproduce the behavior of a gas but is intelligible in the elements of which it is composed. The model of the elastic solid ether, incomplete and objectionable as it became when the subject of optics was enlarged and developed, was intelligible in its elements. The model of electromagnetic operations embodied in Maxwell's formulas is also one which is thus intelligible in its elements. When I say this I do not mean that we know all about electric and magnetic forces, but I mean that we do know enough about such forces to have a clear notion of their variation in space and their variation in time.

This feature of the ideal model or description seems to me to be necessary in order to make the model acceptable as the ultimate or last attainable explanation of phenomena. The elements of which the model is constructed must be of types which are immediately perceived by the

senses and which are accepted by everybody as the ultimate data of consciousness. It is only out of such elements that an explanation, in distinction from a mere barren set of formulas, can be constructed. A description of phenomena in terms of four dimensions in space would be unsatisfactory to me as an explanation because by no stretch of my imagination can I make myself believe in the reality of a fourth dimension. The description of phenomena in terms of a time which is a function of the velocity of the body on which I reside will be, I fear, equally unsatisfactory to me because, try I ever so hard, I cannot make myself realize that such a time is conceivable.

Tried by this test I feel that the principle of relativity does not speak the final word in the discussion about the structure of the universe. The formulas which flow from it may be in complete accord with all discovered truth, but they are expressed in terms which themselves are not in harmony with my ultimate notions about space and time. That this is true is so evident that it is generally admitted. Some writers say that we should not let this circumstance disturb us, because Kant has said that time and space are mere forms of perception, a scheme in which we must arrange occurrences so that they may acquire objective significance. I do not altogether understand what Kant meant by this, but I am sure he did not mean that by the exercise of our wills we can violently eject from our consciousness the notions of space and time which we have in common with the whole race of man, and impose on ourselves other and radically different notions. Planck compares our position before the new notions presented by the theory of relativity to the position of the medieval peoples before the notion of the antipodes. It seems to me that there is no real similarity between the two positions. Many men in the Middle Ages believed that there were no antipodes, but their belief was based on reasons, and so far were they from being unable to conceive of antipodes and to believe in their existence, that there were men who actually maintained their existence, and were pursued therefor as heretics. I do not believe that there is any man now living who can assert with truth that he can conceive a time which is a function of velocity or is willing to go to the stake for the conviction that his "now" is another man's "future" and still another man's "past."

One of the members of this Society, recognizing our present inability to conceive of relative time, and conceiving our intuitions of space and time to be the result of heredity operating through many generations of men who lacked the light of relativity, once proposed to me that everyone who could get even a glimmer of the notion of relative time should persistently exercise his mind therein and teach it to his students, in the hope that in a few generations the notion would emerge with the force of an intuition. It would not be fair to leave the impression that he was solemnly serious when he made this suggestion. When Matthew Arnold was asked to endure the transliteration of Greek names into English in order that the new forms might become familiar to future scholars, he answered that he was not willing to spend his days in a wilderness of pedantry that his children might enjoy an orthographical Canaan; and

mutatis mutandis the same answer may be given in this case. But a more pertinent answer is, to my mind, this, that the attempt to reorganize the perceptions of the human mind in respect to space and time is doomed to failure. "Which of you by taking thought can add one cubit unto his stature?" I believe that these ultimate perceptions are the same for all men now, have been the same for all men in the past, and will be the same for all men in the future. I believe further that this is true because the universe has a real existence apart from our perceptions of it, and that through its relations to our minds it imposes upon us certain common elementary notions which are true and shared by everybody.

Therefore, from my point of view, I cannot see in the principle of relativity the ultimate solution of the problem of the universe. A solution to be really serviceable must be intelligible to everybody, to the common man as well as to the trained scholar. All previous physical theories have been thus intelligible. Can we venture to believe that the new space and time introduced by the principle of relativity are either thus intelligible now or will become so hereafter? A theory becomes intelligible when it is expressed in terms of the primary concepts of force, space, and time, as they are understood by the whole race of man. When a physical law is expressed in terms of those concepts we feel that we have a reason for it, we rest intellectually satisfied on the ultimate basis of immediate knowledge. Have we not a right to ask of those leaders of thought to whom we owe the development of the theory of relativity, that they recognize the limited and partial applicability of that theory and its inability to describe the universe in intelligible terms, and to exhort them to pursue their brilliant course until they succeed in explaining the principle of relativity by reducing it to a mode of action expressed in terms of the primary concepts of physics?

THE KERR ROTATION FOR TRANSVERSE MAGNETIC FIELDS.¹

BY L. R. INGERSOLL.

THE older magneto-optic theory did not anticipate any effect of a magnetic field on light reflected from the surface of a magnetic metal, if the lines of force were normal to the plane of incidence. Some years ago, however, C. H. Wind predicted that an effect should exist for this case although it would be very small, and Zeeman was able to observe it for the most favorable case, viz., steel at 75 degrees incidence.

The author has recently obtained some results by the bolometric method (see *Astrophys. Jour.*, Nov., 1910) which seem to bear out Wind's theory to a considerable extent. The effect observed is a slight change in the azimuth of "restored polarization" of the reflected light, due to a magnetic field which is parallel to the surface and perpendicular to the plane of incidence of the light. The maximum value of this change is about 17 minutes of arc for steel, and smaller for the other metals. Both steel and cobalt give curves closely approximating Wind's computed

¹ Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

curves, the effect increasing from zero at normal incidence to a positive maximum at about 70 degrees, changing to a negative maximum, and becoming zero again at grazing incidence. Nickel on the other hand does not even approximate the theoretical curve but gives values opposite in sign to the other metals. This is similar to its action in the case of the Hall effect, and undoubtedly these two phenomena are related.

The author is also able to show that the only component of vibration effective in producing this effect is the one whose electric vector is in the plane of incidence. This is in agreement with theory.

A NEW TYPE OF APPARATUS FOR MEASURING LINEAR EXPANSION.¹

BY ARTHUR W. GRAY.

THE bar to be measured is supported horizontally in an electrically heated tubular furnace designed especially to secure uniformity of temperature. The tube, of metal and wound with resistance ribbon, has its central portion completely filled by a block of metal pierced by two longitudinal cavities, of square cross-section, symmetrically situated above and below the geometrical axis of the tube. The lower cavity is almost completely filled by the bar to be measured, while the upper one contains a similar dummy bar whose temperature is determined by an electric thermometer placed within a hole following its axis. Properly placed differential thermoelements indicate the distribution of temperature both radially and longitudinally. Heat losses through the ends are prevented by long plugs, each consisting of two thick, solid cylinders of metal united by a thin circumferential tube and separated by a thick layer of heat insulating material. An independent electrical heating coil wound in a groove within the outside cylinder of each plug permits the end to be maintained at a temperature almost the same as that of the interior, thus reducing the longitudinal temperature gradient and thereby the outward conduction of heat. The solid metallic cylinders prevent loss by radiation from the interior, and at the same time aid in producing uniformity of temperature perpendicular to the axis of the tube.

Changes in the length of the lower bar are measured by micrometer microscopes focused on fine wires freely suspended over the ends of the bar and stretched vertically by the weight of vanes immersed in oil, whose viscosity is adjusted to damp any swinging of the wires so that their motions will be almost, but not quite, aperiodic. Disturbances from changes in level are avoided by grinding the ends to form portions of a horizontal cylinder coaxial with the center of the bar. An unheated bar, likewise supporting suspended wires, forms a reference standard for determining any changes that might occur in the distance between the axes of the microscopes. The reference bar is placed below the furnace, the plane of its wires intersecting that of the test bar's wires in a vertical passing through the centers of both bars.

The microscopes are rigidly united by a horizontal slab of marble rotatable

¹ Abstract of a paper presented at the Washington meeting of the Physical Society, Dec. 27-30, 1911.

about the vertical containing the centers of both bars, and provided with the necessary adjustments for rapid focusing.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
December 9, 1911.

THE SPECTRUM OF TITANIUM IN A PARTIAL VACUUM AND THE
PROPORTIONALITY OF DISPLACEMENT TO PRESSURE AT
MODERATE PRESSURES.¹

BY WALTER S. ADAMS AND HENRY G. GALE.

IN an earlier communication we have called attention to the fact that the enhanced lines are not easily reversed, and have referred briefly to the possible bearing of this fact on the spectrum of the chromosphere. On some photographs taken at a pressure of about 10 centimeters we have found the enhanced lines considerably strengthened over what they are at atmospheric pressure. This fact may well account for the strength of the enhanced lines in the upper chromosphere, since the pressures are no doubt low in the higher levels of the solar atmosphere.

The shift of a selected list of 20 titanium lines has been measured at 16, 12, 8, 6, 4 and 2 atmospheres above normal pressure, and at a pressure of 10 centimeters.

The twenty lines selected were all between λ 4300 and λ 4600. They were all reversed and well adapted for measurement. We have used the mean displacement of the twenty lines at each pressure in investigating the linearity of the displacement-pressure law. If we assume a linear relationship and deduce the mean value by a least square solution the following residuals, expressed in Ångströms, are obtained for the observed minus the computed mean shift at each pressure.

P.	Obs.-Comp.	Shift per Atm.
2	+0.0005	0.0042
4	-0.0003	0.0040
6	+0.0003	0.0042
8	-0.0026	0.0038
12	+0.0004	0.0042
16	+0.0028	0.0043
		Mean 0.0041

Rossi gives 0.0045 for the average displacement per atmosphere between 15 and 100 atmospheres for 16 of these same lines. Two plates at a pressure less than 1 atmosphere, give a mean shift of 0.005 Ångströms per atmosphere.

The photographs were taken in the laboratory of the Mount Wilson Solar Observatory. The second order of a 7-inch Michelson grating was used in the 30-foot vertical spectrograph. We are indebted to Miss Lasby for many of the measurements.

¹Abstract of a paper presented at the Chicago meeting of the Physical Society, Dec. 2, 1911.

THE EFFECT OF TEMPERATURE ON THE ABSORBED CHARGE IN ELECTRIC CONDENSERS.¹

BY ANTHONY ZELENY.

THE values of the absorbed charge obtained from a paper condenser were determined at various temperatures between -20° and $+22^{\circ}$ C. The plotted curve shows a sudden change in curvature at and below 0° C., the average temperature coefficient above that point being 0.136, while below it is only 0.037. This change at the freezing point of water suggests that a large part of the absorbed charge may be due to the presence of moisture in the paraffin. The condenser used was made without boiling in vacuo. The cause of this sudden change in curvature will be investigated.

THE ELECTRICAL RESISTANCE AND THE POLARIZATION E.M.F. OF A MIXTURE OF CLAY, FELDSPAR AND QUARTZ.¹

BY A. A. SOMERVILLE AND O. E. BUCKLEY.

POTTERY samples containing English ball clay, feldspar and quartz were moulded in the form of rods about one cm. in diameter and three to five cm. in length and baked at a temperature of 1100° C. Electrical connections were made by passing nickel wires through holes near each end of a rod. Such a piece of pottery is an insulator at ordinary temperatures but becomes a conductor at higher temperatures. It does not follow the most simple form of Ohm's law, $I = E/R$, but due to something like a polarization effect this becomes a conductor to which the form of the equation, $I = (E - e)/R$, is applicable. e is eliminated and R is measured by using an alternating current device, a sechometer, in connection with a dry cell and wheatstone bridge. Temperatures are measured by means of a platinum resistance thermometer and indicated on an automatic recorder. The heating is accomplished by means of a tubular resistance furnace.

The resistance of one of these rods of pottery behaves much like that of glass, porcelain, quartz and the Nernst glower. It is near 10,000,000 ohms at about 550° C. and decreases rapidly as temperature increases until at 1100° C. it measures approximately 15,000 ohms.

When a direct current is passed through a piece of the pottery a polarization or counter E.M.F. is produced which is represented in the above equation by e . The specimen is charged for a given time, at a definite temperature, by a certain voltage. A switch is then thrown to connect it into a potentiometer circuit and a balance obtained giving the value of e as quickly as possible, usually after only a few seconds. The time-decay curve of e is then determined. During the whole experiment the piece of pottery is suspended by means of its lead wires in the furnace. Nickel leads have been used.

It is found that e is a function of the charging voltage, time of charging and temperature.

¹ Abstract of a paper presented at the Washington meeting of the Physical Society, Dec. 27-30, 1911.

1. ϵ increases rapidly as the charging voltage goes up to about 6 or 8 volts and then slowly as the latter moves up to 110 volts.

2. ϵ increases rapidly with time up to about 2 minutes and thereafter slowly.

3. ϵ decreases as temperature increases.

The maximum polarization or counter E.M.F. obtained is about 2 volts.

The rate of discharge is dependent on temperature, time of charging and material and size of electrodes, leads or connectors used. The rate of discharge increases as temperature increases and decreases as time of charging increases. Rate of discharge also decreases as the size or number of nickel wires used as leads are increased. If nichrome, an alloy presumably containing nickel, is used the ϵ obtained is the same as for nickel but in the case of the alloy the decay or discharge is much more rapid. If iron leads are used the ϵ obtained is slightly higher than is the case when using nickel and the discharge also more rapid.

ABSORPTION OF THE β -PARTICLES BY GASES.¹

BY ALOIS F. KOVARIK.

THIS investigation has for its purpose: 1. To obtain a relation between the activities of the soft and hard β radiations as, e. g., from RaD and RaE, Actinium B and Actinium D, Thorium B and Thorium E, etc. This is difficult when the absorbing material used is a metal foil.

2. To determine the coefficient of absorption of the β -particles in terms of gases.

3. To determine the number of soft β -particles emitted from one disintegrating atom which emits soft β -particles.

This preliminary work was done with RaD + E + F. The apparatus consists of an ionization vessel 2 mm. deep placed in a cylinder in which the pressure can be run up to about 20 atmospheres. The RaE β -particles are absorbed exponentially in air and CO₂ down to about 10 per cent. of the initial value. The value of μ for RaE rays is 0.0149 cm⁻¹ air and 0.0296 cm⁻¹ CO₂ (atmospheric pressure).

The RaD β -particles are undoubtedly of two kinds of penetrability. This was already shown by O. v. Beyer, Hahn and Meitner, *Phys. Z. S.*, 12 Jg., 378, 1911. The activity of the harder RaD β -particles, when reduced to no absorption, is about 20 per cent. of the activity of RaE rays and the value of μ for these rays is 0.094 cm⁻¹ air and 0.181 cm⁻¹ CO₂. The softer rays have a much larger value of μ and their activity appears to be several times that of RaE rays, reduced to no absorption.

Eve,² who recently published values for the coefficient of absorption by air of the β -rays from RaC, obtained by an entirely different method, got for the hard rays $\mu = 0.0045$ cm⁻¹ air. In case of aluminum these rays have a value of $\mu = 13.5$ cm⁻¹ Al.

The value of μ for RaE rays is 43.5 cm⁻¹ Al. It will be noticed that the relation between the values of μ in terms of aluminum and air for the rays from

¹ Abstract of a paper presented at the Washington meeting of the Physical Society, Dec. 27-30, 1911.

² Eve, A. S., *Phil. Mag.*, July, 1911.

RaE, as above, and RaC, as found by Eve, is about the same. If this relation holds for RaD rays then the value of μ for the harder β -particles from RaD is 270 cm^{-1} aluminum.

PHYSICAL LABORATORY,
UNIVERSITY OF MINNESOTA.

THE SPECTRA OF IRON AND TITANIUM AT MODERATE PRESSURES.¹

BY HENRY G. GALE AND WALTER S. ADAMS.

AN investigation of the arc spectrum of iron at a pressure of 9 atmospheres, and of the arc and spark spectra of titanium at pressures ranging from a partial vacuum to 17 atmospheres, has been completely by us at the Mount Wilson Solar Observatory.

The range of wave-length included in our photographs extended from λ 3600 to λ 6700. A few plates were also taken for the calcium triplets at λ 3965 and at λ 6192. A total of 250 plates were taken, with from two to eight exposures on a plate. The scale of the plates was approximately 1 mm. per Ångström. A strip of spectrum about 400 Ångströms long was usually measurable on each plate.

The following results have been obtained:

1. Reversal is a function of wave-length, being most frequent in the more refrangible part of the spectrum, and becoming less so toward longer wave-lengths. A similar result was announced by Hale in 1902.

2. The enhanced lines of titanium are not reversed in the arc, and only very faintly in the spark, except in the ultra-violet, even at a pressure of 17 atmospheres. This suggests an explanation of the prominence of the enhanced lines in the lower levels of the chromospheric spectrum.

3. At a pressure of 10 cm. the enhanced lines of titanium are relatively strengthened with respect to the arc lines. This suggests an explanation of the prominence of the enhanced lines in the upper levels of the chromosphere, where the pressure is undoubtedly low.

4. The displacements of the titanium arc lines are found to be accurately proportional to the pressure at pressures of 2, 4, 6, 8, 12 and 16 atmospheres above atmospheric pressure. The proportionality holds also at pressures less than one atmosphere.

5. The 23 enhanced lines of titanium measured in the arc show an average displacement, by a pressure of 8 atmospheres, of 0.034 Ångström. The 103 other lines in the same region of wave-lengths have an average displacement of 0.023 Ångström. On the spark plates the 15 enhanced lines have an average displacement of 0.050 and the 71 arc lines in the same region an average displacement of 0.028. On the average, then, the enhanced lines have a displacement which is about 50 per cent. larger than that of the arc lines. A few of the enhanced lines, however, have small displacements.

6. As shown by the figures above, the displacements in the spark are, on the

¹ Abstract of a paper presented at the Washington meeting of the Physical Society, Dec. 27-30, 1911.

average, greater than in the arc. This difference is most pronounced in the case of the enhanced lines, but is in all probability true for the arc lines as well. In the selected list of 20 arc lines used for testing the proportionality of displacement to pressure, which were measured on many plates in both arc and spark, every line is shifted more in the spark than in the arc. The average difference $\Delta_{\text{spark}} - \Delta_{\text{arc}} = 0.007$ Ångström. For all the enhanced lines measured in both arc and spark the mean difference is 0.014 Ångström.

7. We have been unable to detect any change in displacement with changes in capacity, self-induction, spark gap or current strength, or with a change from air to carbon dioxide.

8. The low temperature or "flame" lines of iron appear to form a distinct group and have small displacements under pressure.

9. There are three other easily recognizable groups in the iron spectrum, for which the ratios of the mean displacements are 1 : 2.3 : 4.5. For the four groups the ratios are 1 : 1.5 : 3.4 : 6.6. If the two groups of smallest displacement are united we have 1 : 2.5 : 4.9 instead of 1 : 2 : 4 as announced by Duffield and by King.

10. The values of the average displacement for the four iron groups at different wave-lengths are well represented by a law of variation of the displacement with the third power of the wave-length.

11. We have been unable to distinguish separate groups in the titanium spectrum, but omitting the enhanced lines, and a few lines of very large shift, similar to the *d* group in iron, the shifts appear to be represented most nearly by a law involving the square of the wave-length.

12. The calcium lines measured in the triplets of the second subordinate series at λ 3965 and at λ 6129 give shifts which are represented by a law involving the first power of the wave-length.

13. There may be some significance in the fact that calcium, titanium and iron occur respectively in the second, fourth and eighth groups of the Mendeléjeff table.

14. In the spectrum of iron there appears to be a direct relationship between pressure shift and Zeeman effect, for lines of the same group, and of the same type of magnetic separation.

15. We have found no evidence to confirm the results of Duffield and King, that, at a given pressure the displacement may have different values for the same source.

16. A number of plates of the titanium arc under a pressure of 5 atmospheres (total) of illuminating gas were taken. There was no appreciable difference in the shift of the 16 arc lines measured, but the 7 enhanced lines showed an average displacement of 0.006 Ångström greater in hydrogen (illuminating gas) than in carbon dioxide.

17. As is well known, the enhanced lines are relatively strengthened in a hydrogen atmosphere. This holds at a pressure of 5 atmospheres as well as at one atmosphere.

18. The small shift of the flame lines of iron may be a property of the flame lines of iron alone. The low temperature line, 4227, of calcium appears to have a shift almost exactly twice that of H and K. The lines of titanium which are strengthened in sunspots, however, appear on the average to have small shifts.

A QUANTITATIVE MEASURE OF DEVELOPMENT IN SCIENTIFIC OBSERVATION.¹

BY OTTO STUHLMANN, JR.

CURVES have been obtained showing the progress in development of accuracy in scientific observation for individuals and for a class of students in a course in laboratory physics.

The errors in an inexperienced observer's results were found to vary periodically but decrease in magnitude as the familiarity with the particular experiment increased. These positive and negative errors were found to be equally probable and if plotted against the trial in which they occurred, it was found that the curve followed very closely the damped oscillatory equation

$$E = ke^{-at} \sin pt.$$

It shows that the maximum error (E) in either direction decays away in geometric progression as the trials (t) increased in arithmetical progression. Here the logarithmic decrement,

$$\log \frac{E_1}{E_2} = \frac{\alpha T}{2} \equiv \delta,$$

or damping of the oscillations per half period, can be shown to be the measure of the development in scientific observation of the subject.

Further curves were obtained for advanced students and for an expert experimenter. They show that while the initial errors, in the results from an unfamiliar experiment, are relatively large for all classes of observers, that the greater the experience in scientific observation the more rapid the damping. The curve eventually deteriorating as in the expert observer's curves into the type generally known as the "dead beat curve,"

$$E = Ae^{a_1t} + Be^{a_2t}.$$

This latter type is commonly known to scientific observers as the "personal equation." Hence the "personal equation" is a curve originally of the form of the first equation but changing gradually to the "dead beat" type as the experience of the subject and hence his accuracy of observation increases.

The same is also here applied to the progress in observation to a class of students in the physical laboratory, although the class as a whole never reaching the "dead beat" type of observational curve.

PHYSICAL LABORATORY,

STEVENS INSTITUTE OF TECHNOLOGY,

HOBOKEN, N. J.

RELATION BETWEEN THE JOULE EFFECT AND THE INDUCTION AND PERMEABILITY IN THE SAME SPECIMENS OF STEEL.¹

BY S. R. WILLIAMS.

A SURVEY of the investigations which have been carried out on the varied phenomena of ferromagnetism indicates that too little comparative work has been accomplished, in which one phenomenon has been compared with another in the same specimen of ferromagnetic substance.

¹ Abstract of a paper presented at the Washington meeting of the Physical Society, Dec. 27-30, 1911.

I have therefore made a collection of rods (80 cm. in length) of ferromagnetic substances in which I propose to study the various magnetic effects possible to be carried out with specimens of this shape. Among these various phenomena may be mentioned the Joule and Wiedemann magnetostrictive effects, the permeability, susceptibility, carbon content, rigidity, crystalline structure by micro-photography, the Kerr effect, etc. From a comparative study on such a group of rods it is hoped that something more definite may be arrived at regarding these inter-relations.

The present paper deals with the relation between the Joule magnetostrictive effect and the induction and permeability of the same rods. The first part deals with the Joule effect and a photographic method of quickly registering the magnetic changes in length with corresponding field strengths. Heat effects are thus eliminated by obtaining the magnetic changes before the temperature changes occur. The second part is a study of the relative position of maximum permeability and maximum increase in length as regards field strength. Accompanying this are micro-photographs of the specimens and analysis showing carbon content.

OBERLIN COLLEGE,
OBERLIN, OHIO.

THE APPLICATION OF STATISTICAL PRINCIPLES TO PHOTOELECTRIC EFFECTS AND SOME ALLIED PHENOMENA.¹

BY O. W. RICHARDSON.

THE chief part of this paper deals with the application of statistical principles and of thermodynamics to the reflection of electrons by bodies and the emission of electrons under the influence of photoelectric action. Considering first of all the case of a body which is devoid of photoelectric activity, the group of electrons which arrive at the surface from outside is balanced, in the steady state, by the reflected electrons together with those liberated by thermionic emission. It is shown that the distribution of velocity among the reflected electrons is in accordance with Maxwell's law. Moreover if $F(u \cdot v \cdot w \cdot u_1)du_1$ denotes the proportion of the electrons reflected, out of the incident group having velocity components between u and $u + du$, v and $v + dv$ and w and $w + dw$, which have their u component of velocity between u_1 and $u_1 + du_1$ then F has to satisfy the equation

$$\int_0^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty u e^{-km(u^2+v^2+w^2)} \{ F(u \cdot v \cdot w \cdot u_1) - 2kmu_1 e^{-kmu_1^2} \} du dv dw = 0. \quad (1)$$

There are similar equations for the functions which determine the proportions of the other components $v_1 w_1$.

The more general problem of the exchange of energy and electricity under the combined influence of ethereal radiation, thermionic emission and photoelectric action, in the state of statistical equilibrium, which characterizes an enclosed system at a uniform temperature, is then considered. By a slight modification of the usual proofs it is shown that the complete (black body) ethereal radiation has the proper-

¹ Abstract of a paper presented at the Washington meeting of the Physical Society, Dec. 27-30, 1911.

ties which have hitherto been deduced for it. It is shown that the distribution of kinetic energy among the electrons in the complete atmosphere is in accordance with Maxwell's law and the average number n of electrons per unit volume at any point is connected with the local value w of the potential energy of an electron by the relation.

$$RT \log n + w = C, \quad (2)$$

where C is a function of T only. It is also shown that in the case of a cavity in the interior of any particular substance n is given by

$$n = Ae^{\int \frac{\phi}{RT^2} dT}, \quad (3)$$

where A is a constant characteristic of the material, but independent of T , and ϕ is the absorption of heat which is associated with the liberation of one electron at the surface. The above relations apply to the complete atmosphere of electrons and do not distinguish between their modes of origin. The electrons originate, in general, from the combined effects of reflection, thermionic emission and photoelectric action. It is clear that there is a very close correspondence between the thermionic emission, and the photoelectric emission arising from the complete ethereal radiation characteristic of any given temperature. It is pointed out, however, that the experimental evidence is against the view that the thermionic emission is nothing more than the aggregate photoelectric effect of the complete ethereal (temperature) radiation.

The properties of an ideal body which is photoelectrically active but which is devoid of thermionic emissivity and of the power of reflecting electrons is then discussed. It is shown that the photoelectric emission from such a body must satisfy the equation

$$\frac{c}{4\pi} \int_0^\infty \epsilon F(\nu T) E(\nu) d\nu = A \left(\frac{R}{2m\pi} \right)^{\frac{1}{2}} T^{\frac{1}{2}} e^{\int \frac{\phi}{RT^2} dT} \quad (4)$$

for all values of the temperature T . Here $E(\nu)d\nu$ is the density of the black body radiation of frequency between ν and $\nu + d\nu$, ϵ is the proportion of it which is absorbed (*i. e.*, not reflected), $F(\nu T)$ is the number of electrons emitted by photoelectric action when unit quantity of energy of this frequency is absorbed at this temperature. A formal solution of (4) under the restrictions that $\epsilon F(\nu T)$, A and ϕ are independent of T , is given, but it appears to be divergent except when ν is very large. It is shown that equation (4) is consistent with the view suggested by Einstein that the light of frequency ν communicates to the electrons an amount of energy $h\nu$, where h is Planck's constant, that part of this is used up by the work ϕ required to liberate an electron and that the rest appears as the kinetic energy of the electrons which have escaped. What appears to be a rather more natural view, however, leads to the conclusion that only one fourth of the energy of the escaped electrons arises in this way, the remainder being the energy they possessed when inside the metal. This view is also consistent with equation (4). It is also shown that the distribution of velocity among the electrons emitted under the influence

of radiation of frequency ν , incident in the altitude between θ and $\theta + d\theta$ and azimuth φ and $\varphi + d\varphi$ has to satisfy equations of the type:

$$\int_0^{\pi/2} \int_0^{2\pi} \int_0^\infty \sin \theta \cos \theta E(\nu) \{ \epsilon_1(\nu\theta) F_1(\nu\theta) [f_1(\nu \cdot \theta \cdot u \cdot v \cdot w \cdot k) - 2kmue^{-kmu^2}] + \epsilon_2(\nu\theta) F_2(\nu\theta) [f_2(\nu \cdot \theta \cdot u \cdot v \cdot w \cdot k) - 2kmue^{-kmu^2}] \} d\theta d\varphi dv = 0. \quad (5)$$

Here the suffixes 1 and 2 refer to radiation polarized in and perpendicular to the plane of incidence respectively. ϵ and F are the same as before except for the restriction as to direction. $f(\nu \cdot \theta \cdot u \cdot v \cdot w \cdot k)$ denote the proportion of the liberated electrons which have velocity components between u and $u + du$, u being normal to the surface of emission. The temperature is $T = (2kR)^{-1}$. There are similar equations for the other components $v \cdot w$. All are true for all values of $u \cdot v \cdot w$ and k .

The consequences of restricting the discussion to the case of an ideal body, which is photoelectrically active, but devoid of thermionic emissivity, and which reflects none of the electrons which reach its surface, are considered. It is shown that in the case of a real body the group of electrons which is made up of the sum of those reflected and those emitted photoelectrically will have Maxwell's distribution of velocity. The writer has not been able to show that this is true of the two subdivisions separately but since reflection and photoelectric emission are independent acts it follows that it will be very nearly, if not exactly, true for each subdivision. Thus the photoelectric properties which have been deduced for the ideal matter will possibly be exactly true and probably be approximately true for real matter.

The paper also contains a discussion of the theory of thermoelectric effects from a wider standpoint than that of the former paper on this subject (PHYS. REV., Vol. 33, p. 448, 1911). It is shown that, except when the internal free electrons can all be considered to have very nearly the same potential energy, and to be capable of occupying nearly the whole volume of the material, it is necessary to replace $\log n$ in the earlier formulæ by $\int n \log nd\tau \div \int nd\tau$, where n is the number which expresses, per unit volume, the local value of the sum of both the electrons which are actually free and those which may become free under the dynamical actions actually occurring. In general, thermoelectric phenomena do not distinguish between the actually free and the temporarily bound electrons. In particular, no very close connection is to be expected between thermoelectric effects and changes in electrical conductivity (for example at the melting point).

The part possibly played in electric conduction by electrons similarly situated to those which give rise to polarization in dielectrics is considered. It is shown that some of the peculiar features exhibited by galvanomagnetic phenomena might arise in this way but it does not appear that all the characteristics of this very complex group of effects can be accounted for by such a simple hypothesis.

The very high electrical conductivity of metals at the very lowest temperatures, observed by Kammerlingh Onnes, would appear to indicate that either the number

of the centers of force considered in the previous paper (PHYS. REV. vol. 34, p. 77, 1912), or their strength, is greatly diminished at these very low temperatures.

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J.

A STUDY OF CRYSTAL RECTIFIERS.¹

BY ROBERT H. GODDARD.

WITH currents of from 0.5 to 9 amperes, there is a tendency, with contacts which show a unilateral effect, for the larger current to flow from substance to metal, even when rectification for smaller currents is in the opposite direction; together with evidences of oxidation. A large number of experiments with various substances and conditions of contact suggested the necessity of performing experiments in high vacuo, with substances cleaned in vacuo.

An apparatus in which this could be accomplished consisted, briefly, of a long glass tube with two side tubes opposite each other. The substances and metals tested were fastened to the ends of metal cylinders of a proper size to slide in these side tubes, which cylinders were, in turn, connected to platinum wires sealed in the ends of the tubes by flexible spirals of closely wound copper wire. The metal cylinder, or holder, could be moved, and rotated, from outside the tube by a magnet, acting on an iron rod in the cylinder—off center. These substances and metals could be filed, or the ends broken off, by a round file which could slide inside the large glass tube, but was prevented from breaking through by springs—one on the end of the file and another at the end of the glass tube. By this means it was found possible to break off a clean section of a soft metal, like aluminium, 1 mm. square. The vacuum was the highest that could be obtained with a Gaede pump, without freezing mixtures.

Experiments were thus performed, with as nearly chemically clean surfaces as possible, with magnesium, aluminium, iron, and lead against tellurium, silicon, galena, and mineral graphite. The effects were found to be practically independent of the particular metal used. With tellurium—melted in vacuo, to obtain a clean surface—the rectification, for currents of from 0.03 or less, to 0.1 ampere, was reduced to 2 or 3 per cent. of the current flowing through the contact, becoming for some contacts actually zero, or being slightly reversed. Ordinary fused silicon gave rectification in vacuo, but small crystals of as pure silicon as could be obtained showed rectification only of the order obtained with tellurium. Galena and carbon showed lack of uniform behavior in the air and vacuo, although the former gave noticeable unilateral effects in vacuo.

Admission of dry hydrogen, nitrogen, and carbon dioxide gave results similar to the above, but in dry oxygen the substances which had lost rectifying properties behaved as in air.

That a surface film of oxygen is necessary for rectification with metals against pure elements is a conclusion at considerable variance with some explanations

¹ Abstract of a paper presented at the Washington meeting of the Physical Society, Dec. 27-30, 1911.

that have previously been suggested, and attempts were made to check it in various ways. Tellurium was melted on a piece of sheet aluminium, and a good contact formed. The tellurium was then touched with a cold aluminium wire, and the whole allowed to cool. The result was a rectifier, the active contact being that between the tellurium and the wire. Secondly, melted tellurium, with 10 volts applied in the direction of greater resistance gave 0.5 ampere, but less than 0.0002 ampere when the E.M.F. was changed to 1.0 volt. When the 1.0 volt was applied in the reverse direction, the current increased to 0.008 ampere. With 10 volts in the other direction, no such high resistance state with 1.0 volt was observed.

Metal filings in air show difference in conductivity in the two directions, and deviation from Ohm's law—the ratio of current to E.M.F. increasing. An apparatus similar to that just described was made in which filings could be produced by the inertia of a file, shaken back and forth in the tube; there being, at the same time, practically no strain on the glass. Filings of the three metals experimented with, copper, aluminium, and magnesium, showed no deviation from Ohm's law, in vacuo or hydrogen, and no apparent asymmetry, but behaved as in air when oxygen was present. The experiments were most satisfactory in the case of copper.

As a still further check, galena, which when powdered in air shows asymmetry and deviation from Ohm's law, was filed to a powder in vacuo, and the two effects were still found to be present.

From the above experiment it is concluded that rectification is of two kinds; due to a surface film of oxygen or oxide in the case of pure elements and metals, and to a film of some other sort in the case of impure elements or compounds; which may be called surface, and body, rectification, respectively. Experiments, in which it is hoped to obtain oscillograms of rectifiers at high frequencies, to test this point further, are now in progress.

CLARK UNIVERSITY,
December 5, 1911.

A SIMPLE SLIT FOR THE SPECTROSCOPE.¹

By J. P. NAYLOR.

SOME time since while arranging a spectrograph for student use the writer devised a slit which is believed to be new in construction and of sufficient merit to be of interest to readers of the REVIEW who are engaged in spectroscopic work. Besides having several advantages over a slit made in the usual way, this slit is so simple in construction that it can be made by anyone having ordinary mechanical skill and using only very simple tools. And yet the slit, by a little care in the making, is suitable for the most exacting work.

The base on which the slit is arranged consists of a piece of one eighth inch sheet brass, one side of which is made flat by grinding it on a piece of emery paper glued to a flat block of wood. A hole of the necessary size to limit the length of the slit is drilled in the center of the base and over this opening the plates are ar-

¹ Abstract of a paper presented at the Washington meeting of the Physical Society, Dec. 27-30, 1911.

ranged that form the jaws of the slit. These plates are made of No. 16 sheet brass or, better still, German silver. The plates are carefully flattened by grinding on the emery paper.

One of the plates, *A* of the figure, is fastened to the base by two flat head screws which pass through holes in the plates which are slightly slotted to permit of a small lateral adjustment of the plate.

The other plate, *B*, is movable so as to regulate the width of the slit. This plate has its two edges inclined to each other in the ratio of 10 to 1. The back edge of this piece can slide endwise under the flat heads of two screws, 2 and 3, so placed as to bring the front edge about parallel with the edge of the fixed plate. A similar screw is fastened in the center of the plate and the head of the screw is raised so as to permit a piece of No. 26 spring brass wire to pass under it. This wire has its ends bent under two screws, 7 and 8, near the ends of the base and forms a spring that holds the plate down to the base and back against the body of the screws, 2 and 3. The opposed edges of the plates are beveled back slightly from the front and are, of course, carefully straightened by filing and finally by grinding on plate glass, first, with fine emery and last with powdered pumice.

To make the edges of the slit perfectly parallel screws 5 and 6 are slightly loosened and the edge of *B* is gently forced down against *A* when, if the screws are again tightened, the edges will remain parallel provided the back edge of plate *B* has been ground straight. A millimeter scale and an index on *B* is provided by which the width of the slit can be readily determined. This is a convenience usually only found on expensively constructed slits provided with a micrometer screw and divided head.

The facility with which the slit can be cleaned is an advantage which will certainly appeal to workers with the spectroscope. By simply slipping off the wire spring plate *B* can be removed; the edges can then be cleaned and the plate returned accurately to its former position. If the edges, by any accident or use, should become damaged the plates can be easily removed, the edges reground, and the plates put back in a very short time.

It will be observed that, by using two movable plates arranged as above, the construction readily permits of the making of a bilateral slit. In this case, the second plate should be backed up with a brass plate provided with screws and slotted holes so as to permit of adjustment for parallelism of the jaws. If by means of a pin on one plate and a projection on the other the plates are made to move together the slit will open symmetrically.

MINSHALL LABORATORY,
DEPAUW UNIVERSITY.

THE DISTRIBUTION OF THE ACTIVE DEPOSIT OF RADIUM IN AN ELECTRIC FIELD.¹

By E. M. WELLISCH AND H. L. BRONSON.

IN this paper an account is given of experiments which have been made to determine the activity that is transmitted to the electrodes in an electric field. This activity has throughout been obtained for each electrode as a percentage of

¹ Abstract of a paper presented at the Washington meeting of the Physical Society, Dec. 27-30, 1911.

the total activity. It has been found that the curve connecting the percentage of the total activity transmitted to the cathode with different electric fields has all the characteristics of the curves for ionization by α -particles. The anomalous results obtained by many previous experimenters are almost certainly due to the fact that the electric fields employed were far from saturating. It has been found that all the so-called anode activity is due to the diffusion of uncharged carriers.

It has been shown that the curves for α -ray ionization do not in general have asymptotes such as the ordinary saturation curves for X-ray ionization, in fact, the more nearly the curve approaches to parallelism with the axis of electric field the farther is the ionization from saturation. The shape of the ionization curves has been explained as being due to the formation by the α -particle of "neutrons" which are afterwards separated into ions by the collision with ions already established in the columns. The results obtained by several workers in the case of different inclinations of the α -ray columns are readily explainable on the present hypothesis.

THE THOMSON EFFECTS IN AND THE THERMAL CONDUCTIVITIES OF TUNGSTEN, TANTALUM AND CARBON AT GLOWING TEMPERATURES.¹

BY A. G. WORTHING.

IN the investigation of the losses in incandescent lamps due² to the cooling effects of the supports and the leading-in wires, a variation was noticed which was ascribed to the Thomson effect. The suggestion was made by Dr. Hyde at that time that the same method might be used to study the Thomson effect. This has been done. The method of observation is the same as that of the investigation noted above, though greatly refined as to accuracy. The observations consist essentially in the determination by an optical pyrometric method, in the case of an incandescent lamp, of the time rate of energy radiation per unit of length for elements of filament in the neighborhood of a support or leading-in wire as a function of their distances from the support or leading-in wire; (a) when the current through the element of filament is flowing toward the cooling support; (b) when in the opposite direction. The difference between these two quantities is in part due to the Thomson effect. From the platted data as a basis, as soon as the temperature of an element is known in terms of the time rate of energy radiation per unit of length, the Thomson effect may be determined by means of the following equation, when elements of filaments at the same temperature are chosen:

$$\sigma = \frac{W_a - W_b}{I \frac{d\theta}{dE} \left[\left(\frac{dE}{dl} \right)_a + \left(\frac{dE}{dl} \right)_b \right]},$$

where σ represents the Thomson E.M.F. per unit of temperature difference,

¹ Abstract of a paper presented at the Washington meeting of the Physical Society, December 27-30, 1911.

² Hyde, Cady and Worthing, Trans. Ill. Eng. Soc., 6, 1911, p. 238; Hyde, Cady and Worthing, Ill. Eng., Lond., 4, 1911, p. 389.

W the time rate of transformation of electrical energy into heat per unit length, I the current, θ the temperature, E the time rate of energy radiation per unit of length, l the distance of the element in question from the support or leading-in wire, and a and b , as subscripts, the two conditions as to direction of current noted above.

The thermal conductivities of the substances may be obtained from the same data as given above. Here the equation is

$$k = \frac{\int_l^{l_m} (W_a - E_a) dl}{S \frac{d\theta}{dE} \left(\frac{dE}{dl} \right)_a} = \frac{\int_l^{l_m} (W_b - E_b) dl}{S \frac{d\theta}{dE} \left(\frac{dE}{dl} \right)_b},$$

where in addition to the above symbols k represents the thermal conductivity, l_m the distance to a point where the rate of heat conduction along the filament is negligibly small, and S the cross section of the filament. Thus far the thermal conductivities have been expressed only in terms of their ratios to the corresponding electrical conductivities.

The lamps used were kindly calibrated by Professor Mendenhall and Mr. Forsythe as to temperature in the neighborhood of the temperatures at which they have their ordinary efficiencies. Other temperatures were obtained by using certain results not yet published which are due to Dr. E. P. Hyde. The uncertainties arising from temperature measurements in this work will not change the order of magnitude of the results obtained. Some results are included in the following table.

Absolute Temperature.	Tungsten.	Tantalum.	Carbon.
Coefficients of Thomson effect in $\frac{\text{microvolts}}{\text{deg.}}$			
1,600° K	16	-5	-20
1,800	22	-6	-20
2,000	29	-8	-20
2,100	33	-9	-20
$10^{-10} \times \frac{\text{thermal conductivity}}{\text{electrical conductivity}}$ in C.G.S. electromagnetic units.			
1,600° K	56	27	280
1,800	73	33	270
2,000	94	46	260
2,100	103	56	240

When these results together with some thermoelectric results on tungsten and tantalum by Coblenz¹ are handled thermodynamically, it is found that a tungsten-tantalum thermocouple at high temperatures is about four times as sensitive as a platinum-platinum rhodium thermocouple.

It is evident that the Peltier E.M.F. between two metals may be measured

¹ Bul. Bur. Stds., 6, 1909, p. 107.

by a method similar to the foregoing. All that is essential is to determine the change in the rate of heat conduction along a filament composed of the two metals welded together, in passing from one side of the junction to the other. Such measurements are contemplated.

THE DIELECTRIC CONSTANT; RESISTIVITY AND ELECTROSTATIC ABSORPTION
OF DIFFERENT CRYSTALS.¹

By H. L. CURTIS.

IN 1881, Rowland and Nichols measured the electrostatic absorption in plates of mica, quartz, selenite and calcite. They found no absorption in calcite, but their results do not show how the absorption of the other substances compares with mica. It seemed desirable, therefore, to measure the absorption of a larger number of crystals, and to compare their absorption with that of mica which has been extensively studied. Also it was thought best to measure the dielectric constant and resistivity of the crystals, thus giving important electrical data concerning them.

Thin plates of the crystals were used as the dielectric in a plate condenser having a guard ring. The same arrangement was used for determining the resistivity.

Two methods of measuring the capacity of the condenser were used; one by direct current and the other by high frequency alternating current. In the direct current method, the condenser is charged 0.5 second, then disconnected from the battery, and about five hundredths of a second later is connected to a tenth microfarad air condenser for a few thousandths of a second. As the capacity of the air condenser is much larger than the condenser which is being measured, nearly all of the charge is transferred to the air condenser. The charging and discharging is accomplished by a mechanism driven by a motor, the gearing being so arranged that the operation of charging and discharging is repeated every two seconds. The air condenser is discharged through a ballistic galvanometer when a sufficient quantity has collected to make a reasonable deflection. A variable air condenser is then substituted for the plate condenser, and its capacity varied until, with the same number of discharges, the deflection of the galvanometer is the same as with the plate condenser. From the measured capacity and the capacity computed with air as a dielectric, the dielectric constant is determined.

In the apparatus just described, a contact is arranged so that a second discharge of the condenser takes place either 0.5 or 1 second after the original discharge. This discharge can be collected upon the air condenser instead of the first discharge. From the measurement of this discharge, the absorption under the given conditions is computed.

In cases where the resistivity is so low that the method just outlined does not give satisfactory results, an approximate measurement of the dielectric

¹ Abstract of a paper presented at the Washington meeting of the Physical Society, December 27-30, 1911.

constant can be obtained from high frequency measurements of the capacity. The plate condenser without a guard ring is placed in parallel with a variable air condenser in a resonating circuit with a known inductance, and the frequency determined by means of a wave meter. The crystal condenser is then removed and the air condenser varied until the resonating circuit has the same frequency as at first.

The resistance is obtained by means of leakage methods such as are commonly used for high resistances. As a guard ring is employed, the measured leakage takes place through the crystal and not over its surface.

Results have already been obtained upon a number of crystals and others will be investigated. The following table gives some of the values which have already been determined. The maximum and minimum value of the index of refraction, determined by the angle of total reflection from the surface, has been given for most of the crystals.

The results show that fluorite has no measurable absorption; *i. e.*, less than 0.1 per cent. The calcite which was used showed some absorption, but the plate was not perfect, having several cleavage cracks across its face. Also the dielectric constant of rutile is much smaller than that found by other observers.

Crystal.	Capacity in Micro- microfarads.	Dielectric Constant.	Index of Re- fraction for Sodium Lines.		Absorption.		Resistivity in Megameg- ohm-cm.
			Max.	Min.	0.5 Sec- ond Dis- charge.	1 Second Dis- charge.	
Mica.....	347.	6.8	1.560	1.598	0.26%	0.30%	80,000.
Fluorite.....	5.3	6.9	1.434		0.	0.	3,000.
Calcite.....	15.5	8.3	1.658	1.486	2.7	3.7	1,600.
Quartz.....	70.	4.9	1.553	1.544	11.	13.	69.
Selenite.....	13.3	4.8	1.531	1.522	4.3	6.5	1,400.
Green beryl ..	14.4	5.6	1.586	1.580	3.9	4.4	200.
White beryl \perp .	27.6	7.4	1.578	1.571	22.	25.	17.
White beryl \parallel .	25.9	9.3	1.580		8.5	9.2	160.
Cryolite.....	15.2	6.2	1.335		8.6	10.	200.
Apatite.....	16.9	7.8	1.654		12.	13.	1,200.
Barite.....	20.6	9.0	1.647	1.635	21.	24.	6.
Spodumene...	35.3	17.6	1.713	1.686	25.	27.	6.
Monocline....	14.8	5.8	1.519		27.	32.	23.
Feldspar.....	17.7	5.7		26.	29.	24.
Celestite.....	11.7	6.3	1.626	1.619	31.	37.	380.
Talc.....	13.0	5.8		32.	36.	50.
Rutile.....	240.	31.	2.903	2.616			0.09

THE INFLUENCE OF TEMPERATURE ON THE PHENOMENA OF PHOSPHORESCENCE IN ZINC SULPHIDE.¹

BY HERBERT E. IVES AND M. LUCKIESH.

THREE phenomena have been the subject of study, namely, the law of decay, the flashing up of brightness under infra-red, and the flashing up with heating. Temperatures were varied between -80°C. and $+200^{\circ}\text{C.}$ The decay curves obtained, if plotted in terms of $1/\sqrt{I}$ against time, vary from strongly concave to the time axis at low temperatures to strongly convex at high temperatures. The formula

$$I^{\alpha} = a + bt$$

fits all the curves obtained, where α varies from -1.6 to $-.2$ over the range of temperatures employed. This formula holds from two or three seconds after the end of excitation to fifteen minutes, the longest time so far investigated. The initial intensity, or fluorescence, is not however given by the formula, which gives too low a value. The change from fluorescence to phosphorescence obeying the above relationship appears to be almost instantaneous.

The flashing up of brightness under infra-red, which the writers have found to be greater the longer the interval since excitation, is now found to be greater at low temperatures and less at high.

Heating always causes a flash of light. In the early stages of decay where infra-red causes an immediate drop in brightness, heat and infra-red act oppositely. An experiment with separate and then simultaneous application of heat and infra-red shows the two to sum algebraically. This is not in harmony with the theory that infra-red is equivalent to high temperature.

The facts here presented are not compatible with the theory that the law of decay is represented by the expression

$$1/\sqrt{I} = a + bt,$$

or by a summation of such terms. Such a summation can account only for curves concave to the time axis (on a $1/\sqrt{I}$ against time plot), not for curves convex to it.

A possible explanation of the experimental facts is that the sample of zinc sulphide is a mixture of components all decaying according to some simple law. Curves of the character found are given by the summation of several terms of the form Ae^{-kt} , which is the law of mass action with one substance changing. k varies with temperature. Since different samples of zinc sulphide decay at different rates at the same temperature it is possible that the variation of the k 's is with respect to temperature zeros which are functions of the composition of the components.

According to this view the later part of a decay curve is due to components at a "low" temperature; the flashing-up phenomenon, a low temperature effect, therefore, becomes more marked as decay goes on.

¹ Abstract of a paper presented at the Washington meeting of the Physical Society, December 27-30, 1911.

SPARK SPECTRA IN THE SCHUMANN REGION.¹

BY THEODORE LYMAN.

THE spark spectra of metals in the region of short wave-lengths have been investigated by Schumann and by Handke.² In both researches the spark was placed in air outside the vacuum spectroscopic. Handke, who worked with aluminum, copper, gold, silver, zinc, magnesium and mercury in this way, made measurements of wave-length as far as the neighborhood of λ 1600.

In the work described in the present paper an improvement was introduced by placing the spark in a stream of pure, dry hydrogen at atmospheric pressure. The mantle of absorbing gases around the spark was thus very much reduced, with the result that the spark spectra of aluminum, magnesium, calcium and strontium-amalgam yielded lines up to the limit set by the transparency of fluorite, λ 1250. The spectrum of calcium proved of particular interest since it showed some new series relations. The work on the spectra of the alkali earths is being continued.

JEFFERSON LABORATORY,
HARVARD UNIVERSITY.

A NEW FORM OF MECHANICAL VACUUM PUMP.¹

BY JOHN JOHNSTON.

THIS pump—known as the May-Nelson Rotary Pump—consists essentially of two plates of metal, one fixed, the other movable; no valve, packing or fluid is required, though a little lubrication is desirable for mechanical reasons. A somewhat defective pump of this type has given a vacuum of 0.01 mm. when connected to the gage by rubber tubing. It has exhausted 6 liters to 0.1 mm. in 2 minutes. After pouring in 1 c.c. of water, the pressure was reduced to 0.5 mm. in 4 minutes, without drawing any air through the pump. The exact mode of working is difficult to show without a diagram, but the principle may perhaps be made sufficiently clear by what follows:

In each of the metal plates are cut a series of concentric grooves, which are so situated that the projections on each plate will lie in the grooves on the other plate. Through the center of the fixed plate passes a shaft, on which is an eccentric carrying the movable plate. Rotation of the shaft thus produces an eccentric motion—but no rotation—of the movable plate; as a result, there is, between each adjacent pair of curved surfaces, a rolling contact which sweeps out the air ahead of it. The pump above mentioned has an inner diameter of 13 cm., a depth of 4 cm., and consists of three rings which pump in series; the speed of rotation of the shaft was 800 r. per minute; $\frac{1}{4}$ H.P. suffices to operate it.

The effectiveness of this form of pump appears to be due to two things: (1)

¹ Abstract of a paper presented at the Washington meeting of the Physical Society, December 27-30, 1911.

² Inaug. diss. u. Berlin, 1909.

A very slight travel of the metal parts produces a relatively rapid motion of the crescent-shaped spaces between the rings on the two plates; (2) the contacts, since they occur between curved surfaces of nearly the same radius, are relatively long; hence the viscosity of the air is sufficient to prevent appreciable leakage backwards.

This pump may also be operated as a blower, or, with slight modifications, can be used as a water motor; indeed, since the rings may be made independent of one another, one can drive the pump by supplying water to one ring and obtain vacuum or blast from the other rings.

GEOPHYSICAL LABORATORY,
CARNEGIE INSTITUTION OF WASHINGTON.

THE PHOTOELECTRIC EFFECT OF PHOSPHORESCENT MATERIAL.¹

BY CHESTER A. BUTMAN.

PHOSPHORESCENT CaS prepared according to the method of Lenard and Klatt and also Sidot blende were examined for the photoelectric effect in a high vacuum.

1. It was found that the photoelectric fatigue and recovery was an inherent property of the material. The fatigue is due solely to the incident light.

2. The velocities of the electrons ejected from these substances is dependent on the photoelectric state of the material. The greatest voltage necessary to keep the electrons from coming off from the CaS was -0.35 volt. In a fatigued state it was sometimes necessary to apply $+4$ volts in order to get any photoelectric effect.

3. The phosphorescent intensity was found to be approximately independent of the state of the photoelectric fatigue.

4. Infra-red light gave no photoelectric effect.

5. The components of the phosphorescent CaS were examined for the photoelectric effect, and none were found to be active with light of longer wave-length than $3,200$ Ång., with the exception of sulphur. It was found that light passed through glass which gave no effect with brass, lead, or platinum, did give a photoelectric effect with flowers of sulphur and brimstone.

6. The following theory is proposed: Phosphorescent light is the light emitted at the time when a charged atom collides with the material that emits the light which is characteristic of that phosphorescent compound. The charge on the atom comes from the electrons sent off from the sulphur. This theory only applies to compounds where there is no phase relation between the exciting and excited light. It is possible to prepare a solid fluorescent (short duration) compound without sulphur. However, here we probably have to do with a *selective emission* due to resonance. This last would be proven to be the case if a phase relation was shown between the exciting and excited light, as is the case with liquids.

¹ Abstract of a paper presented at the Washington meeting of the Physical Society, December 27-30, 1911.

7. The experiments of Ives showing a difference in the action of heat and infra-red would be interpreted as follows: The heat causes more collisions to take place making a flashing up. The infra-red has little effect on the atomic movements, but does dampen the period of the oscillating electrons in the emitting material, hence the extinguishing action.

NOTE ON THE ASCENSIONAL RATE OF THE FREE BALLOONS USED FOR
METEOROLOGICAL PURPOSES.¹

BY WM. R. BLAIR.

THE same type of balloon was used for a number of ascensions. From these ascensions, 20, all of those reaching an altitude of 18 or more kilometers above sea level, were selected, and the ascensional rates of the balloons considered in connection with other data obtained in the ascensions.

It is found that the effect of temperature on the relative densities of the gases outside and inside of the balloon, and consequently, on the ascensional rate of the balloon, needs more consideration than has been given it in the note by A. Mallock, F.R.S., on the "Ascent of Meteorological Balloons and the Temperature of the Upper Air," *Proc. Roy. Soc.*, A80, p. 530. It is also found that the variations in the ascensional rate of these balloons is so great that computations based on the assumption of uniform ascensional rate are much in error, especially at the higher altitudes.

THE VERTICAL TEMPERATURE GRADIENT OF THE ATMOSPHERE.¹

BY WM. R. BLAIR.

THE data considered are those collected by the Blue Hill Meteorological Observatory at St. Louis, Mo., and at Pittsfield, Mass.—59 observations, and those collected by the Mount Weather Observatory at Fort Omaha, Indianapolis, and Huron—79 observations.

These data, together with considerations of convection, nucleation, and absorption, radiation and conduction of heat by the atmosphere seem to point to the conclusion that, while the temperatures themselves are determined mainly by absorption and radiation of heat supplied by the radiators on either side of the atmosphere, the vertical temperature gradient as far up as it has been observed owes not only its local peculiarities but its general character to convection and closely related phenomena. There is evidence that at no great distance above the explored region, the character of the vertical temperature gradient is determined chiefly by gravitational arrangement of the atmospheric constituents.

¹ Abstract of a paper presented at the Washington meeting of the Physical Society, December 27-30, 1911.

NEW BOOKS.

- Vector Analysis.* By J. G. COFFIN. New York: John Wiley and Sons, 1911. Pp. xxii + 262. Price, \$2.50.
- Tatsachen und Theorien der atmosphärischen Polarisation.* By FRIEDRICH BUSCH and CHR. JENSEN. Hamburg: Graefe and Sillem, 1911. Pp. i + 532.
- Physical and Chemical Constants and Some Mathematical Functions.* G. W. C. KAYE and T. H. LABY. New York: Longmans, Green and Co., 1911. Pp. vi + 153. Price, \$1.50.
- Engineering Applications of Higher Mathematics.* By V. KARAPETOFF. New York: John Wiley and Sons, 1912. Pp. xv + 69. Price, \$.75.
- Die Theorie der Wechselströme.* By ERNST ORLICH. Leipzig: B. G. Teubner, 1912. Pp. i + 94. Price, M. 2.40.
- Gabriel Lippman, Biographie, Bibliographie Analytique des Écrits.* By ERNEST LEBON. Paris: Gauthier-Villars, 1911. Pp. i + 70.
- Unsere Kenntnisse von den Seriengesetzen der Linienspektren.* By BERTHOLD DUNZ. Leipzig: Hirzel, 1911. Pp. i + 186. Price M. 2.
- Lehrbuch der Physik.* By E. GRIMSEHL. Leipzig: B. G. Teubner, 1912. Pp. xvi + 1262. Price, M. 15.
- Prinzipien der Atomdynamik.* By J. STARK. Leipzig: Hirzel, 1911. Pp. xv + 286. Price, M. 7.80.
- Vorlesungen über Technische Mechanik. II. Graphische Statik.* By AUG. FÖPPL. Leipzig: B. G. Teubner, 1912. Pp. xii + 419. Price M. 8.
- Annuaire Pour L'An 1912, publié par le Bureau des Longitudes.* Paris: Gauthier-Villars, 1912. Pp. vi + 692.
- Physiography.* By A. L. AREY, F. L. BRYANT, W. W. CLENDENIN and W. T. MORREY. New York: D. C. Heath and Co., 1912. Pp. vi + 450. Price \$1.25.
- Laboratory Problems in Physics.* By F. T. JONES and R. R. TATNALL. New York: Macmillan Company, 1912. Pp. ix + 81. Price, \$.50.

THE PHYSICAL REVIEW.

ON THE FAMILY-TREE ARRANGEMENT OF THE ELEMENTS AND CALCULATION OF ATOMIC WEIGHTS ON THE CORPUSCULAR RING THEORY OF THE ATOM.

BY ALBERT C. CREHORE.

THE atomic theory which has been worked out in some detail by Sir J. J. Thomson¹ assumes that the atoms of matter consist of positive electrification enclosing a number of negatively electrified corpuscles exactly balancing the positive electrification when the atom is neutral. The corpuscles within mutually repel each other and seek positions of equilibrium as the result of the forces acting upon them. It has been shown that the corpuscles will distribute themselves as a result of these forces on concentric rings or shells within the sphere to attain equilibrium, the exact arrangement depending upon the number of corpuscles to be accommodated within it, and upon their velocities. The distribution, in rings, for equilibrium has been partially worked out for moderate velocities, but that in shells presents so many added difficulties that no complete solution is known.

Equilibrium figures closely related to those formed by the rotating electrons of the Thomson atom have been studied by Mayer in his well known experiments with floating magnets. In these experiments small magnetized needles were thrust through pieces of cork and floated upon water, each needle having its negative pole above the water. The needles mutually repel each other in much the same way as the corpuscles within the atom are supposed to, and each needle in the experiment represents a corpuscle in the atom. The positive electrification of the atom is represented by a positive magnetic pole suspended at some distance over the needles above the water. Under these conditions it has been observed

¹ J. J. Thomson, "On the Structure of the Atom," *Philosophical Magazine*, Vol. 7, p. 237, 1904.

that the needles arrange themselves as represented in Fig. 1. This gives the arrangement of the magnets from two to twenty, and the effect of the addition of a single magnet to the collection is well shown. If a single magnet is added there is in general only one point where it can be placed to preserve the equilibrium. It may go into the outer ring,

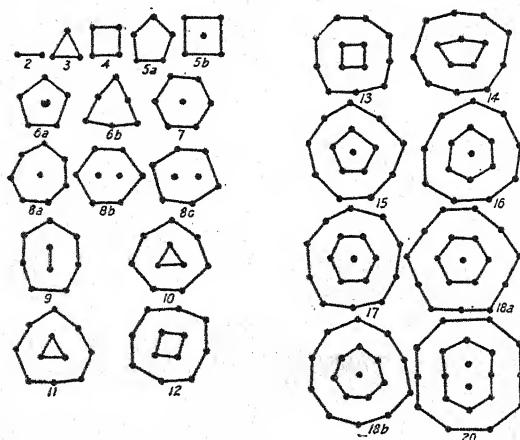


Fig. 1.

the next within it, or the third, etc., or may start to form a new ring at the center of the whole collection of concentric rings. The outer ring cannot receive it until each of the other rings has received its required allotment.¹

The arrangements of the corpuscles for equilibrium have been approximately calculated for moderate velocities by Sir J. J. Thomson.¹ Table I. gives each configuration from one to one hundred corpuscles by intervals of unity. This table is represented graphically in Fig. 2. The broken line OA is obtained by plotting as abscissæ the total number of corpuscles comprising the system, and as ordinates the number of corpuscles in the first or outer ring, A , only. The lines B_2B , C_2C , D_2D , etc., represent in a similar manner the corpuscles in the second, third, fourth, etc., rings.

By adding the number of corpuscles in the first and second rings the line $OA + B$ is obtained, and by adding to this the third ring the line $OA + B + C$ is obtained. When the sixth ring is added the limit of the chart is reached and the whole collection or atom is represented by the line OG_1 , and the broken lines B_1A , $C_1A + B$, etc., separate the atom into its component rings branching off from the atom at the points B_1 , C_1 ,

¹ The Corpuscular Theory of Matter, page 109. J. J. Thomson, 1907.

D_1 , etc. The portion between the line OB_1A and the straight line OG_1 represents the number of corpuscles left in the atom when the outer ring is removed, and the portion between $OC_1A + B$ and OG_1 represents the number left when the two outer rings are removed, and so on.

TABLE I.

[illegible]

These points of departure of the rings, B_1 , C_1 , D_1 , etc., are carefully chosen to represent the points from which curves would begin if drawn to represent the broken lines B_1A , etc., when drawn through the mean positions of the points. The points B_2 , C_2 , etc., upon the horizontal axis are vertically below B_1 , C_1 , etc., and similarly represent the starting points of the mean curves representing the broken lines B_2B , C_2C , etc.

In the theory, if an outside ring can be added to a given configuration of corpuscles without disturbing the original arrangement of the inner rings, the two atoms represented by these collections should possess similar properties, and related spectra, and belong to the same group in the periodic arrangement of the elements, but should fall in the next succeeding series. With this understanding it may be considered that each of the points, B_2 , C_2 , etc., in Fig. 2, represents the starting point of a new

series of elements, and a curve to express this condition may be constructed by laying off as abscissæ the distances OB_2, OC_2, OD_2 , etc., corresponding to ordinates one, two, three, respectively, these numbers representing the number of the ring in the atom. This curve is given in the upper portion of Fig. 3. The origin is taken, however, three units above the origin of the scale indicated at the left to avoid confusing the curve with the others on the same chart. The curve is a hyperbola having the equation,

$$y^2 = .00233x^2 + .213x, \quad (a)$$

and the points B_2, C_2 , etc., are marked to correspond with those in Fig. 2. The units on the vertical scale here correspond to the different series in the arrangement of the elements in the periodic law. The curve itself represents a group in the periodic arrangement of the elements, because the points on

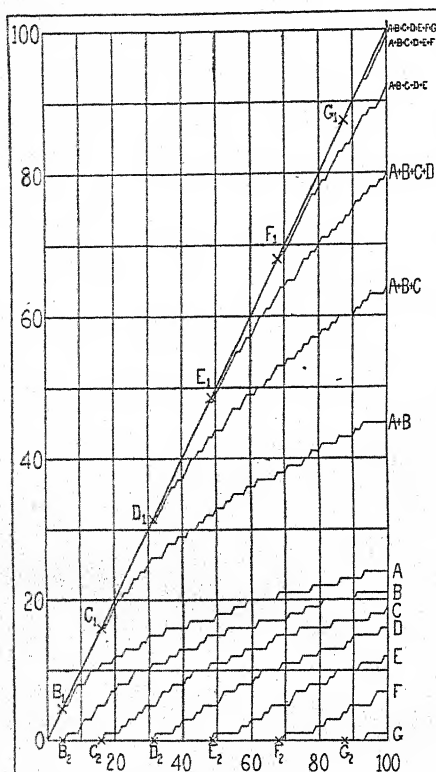


Fig. 2.

this curve correspond to those elements related to each other simply by the addition of successive outer rings.

This curve suggests a scheme for arranging graphically all of the elements as they appear in the table of Mendeleeff. Locate a series of points having abscissæ equal to the atomic weight, which is proportional to the number of corpuscles in the atom, and having ordinates equal to the number of the series in which the element is found in the table.

All of the elements in the 1911 International Table are thus represented in Fig. 3. The roman subscripts after the symbol of the element denote the group in which it is found in the table. Broken lines are drawn connecting elements in the same group. There are thus nine broken lines, each corresponding to one of the nine groups.

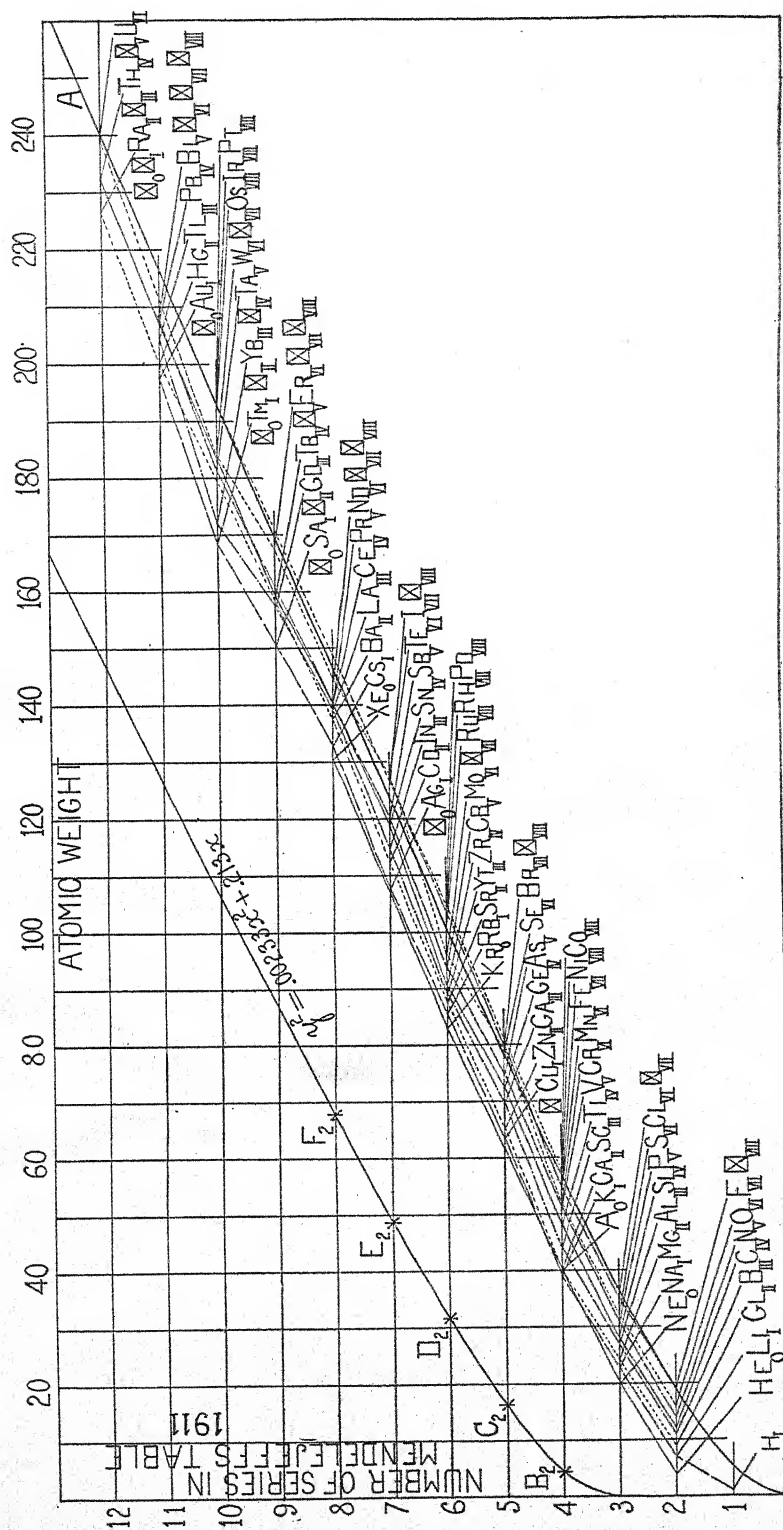


Fig. 3.

DETERMINATION OF THE NUMBER OF CORPUSCLES IN AN ATOM.

To compare the hyperbola B_2F_2 , with these group-curves of the elements it is necessary to know the ratio of the number of corpuscles in the atom to the atomic weight; for it will be remembered that the abscissæ of the hyperbola represent the number of corpuscles in the atom, while those of the group curves of the elements represent atomic weights.

It is permissible, however, without knowing the number of corpuscles in the atom, to change the horizontal scale of the hyperbola to make it fit best the group curves of the elements. By trial it is found that when the abscissæ of the hyperbola are multiplied by the factor 1.16, and then plotted from O as origin, the hyperbola OA is obtained. This curve conforms very closely to the broken lines representing the seventh and eighth groups of the elements.

From these two curves it appears, for example, that the atom consisting of 68 corpuscles, the point F_2 on the curve in the fifth series, corresponds to an atomic weight of $68 \times 1.16 = 79$. An atomic weight of unity, therefore, corresponds to .863 of a corpuscle.

FAMILIES AND GROUPS IN TABLE I.

It is evident from Table I. that any combination of rings in a given series may be obtained from some combination in the preceding series by the addition of an outside ring. For example, consider the combination of 100 corpuscles made up of several rings, the outer ring having 24 corpuscles, the second 21, the third 19, etc. If the first ring of 24 is removed, the combination of 76 corpuscles with six rings is obtained, each ring having precisely the same number of corpuscles as the corresponding ring in the 100 collection. By removing the next ring of 21 corpuscles the collection of 54 is obtained, and by removing the third ring the 36 combination results. Removing the fourth gives 20, the fifth 8, and the sixth one corpuscle only. It makes no difference from which combination we start it is always possible to find a combination in the next lower order or series which has the identical distribution of corpuscles remaining after removing the outer ring.

Table II. is a repetition of Table I. omitting all figures except those giving the number of the series and the total number of corpuscles in the combination. Lines are drawn connecting certain particular numbers in adjoining series for the purpose of indicating those combinations which are identical in the distribution of corpuscles in each ring, when the outer ring in the lower series is omitted. For example, starting with the number 88 and following the lines upward, we pass to 65, 45, 28, 14 and finally to 4, losing an outside ring each time. In the reverse

direction, however, starting with 4 and following the lines downward, we come to 14, at which number the way divides into two alternative paths, the one leading to the numbers 27, 44, 64 and 87, and the other to 28, 45, 65 and 88.

TABLE II.

TABLE II.	
1	0 1 2 3 4 5 6
2	5 6 7 8 9 10 11 12 13 14 15 16
3	17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
4	32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48
5	49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69
6	70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93
7	94 95 96 97 98 99 100

Table II. is represented graphically in Fig. 4, the abscissæ being the total number of corpuscles in the collection, and the corresponding ordinates the number of the series in the table set opposite the horizontal rows of numbers.

The very close resemblance of Fig. 4 to the chart representing the elements, Fig. 3, is too striking to be the result of any chance. It deserves the closest study, but perhaps the most important result is obtained from the suggestion contained in this figure that the proper way to arrange a table of the elements is in tree formation having branches leading away from a common origin. The farther we proceed from this origin the more numerous are the branches. It seems clear that the old way of arranging the elements in Mendeleeff's table, where each

group of elements occupies a column of its own having no alternative column as we descend from series to series, must be incomplete. A need for more spaces in the table to accommodate elements recently suspected to exist has already been felt, and the following family-tree arrangement, based exactly upon Table II. and Fig. 4, supplies this need, and besides gives an arrangement of the elements in tabular form which agrees exactly with the corpuscular ring theory.

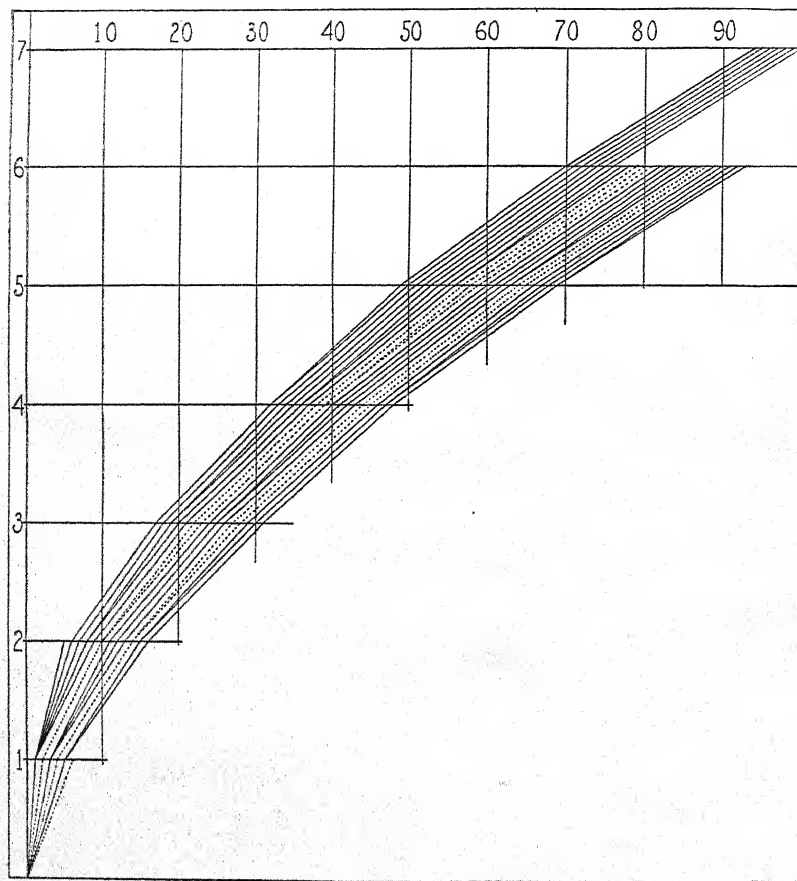


Fig. 4.

An arrangement of squares numbered consecutively from 1 to 100 based exactly upon Table II. and Fig. 4 is presented in Table III. In Fig. 4 five families of lines starting from the first five corpuscles composing the simpler atoms are clearly marked. At the upper limit of the figure there are in all 24 paths, and the upper nine paths, indicated by full lines, all emanate from one corpuscle. The next three, indi-

TABLE III.

[illegible]

cated by dotted lines, start from the group of two corpuscles. Below these the next six full lines emanate from the group of three corpuscles. There are then two dotted lines starting at the group of four corpuscles, and finally a group of five full lines from the group of five corpuscles.

Each of these families is separated very completely from each of the others, as it is impossible to pass from one family to another by any of the lines in Table II. For this reason the vertical double lines are used in Table III. to separate the five families completely.

Although there are but five separate families, yet it is easy to accommodate the nine groups of the elements in Mendelejeff's table. The zero, first and second Mendelejeff groups find their places in the first family, the third group in the second family, the fourth and fifth groups in the third family, the sixth group in the fourth family, and the seventh and eighth groups in the fifth family. In some of the families it is not so difficult to assign elements to their proper squares because there is not much choice. This is particularly true in the fourth family in which there are but two branches in Fig. 4. Oxygen is assigned to the 14th square, sulphur to the 28th, chromium to the 45th, selenium to the 65th, and molybdenum to the 88th square.

The reason for assigning oxygen to the 14th square is because its atomic weight, 16 multiplied by the factor .863 previously determined to be the ratio of the number of corpuscles in the atom to its atomic weight, gives the number 13.8, nearly equal to 14, being only 1.3 per cent. less than 14. Similarly, multiplying the atomic weight of sulphur, 32.07, by .863 we obtain 27.6, a quantity 1.4 per cent. less than 28, which is one of the two numbers immediately under 14 in the 4-family. At this point it is difficult to decide which of the two alternative paths to take because the number 27.6 falls between 27 and 28. By considering the next element in the group, chromium, however, the number 44.8 comes nearer to 45 than to 44, and this consideration places sulphur in the 28th instead of the 27th square. The next two elements, selenium and molybdenum, do not agree as well with the whole numbers 65 and 88, but when all is considered it seems truly remarkable that there is such a close agreement as we find.

A SECOND GRAPHICAL REPRESENTATION OF TABLE II.

If we compare the two outside paths in Table II. the one in family 1 terminating in the number 94, and the other family 5 terminating in

Lowest Curve.	Differences or Number of Paths.	Highest Curve.
93		94
69	24	70
48	21	49
31	17	32
16	15	17
5	11	6
0	5	1

the number 93, it appears that the differences in these two columns are exactly the same.

The same facts are seen in Fig. 4, for, if the lowest curve of the set, beginning at zero and terminating at 93, is moved as a whole one unit to the right and also one unit upward it exactly coincides with the curve beginning with 1 and terminating with 94. This suggests the graphical method of representing the same thing in Fig. 5. If all the lines between

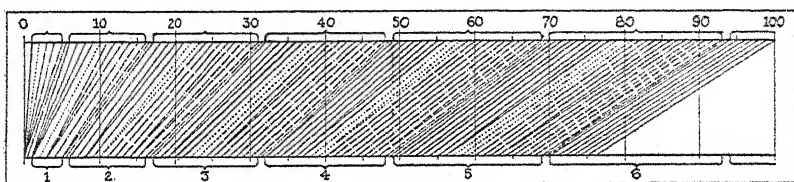


Fig. 5.

the first and second groups in Fig. 4 are lowered one unit without disturbing the horizontal distances, so that all lines on the one unit line will be dropped vertically downwards to the zero line, it is observed that each group of lines exactly fits in with the adjacent groups, adjoining lines being parallel to each other at one unit distance apart.

Fig. 5 is thus a reproduction of Fig. 4 with this difference, that the curves in Fig. 5 may be considered to be drawn upon the surface of a cylinder, each curve following the path of a screw thread having variable pitch, greater as we descend, each turn of the thread corresponding to a new series of the elements. The circumference of the cylinder is equal to one unit on the vertical scale in Fig. 4, that is, to one series. The axis of the cylinder is parallel to the horizontal axis in Fig. 4 which corresponds to the number of corpuscles in the atom.

In this manner all of the atomic weights, or of the number of corpuscles, are brought into one straight line indicated at the top of Fig. 5, and the relationship of any one element to any other may be seen by following the path drawn through that number. Each of the five families is distinguished by a different kind of line, the first family by a full line, the second by a fine dotted line, the third by long dashes, etc.

OUTLINE OF METHOD OF CALCULATING ATOMIC WEIGHTS.

The theory of gravitation proposed by the writer¹ has thrown light on a possible method of calculating atomic weights. The indication is that the gravitational attraction of the earth for the atoms of the different

¹Electrical World, Feb. 10 and 24, 1912. These papers, with the present one, will be included in a series of papers by the writer about to be issued.

elements may be calculated by adding together the attractions and repulsions of the earth for the constituent parts of each atom, namely, the positive electricity and the individual negative corpuscles revolving in rings within the positive electricity. At this time only an outline of the method can be given, but it seems as though efforts in this direction will finally be successful.

Let P denote the attraction of the earth for a charge of positive electricity equal and opposite to the charge carried by one corpuscle or electron, referred to the weight of the hydrogen atom as a unit, and p the total number of corpuscles in the atom. The attraction of the earth for the positive electricity of any atom is then $= Pp$.

To determine the attractions and repulsions upon the negative corpuscles is more difficult because a knowledge of the angular velocity of the corpuscle is required before we can tell which formula to apply in deriving the attraction. The atomic weights themselves may, however, form some guide as to their velocities by finding that formula which gives results in closest agreement with the atomic weight.

Assuming that the angular velocities of the corpuscles in the rings of the different atoms are equal, the values of atomic weights in the only cases as yet calculated come as near as would be expected to the measured atomic weights. Besides a knowledge of the angular velocity, it is also necessary to know the positions of the different rings of corpuscles within the atom. These positions have never been calculated so far as I am aware. It seems worth while in the absence of an accurate solution near the velocity of light to obtain the positions of the corpuscles within an atom on the basis that they are moving slowly compared to the velocity of light, as being possible of calculation for the simpler elements.

An illustration of the method is here given on the basis of equality of angular velocity, and determination of the positions of the corpuscles as if at rest.

The gravitational attraction upon the positive sphere of the atom is given by the formula¹

$$F_p = \frac{\mu q'^2}{6} \frac{ee'}{r^2}, \quad (1)$$

and the force upon a revolving negative corpuscle under this hypothesis by the formula

$$F_n = \frac{\mu q'^2}{6} \frac{(a' - a) ee'}{a' r^2}. \quad (2)$$

The ratio of the force upon a revolving corpuscle to that upon a positive charge of equal amount is the ratio of (2) to (1), namely,

¹ For the derivation of (1) and (2), see *Electrical World*, Feb. 24, 1912.

$$\frac{F_n}{F_p} = \frac{a' - a}{a'}.$$

If n_1, n_2, n_3 , etc., denote the numbers of corpuscles in rings in the atom, number one being the outside ring, the force upon one corpuscle in the outside ring is

$$\frac{a_1 - a}{a_1} P. \quad (3)$$

This is positive and denotes an attraction when a_1 is greater than a , and is a repulsion when a_1 is less than a . a_1 denotes the radius of the outside ring of corpuscles in the atom of the element being attracted, and a the radius of a ring in an average or equivalent atom in the earth. By this is meant the radius of the ring of corpuscles in a hypothetical atom, which has one ring only and represents the equivalent of all the atoms composing the earth. If the earth is supposed to be composed of such hypothetical atoms only, the total number of the corpuscles contained therein being the same as the total number of corpuscles in the atoms making up the earth, then the atomic weights of the elements would not be altered. The determination of the equivalent radius a and of the force P upon a positive charge equal and opposite to that on a corpuscle may be accomplished as follows. If A denotes the atomic weight of any element we obtain by applying formula (1) to the positive electrification and (2) to the corpuscles,

$$A = pP + P \frac{a_1 - a}{a_1} + P \frac{a_2 - a}{a_2} + \dots \quad (4)$$

The first term in the right member denotes the attraction of the earth for the positive electricity, and added to this is one term for each corpuscle within the positive sphere. The quantities a and P are constant because they depend upon the constitution of the earth; and a_1, a_2 , etc., denote the radii of the orbits within the positive sphere of the several corpuscles composing the neutral atom. It is not strictly true that the corpuscles arrange themselves in rings which are exact circles when they form equilibrium figures within the positive sphere, and to apply this method more exactly it will be necessary to locate each corpuscle. If it is assumed, however, that they form circular rings the equation (4) may be written

$$A = pP + n_1 P \frac{a_1 - a}{a_1} + n_2 P \frac{a_2 - a}{a_2} + \dots \quad (5)$$

This applies to the case where there is no corpuscle at the center of the sphere, the first term being due to the positive electricity, the second to

the outside ring and third to the second ring next to the outside, and so on. When there is one corpuscle at the center as in the first family, the repulsion upon it is equal to P , and the first term in the right-hand member of (5) becomes $(p - 1)P$.

Applying the formula to the case of two elements, say hydrogen and glucinum, and assuming that there are two corpuscles in the hydrogen and eight in the glucinum atom, the latter consisting of a ring of seven and one corpuscle at the center of the sphere, we find for hydrogen

$$A = 2P + 2P \frac{.5 - a}{.5} = 1.008 \quad (6)$$

and for glucinum

$$A = (8 - 1)P + 7P \frac{1.19 - a}{1.19} = 9.1. \quad (7)$$

The values of the radii of the rings for hydrogen and for glucinum are referred to the radius of the positive sphere of the hydrogen atom as unity, and the volume of the positive sphere is taken as proportional to the total charge in the atom. The radius of the ring of two corpuscles in hydrogen is .5 or one half the radius of the positive sphere, which is taken as unity. The radius of the ring in glucinum is .747 of the radius of its sphere of positive electricity, but this radius is 1.59 times the radius of the hydrogen sphere, and the ring is therefore $.747 \times 1.59 = 1.19$ times the radius of the positive hydrogen sphere. Dividing (7) by (6) P cancels, and we find that $a = .731$ and $P = .935$.

The formula by which the radius of the outside ring of corpuscles in the atom may be calculated for slow velocities is

$$\left(\frac{a}{b}\right)^3 = \frac{1}{p} \left(\frac{S_n}{4} + m \right),$$

where a = the radius of the outside ring, b = the radius of the sphere of positive electricity, p = the total number of corpuscles in the neutral atom, n = the number of corpuscles in the outside ring, and $m = p - n$ = the remaining corpuscles in rings of smaller radius than a , and in some cases one corpuscle at the center of the sphere.

$$S_n = \operatorname{cosec} \frac{\pi}{n} + \operatorname{cosec} 2 \frac{\pi}{n} + \operatorname{cosec} 3 \frac{\pi}{n} + \cdots \operatorname{cosec} (n - 1) \frac{\pi}{n}.$$

The formula is exact when there is not more than one ring of corpuscles, and approximate for more than one ring.

If it is assumed that the volume of the positive electricity is proportional to the total number of corpuscles contained therein, a relation

which agrees with experiment as nearly as could be expected, and that the radius of the positive electricity in hydrogen is a unit of reference, the radius of the positive sphere for any other atom is

$$b = \left(\frac{p}{2}\right)^{\frac{1}{3}},$$

on the assumption that there are just two corpuscles in the hydrogen atom. This is the only number of corpuscles for the hydrogen atom which seems to give the correct atomic weights.

Table IV. gives the atomic weights for each combination of corpuscles as calculated up to but not including boron, which is the first combination having two rings of corpuscles, assuming that $a = .73$ for the earth, and $P = .933$. It is remarkable that the measured atomic weights of hydrogen, helium, lithium and glucinum coincide almost exactly with the weights calculated for the even numbers of corpuscles, $p = 2, 4, 6$, and 8 respectively, and that carbon is the only element in the second series which has an odd number of corpuscles.

The table has been extended to include all of the second series of elements; but, since the position of the inside ring of corpuscles is not known, the atomic weights have not been calculated as before but assumed to agree with the experimental values, and the radius, c , of the inside ring calculated.

The process of calculating the radii of the rings independently for equilibrium involves much labor, and is being carried out at the present time for two rings only. When there are three or more rings the difficulties rapidly increase.

If it turns out to be true that atomic weights may be calculated in this way, it will be difficult to avoid the conclusion that the fundamental assumptions upon which the calculation is based are nearly correct. One of these assumptions is that the rings in all the atoms which have been calculated revolve with the same angular velocity. It will be interesting to find the value of this velocity. Perhaps some cause which operates to synchronize the electrons of the atoms will be found.

The calculation of the combinations of corpuscles as in Table I. should certainly be carried far enough to include the heaviest element, uranium. The present form of Mendelejeff's table has $12 \times 9 = 108$ squares, and in the eighth group more than one element is placed in one square. With the new arrangement of the family-tree there will be more than 100 additional squares. There is no doubt that all of the squares will never be filled with known stable elements. Many of them may, however, be assigned to certain squares as existing in a temporary state for a brief

time only, the combination serving as a transition station not sufficiently stable for long continued existence.

TABLE IV.

Atomic Weights.

No.	Element.	p	Rings.	a/b	c/b	c/a	a	b	c	Atomic Weights.	
										Calculated.	Observed.
1	Æther . . .	1	—	—	—	—	—	0.794	—	0.000	0.000
2	Hydrogen.	2	2-0	.5000	—	—	.500	1.00	—	1.0076	1.008
3	"	3	3-0	.5773	—	—	.661	1.145	—	2.52	—
4	Helium . .	4	4-0	.6208	—	—	.781	1.26	—	3.98	3.99
5	"	5	5-0	.6505	—	—	.885	1.36	—	5.49	—
6	Lithium . .	6	6-0	.6726	—	—	.97	1.443	—	6.97	6.94
7	"	7	6-1	.739	—	—	1.122	1.518	—	7.55	—
8	Glucinum .	8	7-1	.747	—	—	1.19	1.59	—	9.1	9.1
9	"	9	8-1	.751	—	—	1.24	1.65	—	10.52	—
10	Boron . . .	10	8-2	.783	.222	.284	1.34	1.71	.38	—	11.0
11	Carbon . .	11	8-3	.808+	.243	.300	1.43	1.77	.43	—	12.00
12	Nitrogen .	12	9-3	.808—	.265	.328	1.47	1.82	.482	—	14.01
13	Oxygen . .	14	10-4	.825	.245	.297	1.58	1.91	.469	—	16.00
14	Fluorine .	16	11-5	.838	.266	.317	1.68	2.00	.532	—	19.0

p = total number of corpuscles in the atom, b = radius of sphere of positive electricity, a = radius of outside ring of corpuscles, and c = radius of inside ring each referred to the radius of the positive sphere of hydrogen as unity. $b = (p/2)^{1/2}$, $P = .933$, and $a = .73$ for the earth.

THE ÆTHER.

The first case in the table, which is that of one corpuscle only at the center of the positive sphere having a charge equal to but of opposite sign from the corpuscle, is perhaps the most interesting of all. The formula for this case where there are no rings reduces to

$$A = (p - 1)P$$

and since $p = 1$, the formula makes the atomic weight absolutely equal to zero. That is to say, if such an element exists the earth and sun and planets have absolutely no attraction for it. It would be consequently uniformly distributed throughout space having no tendency to condense anywhere. In this respect it resembles the æther, the medium heretofore supposed to transmit light and electromagnetic waves through space.

It is true that some advocates of the relativity principle deny the existence of any such medium, but it will be nevertheless interesting to put to the test the proposed structure of the æther to ascertain whether it will not by its properties perform all of the functions that are required of it.

Æther according to these indications would be located in the first square of the first series in Mendelejeff's table, and there can be no further use for a zero series as the table is now published. Mendelejeff himself has advocated placing the æther in the table, but I believe that he does not place it in the first series, and also attributes a very small atomic weight to it, not absolutely zero.

COMPARATIVE STUDIES OF MAGNETIC PHENOMENA. II.

RELATION BETWEEN THE JOULE EFFECT AND PERMEABILITY AND
INDUCTION IN THE SAME SPECIMENS OF STEEL.¹

By S. R. WILLIAMS.

INTRODUCTION.

A SURVEY of the work which has been done in investigating the varied phenomena of ferromagnetism indicates that not enough comparative work has been produced. That is, the relations between several magnetic effects have not been established in the same identical specimens where in many cases such comparisons were possible. For instance, one writer may compare magnetic change in length of a steel rod with the permeability while another inquires into the relation between the carbon content and the Villari effect in an entirely different specimen of steel. It is evident that there can be little or no coördination of such material, since one specimen of steel varies from another in degree of hardness, the previous heat and magnetic treatment, to say nothing of the carbon content itself, and other alloys and impurities. If the carbon content, induction, permeability, the Joule and other magnetostrictive effects, the Villari effect, the Kerr magnetic effect, etc., could all be studied in the same specimens, such a comparison would make an important contribution to our knowledge of the subject of ferromagnetism and especially to any attempt to get at a working hypothesis of ferromagnetism. This latter point cannot be too strongly emphasized, because if we had a working theory² which would coördinate such diverse phenomena as the magnetostrictive effects with the Kerr magnetic effect or the Faraday effect in thin films, it would furnish a great impetus to the work now being done in this field. With this phase of the subject in mind I have made a collection of rods and tubes of various ferromagnetic substances in which I propose to study the relation of all the different magnetic effects possible to be investigated in specimens of the rod or tube form.

In a former paper³ I have pointed out that there are indications that

¹ Presented before the meeting of the American Physical Society in Washington, Dec. 29, 1911.

² Knowlton, *Ter. Mag.*, Vol. 15, No. 1, 1910.

³ *PHYS. REV.*, p. 46, Vol. 34, Jan., 1912.

the Joule magnetostrictive effect may well be studied in comparison with other magnetic effects, and so in a series of studies I wish to discuss some of these relations which I have found between the Joule and other magnetic effects. The present paper deals with the relation between the Joule magnetostrictive effect, and the permeability and induction for corresponding field strengths in the same specimens of steel. In a paper¹ on the "Comparison of the Joule and Wiedemann Magnetostrictive Effects in Steel Tubes" I showed that the permeability seemed to play an important rôle in these two phenomena. It therefore seemed worth while to study in greater detail the relation between the Joule effect and the quantities involved in the relation $B = \mu H$. To this will be added data on the carbon content.

THE JOULE EFFECT.

One of the difficulties in measuring the change in length of any ferromagnetic substance due to a magnetic field is the masking of the effect by changes in length due to variation in temperature. By some investigators this has been overcome by compensation devices, by others, temperature changes have been reduced to a minimum by inclosing the specimen in a jacket whose temperature could be maintained fairly constant by allowing water to flow through the system.

To make a series of observations over any great range of field strengths without some temperature control is impossible with the ordinary telescope and scale method, because it takes too long to adjust the field strength and then to read the changes in length which have occurred. In previous work on the Joule effect I found that if in varying the magnetic field the changes were produced quickly enough, the changes due to temperature did not appear in time to vitiate the results. It therefore occurred to me that if some automatic method could be devised for quickly recording the changes in length with the corresponding field strength, it would be a great advantage, as we would then eliminate heat effects by obtaining the desired results before disturbances due to temperature appeared. That this has been accomplished in a very satisfactory way is shown, I think, in the results given and from the observed fact that after a complete series of readings had been obtained by this method some time elapsed before temperature changes manifested themselves.

The general method of measuring the Joule effect is to place the specimen experimented upon along the axis of a solenoid whose ampere-turns are known, and measure by means of some multiplying device the changes in length which occur as the field in the solenoid is varied from zero up-

¹ PHYS. REV., p. 293, Febr., 1911.

wards. Schematically this is shown in Fig. 1, where as a multiplier of the effect I have used a system of lever and mirror more fully described in a previous paper.¹ This system was found very satisfactory and would

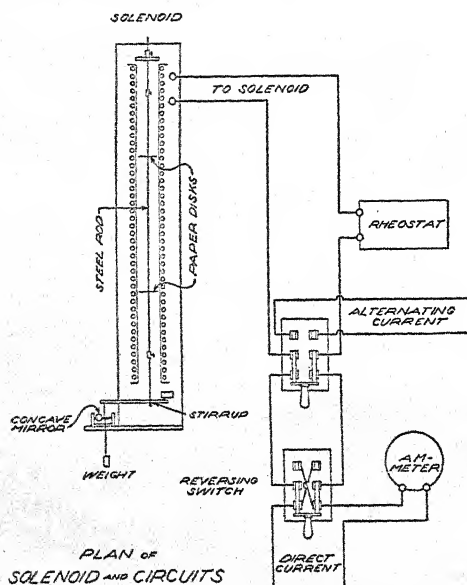


Fig. 1.

could be repeated to one part in ten thousand. In connection with this scheme of multiplying the effect I have devised a simple arrangement for photographing the deflections of the mirror and doing it in a very short space of time. This is shown in Fig. 2. Mr. T. S. Towle, an advanced student in the department, worked out the mechanical details of the arrangement and I am much indebted to him for the success of the method.

MR. TOWLE'S EXPERIMENTS.

The light from a straight filament tungsten lamp, *L*, fell upon the concave mirror, *M*, which formed an image of the filament on the camera, *K*.

The lens was replaced by a diaphragm containing a long narrow slit. The length of the image of the filament was at right angle to the slit so that only a spot of light passed through. As the rod changed its length the mirror was tilted by means of the lever attached to the lower end of the rod and the image of the lamp filament moved up or down the slit

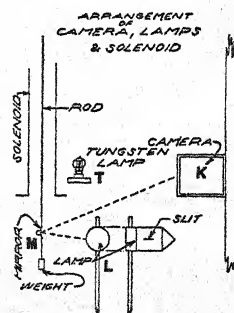


Fig. 2.

¹ PHYS. REV., p. 285, Febr., 1911.

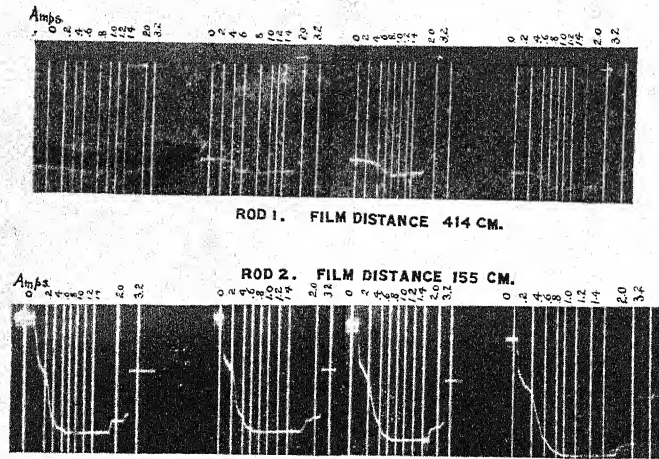


Fig. 4.

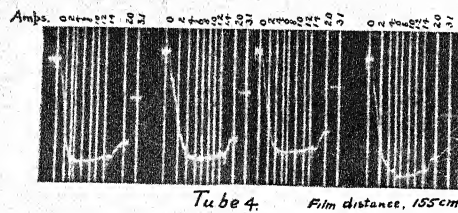
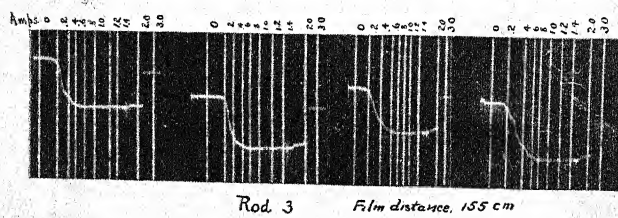


Fig. 5

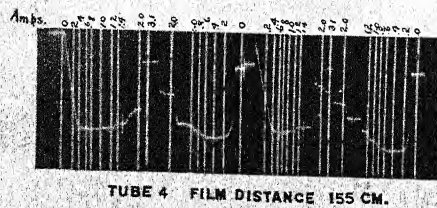
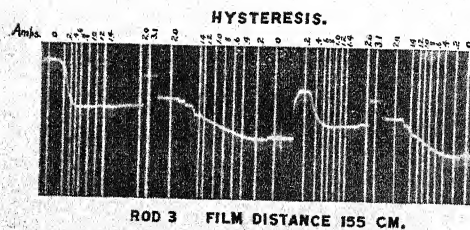


Fig. 6.

S. R. WILLIAMS.
PLATE I.

of the camera depending upon whether the rod shortened or lengthened. Immediately behind the slit was a photographic film which could be moved along by means of a suitable mechanism which is indicated in Fig.

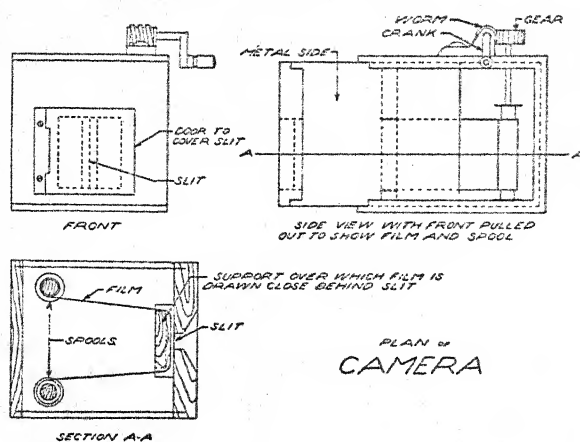


Fig. 3.

3. The graphs obtained by this means are shown in Figs. 4, 5 and 6. The actual deflection of the spot of light can be measured from these graphs and knowing the film distance from the mirror and the multiplying power of the levers, the elongation of the rods could be determined. For recording the field strengths corresponding to a certain elongation of the rod, a tungsten lamp, *T*, Fig. 2, was placed in front at some distance

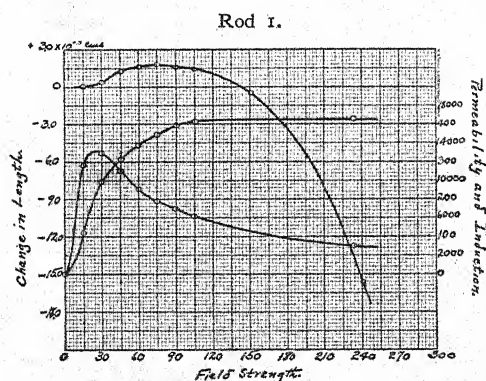


Fig. 7.

away and above the camera. When the current in the solenoid reached a certain known value this lamp was flashed, making an image of the slit as a marker across the photographic film. Knowing the ampere turns

of the solenoid, the field strength for each flash could easily be determined. The values of the deflections are taken at the intersections of the curve and the straight line serving as a marker, because at these points we know the field strengths. The method of taking observations was as

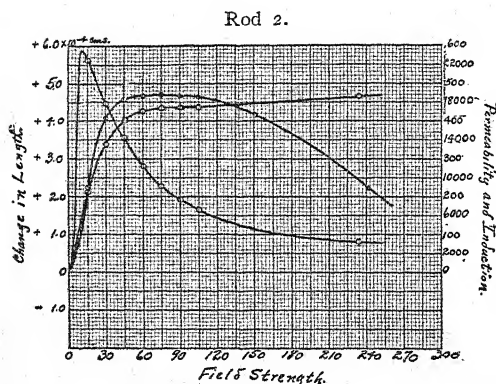


Fig. 8.

follows: A rod was placed in position and magnetized a few times and then demagnetized by sending a decreasing alternating current through the solenoid. The system was then allowed to stand for some little time

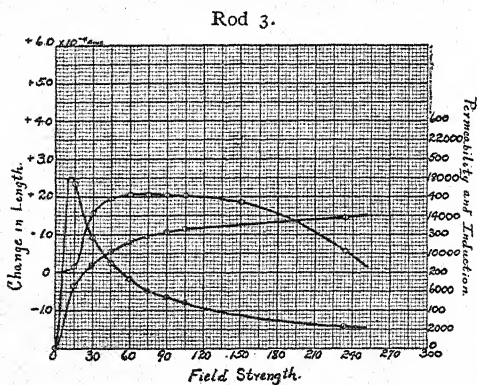


Fig. 9.

until things became settled. At a signal from the operator reading the ammeter and controlling the rheostat in series with the solenoid the assistant would commence to turn the crank operating the motion of the film. The operator at the ammeter would immediately begin to increase the current in the solenoid from zero upwards by means of a liquid rheostat which could be varied steadily and not by steps. As the ammeter pointer moved over the points marked 0, .2, .4, .6, etc., amperes,

the lamp, T , would be flashed by means of a telegraph key in series with it. In this way a whole series of readings could be obtained in a few moments of time and on development of the plates the accurate measurement of the deflections could be made at one's pleasure. The rate of moving the film does not matter because at the instant the lamp flashes

Tube 4.

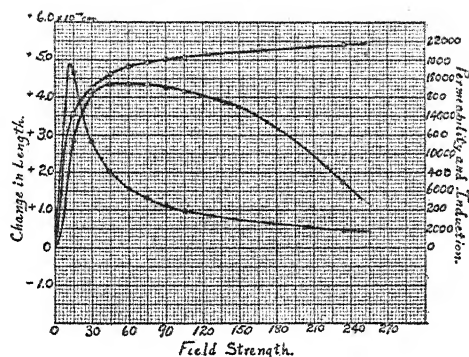


Fig. 10.

the point of intersection of the curve and the line marking the field strength gives the deflection for that particular field strength for which the lamp was flashed. The points are then plotted as shown in Figs. 7, 8, 9 and 10. The film might be operated by clockwork instead of by an assistant, but it was found so convenient to have an assistant that this plan was not carried out.

DESCRIPTION OF RODS AND TUBE USED.

Three rods and one tube of steel were investigated with respect to the Joule effect, permeability and induction, and were numbered 1, 2, 3 and 4 respectively. Their dimensions are as follows:

TABLE I.

	No. 1.	No. 2.	No. 3.	No. 4.
Length.....	80 cm.	80 cm.	80 cm.	80 cm.
Diameters.....	.6386 cm.	.6300 cm.	.5800 cm.	{ O.D. .6409 cm. I.D. .5488 cm.

Specimens in the rod or tube form were studied because it would seem that a greater number of comparisons could be carried out on samples in that shape and at the same time they would be more convenient for

various heat treatments. Again it seemed that a rod 80 cm. long would give a better idea of the character of any particular specimen of steel than a short one, say 10 cm. long. Pierce¹ has shown that the hardness varies greatly along any given bar, hence if ten centimeters from one end of a long rod was compared with ten centimeters from the other end, very discordant results might arise because the hardness plays an important part in the magnetic properties of steel. For a given carbon content the behavior of a long rod may more safely be compared with the behavior of another long rod since one takes the summation effect of the whole. This applies particularly to a change in length study.

Fig. 11 shows the brass extensions used at both ends of the rods so that they would extend beyond the ends of the solenoid, whose length was 100 cm. These extensions also served as support for the upper end of rod and as a connection for the levers at the lower.

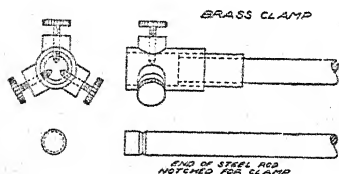


Fig. 11.

Figs. 4 and 5 show sets of graphs for the rods 1, 2 and 3, and the tube 4 respectively. In each set are four graphs and the curves of change in length in

Figs. 7, 8, 9 and 10 are the mean of these four graphs for each specimen. The four graphs in each set show the deflections when field is first down then up, then down and finally up consecutively in the solenoid, demagnetization occurring between each reversal.

I am much indebted to Mr. W. N. Crafts, president of the Crucible Steel Forge Co., Cleveland, Ohio, for the rods 1 and 2 and also wish to express my appreciation of the many valuable suggestions received from him regarding the production and heat treatments of steel. The effect of heat on these specimens of steel will be studied later. Through the courtesy of the chemical department, Mr. Earl Rice, an advanced student, determined the carbon content of the specimens by the chromic acid method. I take this opportunity to express my thanks for this favor. The values found are as follows:

TABLE II.

Rod No.	Carbon Content. Per Cent.
1.....	1.05
2.....	0.20
3.....	1.04
4.....	0.32

¹ Pierce, Proc. Amer. Acad., Vol. 46, No. 8, Oct., 1910.

In addition to the results obtained from my own observations I have calculated and plotted the results obtained by other observers. In Fig. 12

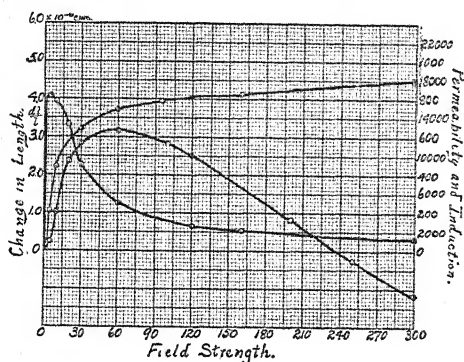


Fig. 12.

are shown the results obtained by Nagaoka and Honda¹ on a specimen of steel, ten centimeters long. I add the curves for the sake of showing how they corroborate the general results of my own work.

HYSTERESIS.

This rapid method of obtaining results is particularly applicable to a study of the hysteresis occurring in the change in length due to a magnetic field. Fig. 6 shows graphs of the hysteresis found in the length change of rod 3 and tube 4, while Fig. 13 is a plot of the results obtained from Fig. 6.

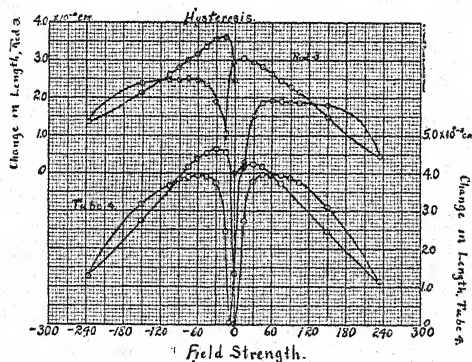


Fig. 13.

DISCUSSION OF RESULTS.

The photographs obtained from rod 1, Fig. 4, were taken at a film distance of 4.14 meters, nearly three times as far away as that for rods

¹ Nagaoka and Honda, Phil. Mag., p. 45, July, 1902.

2 and 3 and tube 4. Even at this large film distance the initial elongation is very small. When working at the film distance used for the other rods I could not definitely detect this initial elongation and so appeared to have a specimen whose behavior was similar to that found by Bidwell¹ in a sample of steel. Others have observed the same phenomenon in steel, viz., the absence of an initial lengthening. From several cases of this sort I have been led to believe that if the magnifying power of the device for showing the Joule effect had been great enough in the case of other observers, they too would have found an initial lengthening for steel. The magnifying power used for rod 1 was about 24,000. If the assumption just made holds good it points to a uniformity in the behavior of iron and steel in the Joule effect, viz., an increase in length for small field strengths and a shortening for strong.²

In Figs. 7, 8, 9, 10 and 11, I have compared the curves for the Joule effect, permeability and induction. What seems to me to be of importance in the relation of these curves is the fact that the maximum elongation occurs at a field strength not far from that at which the knee of the induction curve occurs and necessarily the maximum permeability occurs in all cases at a lower but related field strength, as shown in previous work.

In Winkelmann's *Handbuch der Physik*,³ Auerbach has given an excellent resumé of three important theories of magnetism, (1) *Scheidungshypothese*, (2) *Richtungshypothese*, (3) *Wirbelhypothese*. The relation just spoken of between the change of length and the induction is, in my opinion, a strong argument in favor of the second hypothesis, viz., the orientation theory. As I pointed out in a previous paper⁴ the magnetostrictive effects are easily explained from the standpoint of the orientation of elongated elementary magnets. If these elementary particles possess permeability, as I have assumed, then their rotation will influence the induction and permeability curves and we will get a definite relation between them and the change in length curve as I have shown. Cotton and Mouton⁵ suggest that the phenomena of electric and magnetic double refraction may be explained by means of the orientation of particles.

At present I am studying the permeability and induction of a group of oblate spheroids of soft iron which I can orient mechanically about axes lying in their equatorial planes. Their behavior, I believe, will throw much light on this phase of the subject.

¹ Bidwell, *Proc. Roy. Soc.*, p. 228, Vol. 55, 1894.

² Due to faintness of photograph the elongation of Rod 1 for amperes 3.2, Fig. 4, is indicated by arrows.

³ Vol. 5, Part 2, *Elek. u. Mag.*, pp. 48-49.

⁴ *Loc. cit.*

⁵ Cotton and Mouton, *Comptes Rendus*, t. 147, p. 193, 1908.

SUMMARY.

1. These experiments have developed a very simple method for rapidly securing a series of readings on the Joule magnetostrictive effect, thus eliminating heat effects. The method is not only applicable to this effect but to the Wiedemann magnetostrictive effect by simply rotating the camera through 90° and arranging the lamp filament in a vertical direction. It is also applicable in obtaining permanent records of the magnetometer in magnetometric studies of the magnetic qualities of iron, steel, etc.

2. This study has established certain relations between the induction and change in length which suggest from the standpoint of the orientation theory that the permeability and induction curves are the resultant curves of a group of small elements which also possess the property of permeability, but that the resultant curves of permeability and induction are not the same as that of the elementary particles because they are variously oriented and produce mutual effects which change the character of the resultant curves.

3. The results obtained from rod 1 point to a similarity in the behavior of iron and steel for the Joule effect.

PHYSICAL LABORATORY,
OBERLIN COLLEGE, OBERLIN, OHIO,
December, 1911.

NOTE ON THE OPTICAL EFFECTS IN MOVING MEDIA.

BY Y. LAUB.

IN the second volume (second edition) of his book *Theorie der Elektrizitaet*, 1908, Professor Abraham proposes a modification of Einstein's electrodynamical theory. He abandons the principle of a constant velocity of light, and introduces in addition to the relativity principle the assumption of a non-constant velocity, according to which the velocity of light depends on the motion of the observer even in vacuum. If c denote the velocity of light as measured by an observer at rest, the velocity c' as measured by an observer moving with velocity v is given by the equation

$$c' = c \sqrt{1 - \frac{v^2}{c^2}}.$$

Conversely the time of a watch is independent of the movement of the reference system.

The electromagnetic equations have in spite of the non-constant light velocity, an invariant character, since in them we are only concerned with the product of light velocity and time t , so that the equation

$$c't' = ct \sqrt{1 - \frac{v^2}{c^2}}$$

still holds.

Following on my former work,¹ I shall here show that it is possible to differentiate experimentally between the principle of constant and non-constant light velocity.

From the observations of many experimenters it is definitely established that the canal rays give a displaced line spectrum. Let light rays be propagated through a tube where canal rays are being produced, and consider the dispersion of the light. We form the following picture of the process of dispersion. There are present in the moving canal rays resonating couplets. When light is propagated in the tube, the couplets are set in vibration, but the whole particle remains electrically uncharged,

¹ Y. Laub, *Sitzungsb. d. Heidelberger Akademie, Math. nat. Kl.*, 6 Abh., 1909; *Ann. d. Physik*, Bd. 29, 1909, and *Beibl. d. Phys.*, 1910.

so that neither the direction nor the magnitude of the velocity of the particles suffers any alteration. To obtain the index of refraction of the light we proceed as follows: We introduce two systems of coordinates, K and K' . If matter is present, which is at rest relative to K' , we have the following equations for the system K' :

$$(1a) \quad \begin{cases} \frac{d^2 \mathbf{P}'_x}{dt'^2} = -n_0^2 \mathbf{P}'_x - 2k \frac{d\mathbf{P}'_x}{dt'} + \sum \frac{eN'}{\mu} \mathbf{E}'_x, \\ \frac{d^2 \mathbf{P}'_y}{dt'^2} = -n_0^2 \mathbf{P}'_y - 2k \frac{d\mathbf{P}'_y}{dt'} + \sum \frac{eN'}{\mu} \mathbf{E}'_y, \\ \dots \dots \dots \end{cases}$$

where the vector \mathbf{P}' is the electric polarization, the vector \mathbf{E}' is the electric force, μ the mass of the electron, e the electric charge, n_0 the frequency of the free vibration, k a constant (coef. of viscosity) of the medium.

We now consider a second rectangular system K . The origin of K' moves with a constant velocity v in the direction of the positive x -axis of K . Then from the relativity principle:

$$(2) \quad \begin{cases} x' = \beta(x - vt), \\ y' = y, \\ z' = z, \end{cases} \quad \left(\beta = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right)$$

$$(3) \quad t' = \beta \left(t - \frac{v}{c^2} x \right);$$

where x, y, z and t refer to the system K .

Further we have between the electric and magnetic vectors $\mathbf{E}', \mathbf{D}', \mathbf{H}', \mathbf{B}', \mathbf{P}'$, referred to the system K .¹

$$(4) \quad \begin{cases} \mathbf{E}'_x = \mathbf{E}_x, \\ \mathbf{E}'_y = \beta \left(\mathbf{E}_y - \frac{v}{c} \mathbf{B}_z \right), \\ \mathbf{E}'_z = \beta \left(\mathbf{E}_z + \frac{v}{c} \mathbf{B}_y \right); \\ \mathbf{D}'_x = \mathbf{D}_x, \\ \mathbf{D}'_y = \beta \left(\mathbf{D}_y - \frac{v}{c} \mathbf{H}_z \right), \\ \mathbf{D}'_z = \beta \left(\mathbf{D}_z + \frac{v}{c} \mathbf{H}_y \right); \end{cases}$$

¹ A. Einstein u. Y. Laub, Ann. d. Phys., 26, p. 534, 1908.

From (7) and (8) we have:

$$(9) \quad c^2 \gamma^2 - 1 = \frac{\beta \rho}{n_0^2 - \beta^2 n^2 + 2\beta k n}.$$

If we put

$$(10) \quad c\gamma = \nu - i\kappa,$$

then ν is the index of refraction and κ the coefficient of extinction of the gas.

For simplicity we write:

$$a = \rho \frac{n_0^2 - \beta^2 n^2}{(n_0^2 - \beta^2 n^2)^2 + 4\beta^2 k^2 n^2},$$

$$b = \rho \frac{2\beta k n}{(n_0^2 - \beta^2 n^2)^2 + 4\beta^2 k^2 n^2}.$$

Then equating real imaginary parts

$$(11) \quad \nu^2 - \kappa^2 = 1 + a,$$

$$(12) \quad 2\nu\kappa = b.$$

If we can neglect the absorption and take $k = 0$, we have:

$$(13) \quad \nu^2 = 1 + \rho \frac{\beta}{n_0^2 - \beta^2 n^2}.$$

If we limit ourselves to magnitudes of the second order in v/c we have for the index of refraction the formula:

$$(14) \quad \nu^2 = 1 + \frac{\rho \left(1 + \frac{1}{2} \frac{v^2}{c^2} \right)}{n_0^2 - n^2} - \frac{v^2}{c^2} \frac{\rho n^2}{(n_0^2 - n^2)^2}.$$

In the neighborhood of the free period n_0 of the gas we can put $n = n_0 + d$, where d is very small. Then the formula (14) becomes

$$(15) \quad \nu^2 - 1 = \frac{\rho}{2nd} - \frac{v^2}{c^2} \frac{\rho}{4d^3}.$$

As we see it follows from Einstein's theory that the index of refraction depends on the velocity in the manner indicated in equation (15) when the velocity of the carriers in the canal rays is at *right* angles to the direction of observation. *Analogous effects hold for the absorption.*

If we put $E_x = 0$ in the equation (1) we get an equation whose meaning we can express as follows:

Let n_0 the frequency of the light when the source is at rest.

If the source of light moves with a velocity v perpendicular to the

direction of observation, then the frequency n as measured by an observer at rest is given by the equation:

$$(16) \quad n = \frac{n_0}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

We may call *this the transverse Doppler effect.*

Let us proceed from the standpoint of a *non-constant* velocity of light. Then according to Abraham, the Lorentz-Einstein equations of transformation for space and time coordinates must be written:

$$x' = \beta \left(x - \frac{v}{c} l \right),$$

$$y' = y,$$

$$z' = z;$$

$$l' = \beta \left(l - \frac{v}{c} x \right),$$

where l is the path traversed by the light. On the other hand the equations for the electric and magnetic vectors remain unaltered, as in equations (4)-(6).

Let us put the variable

$$l' = c't'$$

in the differential equations. Proceeding as above and remembering that $c' = c\sqrt{1 - v^2/c^2}$ we finally obtain in place of (16):

$$(17) \quad n = n_0.$$

An experimental decision between the two equations (16) and (17) would be of special importance, for a confirmation of equation (16) would be direct proof of the relativity of time. A transverse Doppler effect follows from the relativity principle only if the time of a watch is dependent on the state of motion of the observer.

Further we could decide, whether the velocity of light in vacuo is a universal constant. It may be here observed that recently it has been questioned whether the velocity of light is truly a universal constant.

Assuming as is now possible, that we have canal rays of a velocity the order 2×10^8 cm. per sec., calculation shows that we could expect as transverse Doppler effect a displacement of the D -lines of some hundreds Angström units.

For dispersion measurements it would be best to use canal rays in vapors.

In conclusion let us consider the direct longitudinal Zeeman effect from both standpoints. In the system K' , the following equations hold:

$$(18) \quad \begin{cases} \frac{d^2 \mathbf{P}_x'}{dt'^2} + \eta H' \frac{d\mathbf{P}_y'}{dt'} + n_0^2 \mathbf{P}_x' = 0, \\ \frac{d^2 \mathbf{P}_y'}{dt'^2} - \eta H' \frac{d\mathbf{P}_x'}{dt'} + n_0^2 \mathbf{P}_y' = 0, \\ \frac{d^2 \mathbf{P}_z'}{dt'^2} + n_0^2 \mathbf{P}_z' = 0, \end{cases}$$

where H' is the extremal magnetic field and $\eta = \frac{[l]}{c\mu}$.

If we transform the equations to the system K , we obtain

$$\begin{aligned} & \beta^2 \frac{\partial^2 \mathbf{P}_x}{\partial t^2} + \beta^2 v^2 \frac{\partial^2 \mathbf{P}_x}{\partial x^2} + 2\beta^2 v \frac{\partial^2 \mathbf{P}_x}{\partial x \partial t} + \beta^2 \eta \left(H - \frac{v}{c} \Theta \right) \frac{\partial \left(\mathbf{P}_y + \frac{v}{c} \mathbf{Q}_z \right)}{\partial t} \\ & \quad + \beta^2 \eta v \left(H - \frac{v}{c} \Theta \right) \frac{\partial \left(\mathbf{P}_y + \frac{v}{c} \mathbf{Q}_z \right)}{\partial x} + n_0^2 \mathbf{P}_x = 0, \\ & \beta^2 \frac{\partial^2 \left(\mathbf{P}_y + \frac{v}{c} \mathbf{Q}_z \right)}{\partial t^2} + \beta^2 v^2 \frac{\partial^2 \left(\mathbf{P}_y + \frac{v}{c} \mathbf{Q}_z \right)}{\partial x^2} + 2\beta^2 v \frac{\partial^2 \left(\mathbf{P}_y + \frac{v}{c} \mathbf{Q}_z \right)}{\partial x \partial t} \\ & \quad - \beta \eta \left(H - \frac{v}{c} \Theta \right) \frac{\partial \mathbf{P}_x}{\partial t} - \beta \eta v \left(H - \frac{v}{c} \Theta \right) \frac{\partial \mathbf{P}_x}{\partial x} + n_0^2 \left(\mathbf{P}_y + \frac{v}{c} \mathbf{Q}_z \right) = 0, \\ & \beta^2 \frac{\partial^2 \left(\mathbf{P}_z - \frac{v}{c} \mathbf{Q}_y \right)}{\partial t^2} + \beta^2 v^2 \frac{\partial^2 \left(\mathbf{P}_z - \frac{v}{c} \mathbf{Q}_y \right)}{\partial x^2} + 2\beta^2 v \frac{\partial^2 \left(\mathbf{P}_z - \frac{v}{c} \mathbf{Q}_y \right)}{\partial x \partial t} \\ & \quad + n_0^2 \left(\mathbf{P}_z - \frac{v}{c} \mathbf{Q}_y \right) = 0, \end{aligned}$$

where Θ is the "extremal electric displacement." Remembering that for an observer moving with the magnetic field the *electric displacement is zero*, we obtain according to Einstein the following equations:

$$(18a) \quad \begin{cases} \beta^2 \frac{\partial^2 \mathbf{P}_x}{\partial t^2} + \beta^2 v^2 \frac{\partial^2 \mathbf{P}_x}{\partial x^2} + 2\beta^2 v \frac{\partial^2 \mathbf{P}_x}{\partial x \partial t} + \frac{1}{\beta} \eta H \frac{\partial \mathbf{P}_y}{\partial t} + \frac{1}{\beta} \eta v H \frac{\partial \mathbf{P}_y}{\partial x} + n_0^2 \mathbf{P}_x = 0, \\ \beta^2 \frac{\partial^2 \mathbf{P}_y}{\partial t^2} + \beta^2 v^2 \frac{\partial^2 \mathbf{P}_y}{\partial x^2} + 2\beta^2 v \frac{\partial^2 \mathbf{P}_y}{\partial x \partial t} - \beta \eta H \frac{\partial \mathbf{P}_x}{\partial t} - \beta \eta v H \frac{\partial \mathbf{P}_x}{\partial x} + n_0^2 \mathbf{P}_y = 0, \\ \beta^2 \frac{\partial^2 \mathbf{P}_z}{\partial t^2} + \beta^2 v^2 \frac{\partial^2 \mathbf{P}_z}{\partial x^2} + 2\beta^2 v \frac{\partial^2 \mathbf{P}_z}{\partial x \partial t} + n_0^2 \mathbf{P}_z = 0. \end{cases}$$

If we solve the differential equations (18a), we obtain, on Einstein's theory for the vibration frequency of the resolved spectral lines:

$$(19) \quad n_2 - n_1 = \eta H \left(1 - \frac{v^2}{c^2} \right),$$

According to Abraham's theory we get

$$(20) \quad n_2 - n_1 = \eta H \sqrt{1 - \frac{v^2}{c^2}},$$

when n_1 the frequency of the slower, n_2 of the quicker vibrations.

If we limit ourselves to magnitudes of the second order in v/c we have according to Einstein:

$$n_2 - n_1 = \eta H \left(1 - \frac{v^2}{c^2} \right),$$

and according to Abraham

$$n_2 - n_1 = \eta H \left(1 - \frac{1}{2} \frac{v^2}{c^2} \right).$$

Then it seems that the longitudinal Zeeman effect also affords a possibility of deciding experimentally concerning the relativity of time. The optical phenomena here considered are especially noteworthy, as these not only give information on the relativity principle in the more restricted sense, but also enable us to decide whether the velocity of light is a universal constant.

LA PLATA,

DEPARTMENT OF GEOPHYSICS OF THE NATIONAL UNIVERSITY,

August, 1911.

A DETERMINATION OF THE RATIO OF THE SPECIFIC HEATS
AND THE SPECIFIC HEAT AT CONSTANT PRESSURE
OF AIR AND CARBON DIOXIDE.

By H. W. MOODY.

I. INTRODUCTION.

AS a result of some work done by Millikan, Chapman, and Moody (1) in investigating the temperature changes in a Wilson fog chamber and a later extension by Chapman (2) of the method to the observation of the temperatures produced, upon expansion, in chambers filled with saturated vapors, it became evident that the thermocouple was exceedingly well adapted to the measurement of instantaneous temperatures. The smallness of the wires now obtainable and, hence, the smallness of the heat capacities and heat conductivities of the leads give it decided advantages over other schemes for the direct measurement of the temperature changes produced by adiabatic expansions.

A marked advance in the accuracy of the determination of γ was made by Lummer and Pringsheim (3) when they first proposed to make this determination depend upon a direct temperature measurement. In their work, they made use of a platinum bolometer strip. Later, Makower (4) experimented according to the same method, save that he employed a platinum thermometer for the measurement of the temperature changes. In view of the fact that there is so little agreement in the values of γ found by earlier observers, it has seemed worth while to make a new determination, using the thermocouple for the detection of the changes in temperature.

II. EXPERIMENTAL WORK.

Method of Experiment.—The method used rests directly upon the law of adiabatic expansion of the gas considered. If p and θ are taken as variables, we have, for the case of a perfect gas expanding adiabatically, from an initial pressure, p_1 , to a final pressure, p_2 ,

$$(1) \quad \left(\frac{\theta_1}{\theta_2}\right)^\gamma = \left(\frac{p_1}{p_2}\right)^{\gamma-1},$$

wherein θ_1 and θ_2 are the absolute temperatures before and after expansion and γ is the ratio of the specific heats by constant pressure and by constant volume. Upon the solution of equation (1) for γ , we obtain

$$(2) \quad \gamma = \frac{\log \frac{p_1}{p_2}}{\log \frac{p_1}{p_2} - \log \frac{\theta_1}{\theta_2}}.$$

Hence, to compute γ , we need to be able to measure p_1 , p_2 , θ_1 , and θ_2 . The first three of these quantities present little difficulty, since each can be measured with sufficient leisure to permit of careful measurement. To get θ_2 , we need at our command such a device as will give us the instantaneous value of the temperature assumed by the gas in the adiabatic expansion, since this temperature changes rapidly from heat flowing in from the walls of the surrounding vessel.

Arrangement of Apparatus.—The arrangement of the apparatus and its operation are shown in Fig. 1. To hold the quantity of gas to be experimented upon, a large glass carboy, C , of about 60 liters capacity was employed. In this were three openings, one of which, O_1 , communicated, by the way of the drying system, D , and the manometers, M_1 , M_2 , with the compression pump or other source of gas. For air, the drying system consisted of a number of bottles of sulphuric acid in series with several long tubes filled with granular calcium chloride. For carbon dioxide, the calcium chloride was replaced by tubes of phosphorus pentoxide, which was renewed whenever it showed any signs of moisture.

The second opening, O_2 , was used for the introduction of the thermocouple into the carboy. The thermocouple, made of 1 mil copper and constantan wire, formed a part of one circuit in a potentiometer system. These wires, in order that the couple, c_1 , might be placed approximately at the center of the body of gas, were led in through glass tubes. These latter were bent, just inside the walls of the vessel, so that they were spread, leaving a length of 20 or 25 cm. of bare wire with one junction in the middle. The inner end of these tubes were sealed with heavy wax in order to shut off communication with the outside. The second junction of the couple was enclosed in a small capsule, c_2 , just outside the carboy, and so was always at the temperature of the bath, B , in which the carboy itself was placed. The lead wires continued from these capsules through glass tubes to the surface of the water bath, thence to the potentiometer system.

The other opening, O_2 , was the avenue of outflow for the gas at expansion. It was closed by a large valve which, when opened, formed a vent of 8 sq. cm. and so made possible a very sudden expansion since this valve could be operated rapidly.

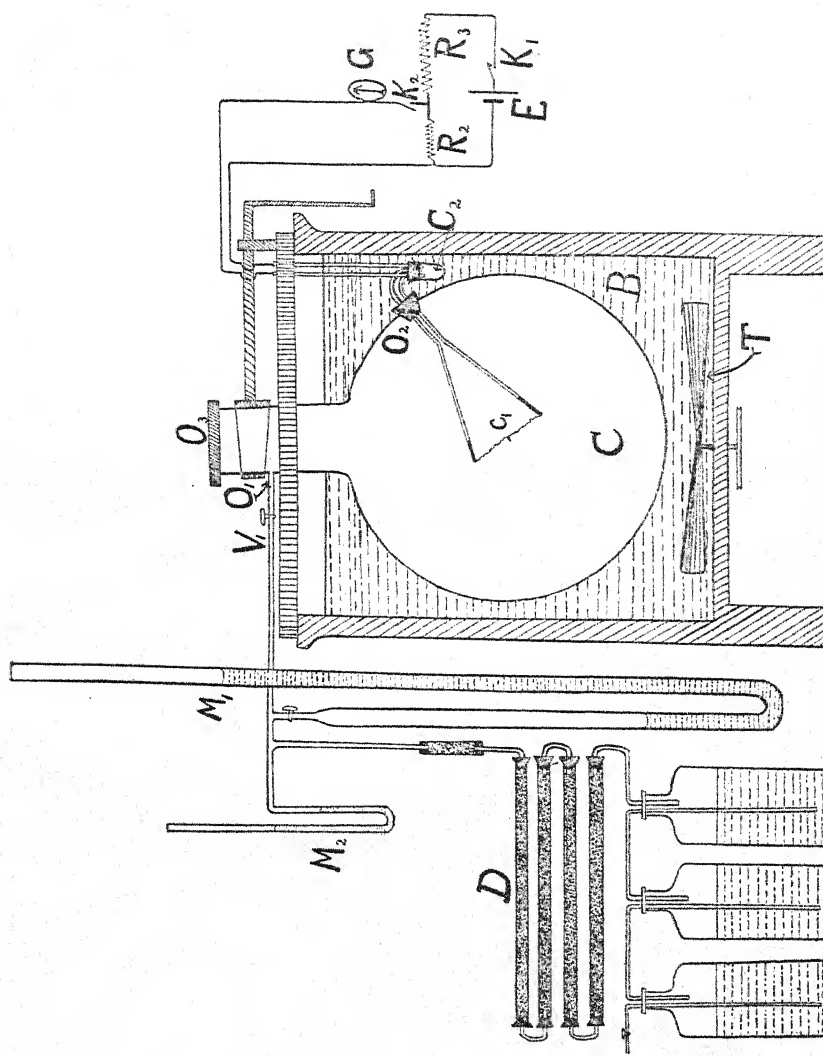


Fig. 1.

The carboy was supported in the center of a large water bath. The stirring device consisted in a large turbine fan, T , motor driven and placed in the center of the bottom of the tank. That the circulation of the water was excellent was evidenced by the movement of particles in the

water. To test the uniformity of temperature throughout the tank, a sensitive thermometer was carried about, readings being taken at many points, and in no place was a variation noticed in excess of a few thousandths of a degree, an amount which could be accounted for by the effect of the variation of pressure on the thermometer bulb. That the temperature of the bath might be controlled, it was equipped with a mercury thermostat, not shown in the figure; the mercury tube being 2.5 cm. in diameter and 150 cm. in length and placed so that it extended from top to bottom and across the bottom of the tank. The thermostat system consisted in the mercury tube, in which the mercury, rising with the rise in temperature of the bath, bridged a circuit, I., through a battery, or other source of current, and an electromagnet. This latter, in turn, lifted an iron core to which was attached a key in circuit II., containing a set of incandescent lamps. Thus was comprised a system by means of which the temperature of the bath could be maintained constant to $.01^{\circ}$ for any length of time.

Measurement of Pressures.—In the measurement of pressure, the final pressure, p_2 , was that of the atmosphere, registered upon a Schneider standard mercury barometer. This instrument had but recently been compared with the standard barometer of the U. S. Weather Bureau service, kept in the Federal Building at Chicago. During the experiments, it was kept in the room, near the manometers, and its reading taken immediately after each observation. Since this reading could be made with an accuracy of .1 mm. mercury, its influence could not introduce an error of more than .01 of 1 per cent. in the final result.

The initial pressure, p_1 , was equal to that of the barometer reading plus an excess pressure which was registered upon a "transformer" oil manometer, M_1 , and, as a check, upon a mercury manometer, M_2 . The readings upon this oil manometer were made by means of a reading glass or microscope, held perpendicularly to the tube. Since the manometer had an internal diameter of 1 cm., a fair portion of the surface was horizontal so that by the above scheme the excess pressure could be measured to .1 mm. of oil. This made the possible error in p_1 , expressed in mercury, not more than .06 of 1 per cent. at the lower pressures and .03 of 1 per cent. at the higher pressures.

For the conversion of the manometer readings in oil into terms of mercury, a temperature-density curve for the oil was plotted over a range of 10° , from 16° C. to 26° C. The mercury temperature-density curve was plotted from data taken from Castell-Evans Physico-Chemical Tables (5).

The valve, V_1 , was closed immediately before an expansion and the reading taken immediately after, if the observation was satisfactory. Then the adjusting of pressures for a new trial was done, approximately, by the aid of a mercury manometer, M_2 , so that the oil was not allowed to move up and down the tube for more than a few cms. This avoided errors in subsequent readings due to the gradual running down of oil that had adhered to the walls of the manometer tube.

Measurement of Temperature.—The temperature, θ_1 , was that of the constant temperature bath in which the carboy was submerged. This was observed by the reading of a Baudin standard mercury thermometer. The readings upon the standard were made by the aid of a microscope, having a comparison scale in the eye piece. In this way, θ_1 was obtainable to $.01^\circ \text{C.}$, with interpolation to $.001^\circ$ possible. The error in γ introduced thus was not more than $.01$ to 1 per cent.

To obtain θ_2 , the procedure is as follows: The thermal electromotive force developed in the couple circuit by a change in temperature incident to an expansion was evaluated in terms of the E.M.F. of a storage cell by means of a potentiometer made up from resistance boxes. The boxes had been standardized at the Bureau of Standards, Washington, D. C., and afforded a total of 120,000 ohms in the battery circuit and read by $.5$ ohm up, in the couple circuit, so that the thermal E.M.F. was known in microvolts. Immediately after each observation, the E.M.F. of the storage cell was evaluated by comparison with a Weston standard cell, which had a certificate from the Physikalische Technische Reichsanstalt.

Having, now, the value of the fall in temperature in terms of the thermal E.M.F., it remained to determine, by direct observation, the difference of temperature necessary to develop the same E.M.F. This was done by removing the couple from the carboy and placing the constant temperature junction in a bath at 25°C. and the second junction in another bath at a lower temperature. By this means, a temperature E.M.F. curve was plotted. For succeeding observations, it was only necessary to measure the thermal E.M.F. and, at once, θ_2 could be read from this calibration curve. This curve was plotted to such a scale that θ_2 could be read directly to $.01^\circ \text{C.}$ and, by interpolation, to $.001^\circ \text{C.}$ The error introduced in γ , thus, was not more than $.01$ of 1 per cent.

Systematic Errors.—Since some heat inflow will occur, the conditions of the experiment do not furnish a perfectly adiabatic expansion. The heat brought in by gaseous conduction is certainly negligible. That due to convection is small since the temperature is uniform on all sides and, so, there is little cause for circulation, save for that which is due to the

surging and swirls arising at the sides of the outlet valve. Of these more will be said. Since we are dealing with wires of only .000009 sq. cm. cross section, it is seen that the heat capacity is very small and, likewise, the amount of heat which would actually be brought in by metallic conduction.

A greater source of error lies in the fact of internal radiation. The amount of this is a function of the emission coefficient of the glass and of the absorption coefficients of the copper and constantan wires. An experimental attempt at evaluating this was made. A determination of γ was made for a certain pressure with a bright couple and, then, again, after the couple had been blackened with a thin coat of "optical black." In the latter case, a smaller value of γ was obtained, owing to a higher coefficient of absorption and, hence, an apparently less fall in temperature registered. Then other couples were made, one bright and one blackened, and the two were placed in a vacuum tube. The ratio of the absorption coefficients was determined by comparing the times required for each, in turn, to acquire the same temperature when exposed to the same source of radiant energy. This ratio, as a mean of 12 observations, ranging from 9.2 to 10.3, was found to be 10. From this, it follows that we will make a close approximation to the correct value of γ , if we add to the value obtained with the bright couple one ninth of the difference between the values obtained with the bright and blackened couples. That this correction might not be a function of a particular couple, the difference was obtained on several couples, as shown in the following table:

TABLE I.

Pressure.	Bright Couple.	Blackened Couple.	Difference.
12.540	1.3980	1.3800	.0180
43.531	1.3932	1.3757	.0175
12.501	1.3962	1.3775	.0187
43.60	1.3926	1.3748	.0178
Mean difference.0180			

Hence the radiation correction is $+.0020$.

It was found advantageous, in order to avoid errors due to swirling eddies in the gas, following an expansion, to close the potentiometer circuit and to open the galvanometer damping key after an interval of two or three seconds. During this time, more or less heat had flowed in, according to the fall in temperature. To eliminate this, γ was measured at different levels of excess pressures and a pressure- γ curve plotted.

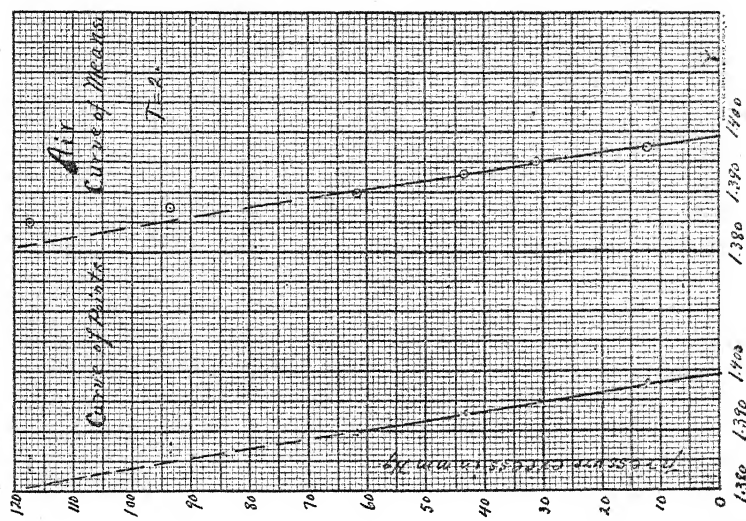


Fig. 2.

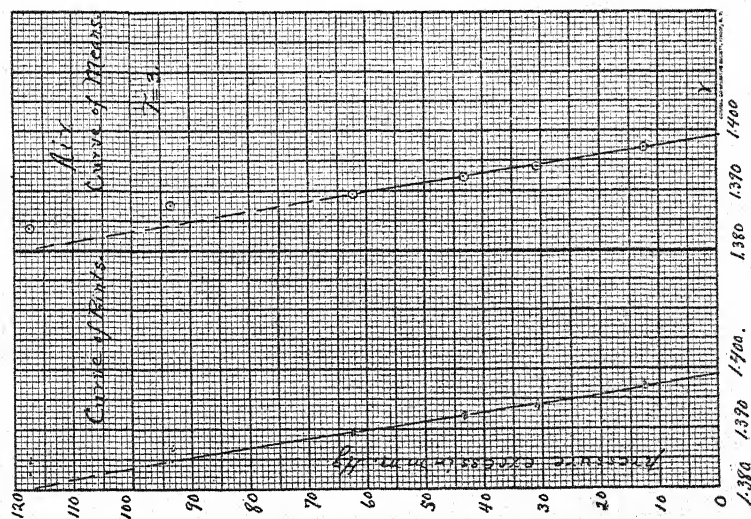


Fig. 3.

TABLE II.

Data on γ for Air, for $\tau = 3$.

p_2	dp	p_1	θ_1	θ_2	γ	D
751.228	12.470	763.698	298.006	296.614	1.3966	-.0006
748.113	12.375	760.448	298.007	296.620	1.3973	+.0001
742.005	12.323	754.328	298.007	296.617	1.3965	-.0007
748.365	12.489	760.834	298.007	296.607	1.3976	+.0004
753.121	12.560	765.681	298.006	296.608	1.3972	.0000
741.027	12.493	753.520	298.008	296.593	1.3980	+.0008
753.058	12.322	765.386	298.007	296.633	1.3972	.0000
746.795	30.868	777.663	298.007	294.617	1.3936	-.0003
750.293	31.032	781.325	298.007	294.612	1.3941	+.0002
749.113	30.893	780.006	298.007	294.617	1.3949	+.0010
753.121	30.996	784.117	298.006	294.626	1.3943	+.0004
754.227	30.942	785.219	298.006	294.646	1.3932	-.0007
740.648	31.002	771.650	298.008	294.575	1.3938	-.0001
754.503	30.836	785.339	298.007	294.652	1.3940	+.0001
744.515	43.520	788.035	298.037	293.307	1.3919	-.0005
751.230	43.436	794.666	298.006	293.316	1.3929	+.0005
745.475	43.372	788.847	298.017	293.317	1.3917	-.0007
750.070	43.500	793.570	298.007	293.305	1.3929	+.0005
734.291	43.501	777.792	298.009	296.214	1.3924	.0000
750.953	43.469	794.422	298.007	293.317	1.3925	+.0001
753.122	43.341	796.463	298.006	293.341	1.3927	+.0003
740.828	43.434	786.262	298.008	293.270	1.3916	-.0008
755.138	43.277	798.415	298.007	293.357	1.3931	+.0007
748.019	62.147	810.166	298.007	291.407	1.3900	+.0005
751.152	62.097	813.249	298.006	291.448	1.3891	-.0004
749.857	62.110	811.967	298.007	291.427	1.3899	+.0004
752.001	62.086	814.087	298.007	291.452	1.3897	+.0002
753.122	61.999	815.121	298.006	291.476	1.3890	-.0005
753.883	62.062	815.945	298.006	291.471	1.3894	-.0001
740.963	62.064	803.027	298.007	291.357	1.3894	-.0001
740.864	62.066	802.930	298.008	291.363	1.3894	-.0001
755.437	62.113	817.433	298.007	291.487	1.3897	+.0002
745.597	93.339	838.936	298.002	288.354	1.3870	+.0004
749.435	93.377	842.812	298.007	288.407	1.3866	.0000
752.001	93.361	845.362	298.007	288.467	1.3849	-.0017
753.221	93.246	846.467	298.006	288.461	1.3868	+.0002
755.763	93.240	849.003	298.007	288.477	1.3876	+.0010
748.679	117.494	866.173	298.017	286.197	1.3843	+.0006
748.350	117.597	865.947	298.007	286.217	1.3823	-.0014
750.307	117.539	867.846	298.007	286.267	1.3822	-.0015
740.704	117.496	858.200	298.008	286.058	1.3849	+.0012
755.813	117.359	873.172	298.007	286.287	1.3852	+.0015

TABLE III.

Data on γ for Air, for $\tau = 2$.

p_2	dp	p_1	θ_1	θ_2	γ	D
751.290	12.430	763.720	298.006	296.616	1.3980	+.0002
748.760	12.440	761.200	298.006	296.614	1.3971	-.0007
753.058	12.322	765.380	298.007	296.632	1.3984	+.0006
751.290	30.981	782.271	298.006	294.616	1.3949	-.0001
748.660	30.942	779.602	298.007	294.607	1.3952	+.0002
754.503	30.836	783.339	298.007	294.647	1.3949	-.0001
751.455	43.551	795.006	298.006	293.306	1.3930	.0000
747.464	43.298	790.762	298.007	293.312	1.3927	-.0003
755.138	43.264	798.402	298.007	293.357	1.3933	+.0003
751.790	62.157	813.947	298.006	291.446	1.3896	-.0001
747.164	61.887	808.051	298.007	291.437	1.3891	-.0006
755.437	61.871	817.308	298.007	291.487	1.3908	+.0009
752.387	93.222	845.609	298.006	288.476	1.3882	+.0007
747.153	93.269	840.422	298.007	288.367	1.3865	-.0010
755.813	93.187	849.000	298.007	288.477	1.3880	+.0005
752.622	117.470	870.092	298.006	286.256	1.3838	-.0013
747.129	117.375	864.504	298.007	286.137	1.3860	+.0009
755.813	117.359	873.172	298.007	286.272	1.3856	+.0004

Having measured γ for excess pressures varying from 12 cm. mercury down to 12 mm., a smooth curve, drawn through the plotted points and extended back to the γ axis, gave the value of γ for $dp = 0$, *i. e.*, when the effect on γ , due to the heat inflow was nil.

Results.—In Table II. is given the data on γ , for the case where the interval between the time of expansion and the time of closing the potentiometer circuit, τ , was 3 sec. These are the observations of several complete series, from high to low pressures, arranged in groups, according to the pressure levels. In these tables, D means the variation of that observation from the mean value for that pressure level.

In Fig. 2 are the graphs of these data, where dp is taken as the ordinate and γ plotted as abscissa. Since the variation in γ is far more than that due to the slight differences in pressure level, it was thought worth while, as a check, to plot a curve of means, *i. e.*, for each level, the mean pressure, for that level, was plotted against the mean γ for that level.

Table III. contains, similarly, the data on γ for air, for the trials in which $\tau = 2$. In Fig. 3 are the corresponding graphs.

In Table IV. will be found the data on γ for carbon dioxide, for $\tau = 3$.

TABLE IV.

Data on γ for Carbon Dioxide, for $\tau = 3$.

p_2	dp	p_1	θ_1	θ_2	γ	D
742.849	12.824	755.673	297.904	296.739	1.2968	+.0002
746.693	12.753	759.446	297.884	296.738	1.2961	-.0005
745.832	12.723	758.555	298.014	296.863	1.2966	.0000
750.213	12.606	762.819	298.034	296.899	1.2967	+.0001
743.674	12.612	756.286	297.944	296.799	1.2969	+.0003
751.052	12.257	763.309	298.004	296.902	1.2967	+.0001
747.103	12.200	759.303	298.004	296.902	1.2965	-.0001
743.299	31.333	774.632	298.074	295.294	1.2934	+.0006
747.172	31.212	778.384	298.014	295.254	1.2942	+.0014
745.398	31.266	776.664	298.014	295.254	1.2927	-.0001
750.858	31.027	781.885	298.014	295.314	1.2913	-.0015
743.574	31.035	774.609	298.014	295.254	1.2931	+.0003
751.253	30.812	782.065	298.004	295.304	1.2926	-.0002
747.103	30.575	777.678	298.004	295.309	1.2928	.0000
743.349	43.692	787.041	298.064	294.259	1.2902	-.0005
747.452	43.642	791.094	298.004	294.219	1.2907	.0000
744.887	43.673	788.560	298.024	294.212	1.2916	+.0009
750.816	43.379	794.195	298.004	294.264	1.2900	-.0007
743.574	43.462	787.036	298.014	294.214	1.2918	+.0011
751.236	43.200	794.436	298.004	294.274	1.2907	.0000
747.191	43.103	790.294	298.004	294.274	1.2899	-.0008
743.374	62.005	805.379	298.014	292.674	1.2886	+.0010
750.836	61.832	812.668	298.004	292.779	1.2878	+.0002
747.079	61.803	808.882	298.004	292.779	1.2868	-.0008
743.349	62.282	805.631	298.064	292.759	1.2873	-.0003
746.920	62.256	809.902	298.004	292.714	1.2881	+.0005
744.558	62.285	806.843	298.044	292.744	1.2875	-.0001
750.816	62.056	812.872	298.004	292.794	1.2873	-.0003
743.349	93.503	836.852	298.064	290.329	1.2852	-.0001
746.741	93.138	839.879	298.014	290.319	1.2862	+.0009
744.558	93.464	838.022	298.044	290.319	1.2854	+.0003
750.134	93.281	843.415	298.004	290.374	1.2841	-.0012
743.062	93.224	836.286	298.014	290.314	1.2850	-.0003
750.836	93.027	843.863	298.004	290.370	1.2855	+.0002
746.967	93.126	840.093	298.004	290.314	1.2862	+.0009
743.049	117.482	860.531	298.054	288.554	1.2831	+.0002
744.628	117.376	862.004	298.014	288.549	1.2842	+.0013
744.567	117.486	862.053	298.044	288.544	1.2838	+.0009
750.134	117.327	867.461	298.004	288.629	1.2820	-.0009
751.048	117.228	868.276	298.004	288.604	1.2826	-.0003
746.967	117.208	864.175	298.004	288.599	1.2820	-.0009

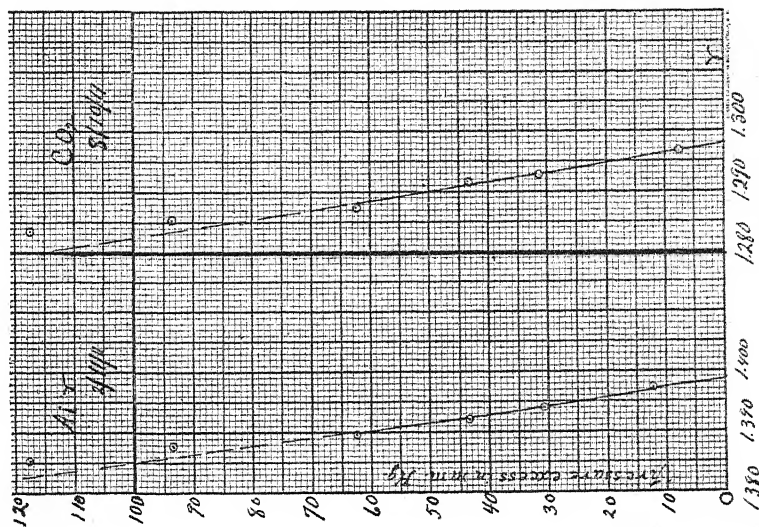


Fig. 5.

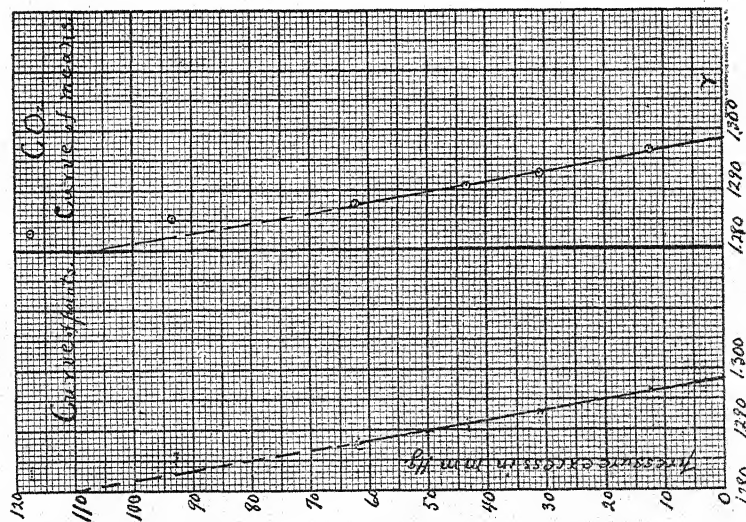


Fig. 4.

This was the only interval used in the case of carbon dioxide. The notation is the same as for the tables on air. Fig. 4 contains the graphs for these data.

Thinking it would be of interest to some, we have inserted Fig. 5 to show the degree of consistency which was found in most all cases. These curves are the graphs of single complete series, all points of which were determined within three or four hours of each other, so that all conditions could be maintained practically constant throughout the series.

The mean of all determinations, made in the manner described above, gives for air, $\gamma = 1.3991$, and for carbon dioxide, $\gamma = 1.2983$. Whence, upon the addition of the radiation correction, we have for air, at 25°C ., 76 cm. , $\gamma = 1.4011$ and for carbon dioxide, at the same temperature and pressure, $\gamma = 1.3003$, in which the probable error is $\pm .0005$.

III. A REVIEW OF RECENT DETERMINATIONS OF γ FOR AIR AND CARBON DIOXIDE.

In order to compare these values with those found by earlier observers and to draw conclusions as to the probable value of γ , for air and carbon dioxide, we may refer to the summary of A. Winkleman (6) and the critical review of M. G. Maneuvrier (7), both of which appeared about 1895. The results there given show such a wide range of variation in values that we can draw no safe conclusion. It is but natural, however, to expect that, with the improved methods and apparatus of the modern laboratory, the more recent determinations of γ should show a closer agreement. In order to see to what extent this expectation has been realized, a brief review of the work done since the time of the above mentioned summaries will be made.

Indirect Methods.—All indirect methods for the determination of γ are based upon the application of the formula for the velocity of sound,

$$(3) \quad V = n\lambda = \sqrt{\frac{p\gamma}{\rho}}.$$

(a) *Long Distance Measurements in Free Air.*—To this method there are the following serious objections: (1) The difficulty of making the proper corrections for the effect of humidity, (2) lack of uniform temperature throughout the path, (3) the relatively high intensity required at the source. This latter must die out as the wave recedes from the source and it is probable that the velocity of the wave diminishes with the intensity. By far the most reliable determination made by this method was that of the French Academy (8), in 1822. They found $V_0 = 331.2 \text{ m. per sec.}$, from which $\gamma = 1.3999$.

(b) *Hebb's Method.*—In 1905, Hebb (9), by a very ingenious method, suggested by Professor Michelson, eliminated the objections mentioned above as well as those which apply to the measurements made in tubes, viz., the “personal equation”¹ and the effect of the walls.

His measurements were made in the halls of Ryerson Laboratory. He applied the principle of interference by means of a system of parabolic mirrors, having a tuning fork source placed at the focus of one and a telephone receiver at the focus of the other mirror. He found $V_0 = 331.29$, with a probable error of .06 m. from which $\gamma = 1.4003$.

(c) *Measurements in Long and Large Tubes.*—The objections to the measurements in free air have also been eliminated by observers using long and large tubes, though they have introduced a new one, viz., the effect of the walls of the tubes. Violle and Vautier (10) made a first determination at Grenoble, in 1885, in a U-shaped conduit, 70 cm. in diameter and about 13 km. in length. In 1895, they made a second determination in another sewer conduit, just constructed, at Argenteuil; this one being a straight conduit, 3 m. in diameter and 3 km. in length. Very careful corrections were made for temperature, pressure and hygrometric conditions. They found $V_0 = 331.15$ m. per sec. with a probable error of 2 or 3 cm. A final correction for the effect of the walls was made by Violle (39), in 1900, and he gives $V_0 = 331.36$ m. per sec., whence $\gamma = 1.4009$.

(d) *Measurements in Short Tubes.*—Many observers have employed the interference method of Quincke with varying success. The difficulties with this method may be most easily pointed out under the discussion of the individual works. (1) J. Webster Low (11) found for air, at 10°C. , $V = 330.88$, whence $\gamma = 1.3968$. The influence of the “personal equation” is evident here in the fact that he used maxima of tone strength instead of minima. Later, Stevens has corrected Low's value for humidity and effect of the size of the tubes. He gives as the corrected value, $V_0 = 331.4$, from which we get $\gamma = 1.4012$. (2) O. Buckendahl (12) found certain disturbing resonance phenomena, between the entire tube system and the body of gas, whose existence was not easily discoverable. Then, too, errors often entered because of diffusion with the outside air, so that some question holds against his results. He obtained, for carbon dioxide, at 0°C. , 76 cm., $\gamma = 1.2990$. (3) E. H. Stevens (13), using this method and making very careful corrections for humidity, size of tubes, etc., found for air, $V_0 = 331.32$, from which $\gamma = 1.4006$. This seems to be the most reliable determination of all the recent ones according

¹ This expression relates to the error in judgment involved in setting on a maximum or minimum of tone strength.

to this method. (4) R. Fürstenau (14) made some improvements on the method and was able to eliminate diffusion with outside air and the "personal equation" to some extent, still he encountered uncertainties as to the correction for the wave-length because of lack of uniformity in the heating of the tube. He, too, met with certain resonance phenomena in the surrounding tubes. He obtained for carbon dioxide, at $13.5\text{--}16.5^\circ\text{C.}$, $\gamma = 1.3008$, and made no measurements on air.

Direct Methods: (a) Assman's Method.—The mechanical manipulation of this method is simple, but it cannot be considered as a serviceable method because (1) the period (about one half sec.) is so long that the process is not adiabatic and (2) because the volume of the bulbs of gas is too small. The evidence of this is found in the work of (1) Assman (15), himself, who, in 1852, obtained for air, $\gamma = 1.421$ and $\gamma = 1.427$; (2) P. A. Müller (16), who, in 1883, obtained for air, $\gamma = 1.4046$ and for carbon dioxide, at 20°C. , $\gamma = 1.2653$; and of B. Hartman (17) who, in 1905, found $\gamma = 1.4137$, as a mean of readings varying from $\gamma = 1.4084$ to $\gamma = 1.4201$. He found for carbon dioxide, $\gamma = 1.3064$, as a mean of values varying from $\gamma = 1.302$ to $\gamma = 1.311$.

(b) *The Method of Clement and DesOrmes.*—In this classical method, the determination of γ rests upon the measurement of pressure. The results obtained by experienced observers are exceedingly discordant, varying between 1.302 (Kohlrausch) and 1.42 (Masson). Roentgen's (18) determination has usually been considered the most reliable, but since a number of unsuccessful attempts have been made to duplicate it, some of them in Ryerson Laboratory, under conditions practically identical with those under which Roentgen worked, we are inclined to attach less weight to it than has sometimes been done, and to leave out of consideration all measurements made by the Clement and DesOrmes method.

A slight modification of the above method was made by Maneuvrier and Fournier (19) when they made use of Reech's theorem and they were able to get more consistent results than earlier observers who used the method. Reech's theorem, $\gamma = \partial p_0 / \partial p_\theta$, holds only for infinitesimal changes while, in practice, the observer must work with finite changes and then extrapolate back, graphically, to the limiting values. Again, there must be surging in the body of the gas upon expansion—a fact which introduces large uncertainties into the measurement of the final pressure. These, together with the criticisms which apply to any method depending upon a measurement of pressures (see below) make it seem that their values are not entirely reliable. They obtained for air, at room temperature and atmospheric pressure, $\gamma = 1.3924$ and for carbon dioxide, $\gamma = 1.2993$.

(c) *Direct Application of the p - θ Adiabatic Equation.*—This is a remarkably simple and direct method, requiring only some device for the direct observation of the final temperature, θ_2 , all other quantities being such that they can be measured with the highest degree of accuracy. This method possesses a great advantage over other direct methods in that it makes the determination of γ rest upon a temperature measurement rather than upon a pressure measurement. A pressure measurement is, at best, a measurement of the mean temperature throughout the vessel at the instant of measurement, while the direct temperature measurement can be made upon a small quantity of gas in the very center of the vessel. By the pressure scheme, the temperature observed is always too high because of the inflow of heat which begins at the instant of expansion, whereas the temperature at the center is not affected until a finite time after the expansion. (1) Lummer and Pringsheim (3) were the first to use the method. They used a platinum bolometer strip to indicate the temperature change by the change in its resistance, this latter being measured by means of a Wheatstone bridge. They obtained for air, at 15° C., 76 cm., $\gamma = 1.4025$ and for carbon dioxide, under the same conditions, $\gamma = 1.2995$. (2) W. Makower (4) used a platinum thermometer to detect the temperature change. While this work was very carefully done, the consistency obtained was not all that could be desired. Working under ordinary conditions of temperature and pressure, he obtained for air, $\gamma = 1.401$. (3) The author's work gives for air at 25° C., 76 cm., $\gamma = 1.4011$, and for carbon dioxide, under the same conditions, $\gamma = 1.3003$.

Summary and Tables.—Several attempts at computing the value of γ , from theoretical considerations, have been made, *e. g.*, by Boynton (20), Capstick (21), and Le Duc (22), but in all cases, because of approximations made or uncertainties in the values of certain constants needed, these results do not agree sufficiently to aid us.

In the following table will be found what seem to be the most reliable determinations made during the period covered by this review.

TABLE V.
Table of Observations on γ for Air.

Observer.	Method.	Temperature.	
Violle and Vautier.....	Velocity of sound.	0° C.	1.4009
Hebb.....	" " "	"	1.4003
Low (corrected by Stevens) ..	" " "	"	1.4012
Stevens.....	" " "	"	1.4006
Lummer and Pringsheim.....	Application adiabatic equation.	15° C.	1.4025
Makower.....	" " "	"	1.401
Moody.....	" " "	25° C.	1.4011

These values have not been reduced to a common temperature because in the case of air the variation of γ with temperature is inappreciable for the range covered.

TABLE VI.

Table of Observations on γ for Carbon Dioxide.

Observer.	Method.	Temperature.	
Fürstenau.....	Velocity of sound.	13.5:16.5° C.	1.3008
Lummer and Pringsheim.....	Application adiabatic equation.	13° C.	1.2995
Moody.....	" " "	25° C.	1.3003

If we make use of the equation developed by Wüllner (23) and reduce the above values to the same temperature, 20° C., the values of Fürstenau (14) and of Lummer and Pringsheim (3) would each be lessened by about .001, while that of the author would be increased by a like amount. These corrected values give a mean for carbon dioxide, at 20° C., $\gamma = 1.2997$.

IV. A COMPUTATION OF THE SPECIFIC HEAT AT CONSTANT PRESSURE FOR AIR AND CARBON DIOXIDE.

Upon the basis of the first law of thermodynamics, we may deduce the expression

$$(4) \quad C_p = C_v + \frac{1}{J} \left\{ \left(\frac{\partial u}{\partial v} \right)_\theta + p \right\} \left(\frac{\partial v}{\partial \theta} \right)_p.$$

(See Buckingham, § 72.) From equation (4), we have, at once,

$$(5) \quad \lambda \equiv \left(\frac{\partial u}{\partial v} \right)_\theta = J C_p \cdot \frac{\gamma - 1}{\gamma} \cdot \left(\frac{\partial \theta}{\partial v} \right)_p - p.$$

This equation gives us a means of determining λ , the latent heat of expansion or the internal work of free expansion. The second law furnishes us with the further relation

$$(6) \quad \lambda = \theta \left(\frac{\partial p}{\partial \theta} \right)_v - p.$$

(See Planck, § 153.) In these equations, (5) and (6), we have two independent means of obtaining λ . In (5), the value of λ depends upon the specific heat at constant pressure, the ratio of the specific heats, and the volume coefficient of expansion. In (6), its value is determined by the thermodynamic temperature, obtained from values of the Joule-Thomson effect, and the pressure coefficient of expansion.

Now, if we make use of the defining equations,

$$(7) \quad \alpha_p = \frac{1}{v_0} \left(\frac{\partial v}{\partial \theta} \right)_p$$

and

$$(8) \quad \alpha_v = \frac{1}{p_0} \left(\frac{\partial p}{\partial \theta} \right)_v$$

we can rewrite our equations. From (5) we have

$$(9) \quad \lambda = J C_p \cdot \frac{\gamma - 1}{\gamma} \cdot \frac{1}{v_0 \alpha_p} - p,$$

and from (6) we get

$$(10) \quad \lambda = \theta p_0 \alpha_v - p.$$

When we insert the values, found, experimentally, for the different quantities, *i. e.*, $C_p = .2375$ and $\gamma = 1.401$, equation (9) gives us a relatively large negative value for λ , while equation (10) gives us a small positive value, which is as we know to be true for most real gases. If an error in γ were the cause of this, it would be necessary for $(\gamma - 1)$ to be about 2 per cent. larger, or $\gamma = 1.408$, in the case of air, in order that we obtain for λ a proper positive value. Since such a value is at variance with all recent reliable determinations of γ , and since it is even more probable that the total errors in all the other quantities is not so great, it seems certain that the generally accepted value of C_p , .2375, is much too small. For this reason, we have thought it worth while to compute it.

Upon the elimination of λ from equations (9) and (10) and the solution for C_p , we obtain an expression for C_p in terms of experimentally determined quantities, *viz.*,

$$(11) \quad C_p = \frac{\theta \gamma}{\gamma - 1} \cdot p_0 \alpha_v \cdot v_0 \alpha_p$$

which gives C_p in cal/(gm. \times deg.). As was pointed out above, this value of C_p is just as accurate as that of γ , since the values of all other factors are known with a very high degree of accuracy.

In these equations, the symbols used are defined as in the following table:

Symbol.	Definition.
C_p ,	Specific heat at constant pressure, in cal./(gm. \times deg.).
C_v ,	Specific heat at constant volume.
γ ,	Ratio of the two specific heats.
θ ,	Temperature on the thermodynamic scale.
θ_0 ,	Temperature of the freezing point of water on thermodynamic scale.
v_0 ,	Specific volume, in cm./gm. at the temperature θ_0 .
p ,	Pressure of one atmosphere in dynes.

J , Mechanical equivalent of heat.

α_p , Volume coefficient of expansion, at temperature θ .

α_v , Pressure coefficient of expansion, at temperature θ .

Since the average room temperature is 20°C. , and since all the quantities needed are known more accurately at 20°C. than at 0°C. , we have computed the value of C_p for 20°C. and one atmosphere's pressure. In making these computations for 20°C. , it must be borne in mind that v_0 , at 20°C. and one atmosphere's pressure is not equal to v_0 at 0°C. if we, as we may, arbitrarily define

$$\alpha_v = \frac{1}{p_0} \left(\frac{\partial p}{\partial \theta} \right)_v.$$

In this case, we must define

$$\alpha_p = \frac{1}{v_0'} \left(\frac{\partial v}{\partial \theta} \right)_p,$$

where

$$v_0' = v_0 \frac{\theta_0}{\theta},$$

on the assumption that Boyle's law holds. This $v_0 = v_0$, at 0°C. and one atmosphere.

As to the available data for use in these computations, we have the following:

1. On the thermodynamic temperature of the freezing point of water, or 0°C. , Makower and Noble (24), Rose Innes (25), Pellat (26), Chappuis and Harker (27), Buckingham (28), Callendar (29), and Berthelot (30) have given results, all agreeing fairly well. The more reliable of these results are, probably, those of Callendar, Berthelot, and Buckingham, from which we conclude that $\theta_0 = 273.11^\circ$ on the thermodynamic scale, with an accuracy of $\pm .01^\circ$. The corrections for the interval are negligible, so that $20^\circ \text{C.} = 293.11^\circ$ on the thermodynamic scale.

2. On C_p , the results for air cluster about those of Regnault (31), viz., $C_p = .2375$, as a mean value. But all these earlier observations have been taken over relatively long ranges of temperature. More recently, Swann (32) has made a determination, covering a short range, and has obtained a value higher than that of Regnault's by nearly 2 per cent. He obtained the value $C_p = .2417$, at 20°C. Just as this paper is ready for the press, an abstract has appeared in the *Physikalische Zeitschrift* of the work of Scheele and Heuse (33), who find $C_p = .2408$ at 20°C. For carbon dioxide, a better agreement is found, but, for this, γ seems to be well determined so that we have computed C_p . For carbon dioxide, Regnault (31) gives $C_p = .2025$; Lussana (34), $C_p = .2013$; Holborn and Hennings (35), $C_p = .2039$; Swann (32), $C_p = .2020$, at 20°C.

3. Values of γ have been discussed in Part III. (see above).

4. Since $v = 1/\rho_0$, we need ρ_0 , the density at 0° C. The density of air at 0° C., 76 cm., and sea-level has been taken from Landolt-Bornstein's tables. From the same authority, we have taken the value of g , at sea-level, as 980.617. For CO_2 we have taken the value of ρ_0 as given by Lord Rayleigh (36).

5. For the value of J , we need only refer to the summary of Ames (37).

6. Upon the expansion coefficients, the splendid work of Chappuis (38) has been generally accepted as the best yet made, so that his values have been used in this work.

In Table VII. are given the quantities needed in equation (II) and their numerical value in the case of the two gases studied. In Table VIII., we have compared the values computed with Regnault's and the more recent determinations.

TABLE VII.

Table of Values Used in the Computation of C_p .

Magnitude.	Air.	Carbon Dioxide.
γ	1.401	1.300
θ_0	273.11	273.11
ρ_000129278	1.52909 \times .00129278
J , at 20° C.....	4.181×10^7	4.181×10^7
α_p , at 20° C., 76 cm.....	.00367812	.00373447
α_v , at 20° C., 76 cm.....	.0036722	.0037067

TABLE VIII.

Values of C_p , Computed and Observed.

Gas.	C_p (Computed).	C_p (Observed).
Air.....	.24118	$\left\{ \begin{array}{l} .2375 \text{ Regnault.} \\ .2417 \text{ Swann.} \\ .2408 \text{ Scheele and Heuse.} \end{array} \right.$
Carbon dioxide.....	.2008	$\left\{ \begin{array}{l} .2025 \text{ Regnault.} \\ .2020 \text{ Swann.} \\ .2013 \text{ Lussana.} \end{array} \right.$

It will be seen that our computed value of C_p in the case of air agrees remarkably well with the mean of the two most recent determinations. It is to be noted that the equation used in the computations is perfectly rigorous and all quantities involved are capable of measurement with the highest degree of accuracy. The one in whose value there is the largest uncertainty is γ and there can be little question but that its value is known, accurately, to the third decimal place. In the case of carbon

dioxide, our value agrees within the limits of error with the value given by Lussana, though it is slightly smaller than those found by other observers. Hence it seems that the correct value of C_p for carbon dioxide must lie close about the value found by Lussana.

V. CONCLUSIONS.

1. Our work gives for air, at 20°C ., 76 cm., $\gamma = 1.4011$, and for carbon dioxide, under the same conditions, $\gamma = 1.3003$, with a probable error in each of $\pm .0005$. These values agree well with the best determinations of recent years, though the value for air is smaller than some of those of earlier date which have previously been accepted.

2. We find for air, at 20°C ., 76 cm., that $C_p = .24118$. This value is so reliable that there can be little doubt but that the values accepted, heretofore, are too small. In fact, a striking confirmation of the correctness of our value of γ is had in the fact that the above value of C_p agrees to within .03 of 1 per cent. with the mean of the recent determinations of Swann and of Scheele and Heuse. For carbon dioxide, at 20°C ., 76 cm., we find $C_p = .2008$, which agrees within the limits of error with the value of Lussana and is slightly lower than the values given by other reliable determinations.

In conclusion, the author desires to express his appreciation to the staff of the Physics Department for their kindly interest in this work and especially to Professor Millikan, at whose suggestion and under whose direction the investigation was carried out.

RYERSON PHYSICAL LABORATORY,
THE UNIVERSITY OF CHICAGO,
January 2, 1912.

VI. BIBLIOGRAPHY.

1. Millikan, Chapman, and Moody, *PHYS. REV.*, 30: 286, 1910.
2. Chapman, *PHYS. REV.*, 32: 561, 1911.
3. Lummer and Pringsheim, *Smithsonian Contrib. to Know.*, XXIX., No. 1126; *Wied. Ann. d. Phys.*, 64: 555, 1898.
4. Makower, *Phil. Mag.* (6), 5: 226, 1903.
5. Castell-Evans, *Physico-Chem. Tables*, p. 218.
6. A. Winkelman, *Handbuch d. Phys.*, 2: 381, 1896.
7. M. G. Maneuvrier, *Jour. d. Phys.* (3), 4: 341 and (3), 4: 445, 1895.
8. French Academy, *Ann. d. Chim. et d. Phys.*, XX., 210, 1822.
9. Hebb, *PHYS. REV.*, 20: 89, 1905.
10. Violle and Vautier, *Ann. d. Chim. et d. Phys.*, XIX., 306, 1890.
11. J. Webster Low, *Phil. Mag.* (5), 38: 249, 1894.
12. O. Buckendahl, *Diss. Heidelberg*, 1906.
13. E. H. Stevens, *Ann. d. Phys.*, 4, 7: 285, 1902.
14. R. Fürstenau, *Verh. d. Deutsch. Gesell.*, 10: 968, 1908.
15. Assman, *Pogg. Ann.*, 85: 1, 1852.
16. P. A. Müller, *Wied. Ann.*, 18: 94, 1883.

17. B. Hartman, Diss. Gottingen, 1905.
18. Roentgen, Pogg. Ann., 148: 580, 1873.
19. Maneuvrier and Fournier, C. R., 123: 228, 1896.
20. Boynton, PHYS. REV., 12: 353, 1901.
21. Capstick, Proc. Roy. Soc., 57: 322, 1895.
22. Le Duc, C. R., 127: 659, 1898.
23. Wüllner, Wied. Ann., 4: 321, 1878.
24. Makower and Noble, Proc. Roy. Soc., 72: 379, 1903.
25. Rose-Innes, Phil. Mag. (6), 2: 130, 1901.
26. Pellat, C. R., 136: 809, 1903.
27. Chappuis and Harker, Trav. du Bur. Int. des Poids et Mes., XII., 1902.
28. Buckingham, Bull. Bur. Standards, 3: 237, 1907.
29. Callendar, Phil. Mag. (6), 5: 48, 1903.
30. Berthelot, Trav. et Mem. du Bur. Int. des Poids et Mes., XIII., 1907.
31. Regnault, Mem. de l'Acad., 26: 1, 1862.
32. Swann, Phil. Trans. Roy. Soc., 210: 199, 1909.
33. Scheele and Heuse, Phys. Zeit., 12, 24: 1674, 1911, and 13, 2: —, 1912.
34. Lussana, Nuovo Cimento, 3, 36: 5, 70, 130, 1894.
35. Holborn and Hennings, Ann. d. Phys., 23: 809, 1897.
36. Lord Rayleigh, Proc. Roy. Soc., 62: 204, 1897.
37. Ames, Congres Int. d. Phys., 1: 178, 1900.
38. Chappuis, Trav. et Mem. du Bur. Int. des Poids et Mes., XIII., 1903.
39. Violle, Congres Int. d. Phys., 1: 228, 1900.

ON THE PASSAGE OF γ RAYS OF RADIUM THROUGH MATTER.

BY S. J. ALLEN.

INTRODUCTION.

DURING the last few years a considerable amount of work has been done on this subject by various experimenters, and if one reads through their papers one is struck with the wide differences in the results. In very few cases can these be due to experimental errors, but must be ascribed to the effect of the so-called secondary rays produced when the γ rays strike upon and emerge from the surrounding objects.

A very good summary and discussion of past work is given in several papers by Soddy and Russell,¹ to which the reader is referred. They have examined very carefully the effect of different experimental dispositions of the apparatus, and also of different kinds of electroscopes. They are of the opinion that the effects of secondary rays can be eliminated by special arrangements, and that when this has been produced the γ rays are homogeneous throughout their entire range. The absorption of γ rays by matter can be roughly divided into two groups; a very irregular initial portion in which λ/d varies widely with the thickness traversed, and a portion where after a considerable thickness has been passed the value of λ/d is more constant. It has generally been explained in the past that these phenomena are due to the fact that the rays are originally heterogeneous and as successive thicknesses are passed the softer rays are absorbed more rapidly than the harder, thus rendering the remaining rays more homogeneous.

Soddy and Russell's view that the rays are homogeneous is questioned in a paper by R. D. Kleeman.²

Before one can decide finally it is necessary that a large number of results on many different materials should be at hand, and that these results should agree quite closely among themselves, which is not the case at present. It can be seen from Soddy and Russell's work that λ/d is not a constant for different elements, but increases largely as the high atomic weight elements are reached. It is also evident that a hardening effect can be produced by first passing the rays through certain elements. These results have also been obtained by the author, and a brief abstract

¹ Soddy and Russell, *Phil. Mag.*, 1909-1910.

² R. D. Kleeman, *Phil. Mag.*, 1910.

has appeared in the PHYSICAL REVIEW. The present paper gives the results in more detail and also additional ones.

EXPERIMENTAL ARRANGEMENTS.

Both the balanced electrometer, and the electroscope methods were tried, and from preliminary experiments it was decided to use the latter. The general arrangement is shown in Fig. 1. The electroscope *B* was of brass, cylindrical in form, with sulphur bead insulation and a fine gold leaf, the movement of which was read by a reading telescope with micrometer scale. The author had the same trouble with irregular initial readings as is mentioned by Soddy and Russell, though not to so great an extent. This trouble is probably due to several reasons, chief among which are the leak, and "soaking in" effect over the sulphur bead, and the heating effect of the electric lamp used to illuminate the leaf. By removing the lamp to a considerable distance and reflecting the light into the electroscope by a mirror, and making the sulphur bead as small as possible, this trouble was eliminated to a great extent. The leaf was charged by means of a battery of lead accumulators, and the full potential was kept on the leaf for an hour or two before taking readings. After sufficient time was allowed to elapse the conditions always became steady for consistent readings.

The readings could always be repeated to within two tenths of one per cent. The values of λ (the so-called coefficient of absorption) were calculated from the equation,

$$\lambda = \frac{\log_e I_0 - \log_e I_t}{t},$$

where I_0 = the initial intensity of rays, I_t = the intensity after passing through a thickness t of the given material. Wherever possible the successive readings of $\log I$ and t were plotted in a curve. In such a case the criterion of the constancy of λ is a straight line. This is the method used by Soddy and Russell. Small variations would however scarcely be distinguished from a straight line, so that it seemed better to the author to test the constancy of λ by successive calculations from the

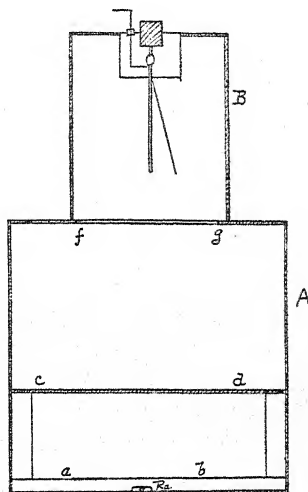


Fig. 1.

formula. If the results merely oscillated about a general mean, λ could be considered constant, but if λ varied uniformly in one direction or the other, the rays were not homogeneous. Since the γ rays are very penetrating, unless thick layers of the materials are used the values of I will not differ by much and consequently the calculated value of λ may be considerably in error. For example if two successive readings of I do not differ by more than 10 per cent. and since two tenths per cent. error is possible in each reading the value of λ might be out as much as 2 per cent. For some of the materials the author had not sufficient to form into thick layers, so the errors in these will be greater than others.

The electroscope was placed upon a brass box *A*, which contained an aluminum platform *cd* capable of being placed at any desired distance between the base of the electroscope *fg* and the bottom of the box *ab*. The radium *Ra* was placed at the bottom of the box in a small thin glass tube. The rays from this radium practically all came from a point source.

EFFECT OF VARYING THE POSITION OF ABSORBING LAYER, AND THE THICKNESS OF ELECTROSCOPE BASE.

A large number of experiments were made upon the effect of different dispositions of the radium and absorbing layer, and the variation in thickness of the electroscope base. These experiments are general in character, and the results show very clearly how it is possible to obtain such widely divergent values of λ under different experimental conditions.

The results of these are tabulated in Table I. The first column gives the substance used as the absorbing layer, the second the thickness of the layer from which λ was calculated, the third the value of λ , and the fourth the value of $\lambda/\text{density} \times 100$.

The ratio $\lambda/\text{density}$ can be considered as proportional to the specific absorption per unit of mass for any one material, and will serve as a relative comparison of the absorbing power of different materials according to their atomic weights, since the cone of rays will be approximately the same for each. The true specific absorption would be defined as the stopping power of a cylinder of the material, one square centimeter in cross section, and one gram in mass. If the stopping power is independent of the nature of the atoms of the material then we should expect to find λ/d constant for each material, provided each is tested under the same conditions and with the same kind of ray.

The different experiments will be taken up in detail so as to bring out the distinguishing features.

Experiment 1.—In this experiment the radium was covered with 7

TABLE I.

Substance.	Thickness, Mm.	λ	λ/d $\times 100$	Substance.	Thickness, Mm.	λ	λ/d $\times 100$
<i>Experiment 1.</i>				<i>Experiment 4.</i>			
Platinum.....	0-1.0	1.043	4.85	Radium covered with 10.5 mm. lead.			
Mercury.....	0-4.0	.669	4.92	Lead.....	0-3.36	.644	5.65
".....	4-12	.589	4.32	Radium covered with 14 mm. lead.			
".....	12-16	.538	3.96	Lead.....	0-3.36	.620	5.44
".....	16-20	.466	3.43	Radium covered with 21.1 mm. lead.			
Bismuth.....	0-3.25	.515	5.25	Lead.....	0-3.36	.584	5.12
Lead.....	0-3	.621	5.44	Radium covered with 31.2 mm. lead.			
".....	3-6	.554	4.87	Lead.....	0-3.36	.494	4.33
".....	6-12	.518	4.54	Radium uncovered.			
".....	12-15	.509	4.47	Lead.....	0-3.36	.664	5.82
".....	15-18	.503	4.41				
".....	18-21	.501	4.39	<i>Experiment 5.</i>			
Palladium.....	0-1	.471	4.09	Mercury.....	0-4.08	.666	4.89
Silver.....	0-3.2	.344	3.27	".....	4.08-8.16	.598	4.39
Copper.....	0-6	.242	2.72	".....	8.16-12.24	.564	4.15
".....	6-9	.273	3.07	Bismuth.....	0-3.25	.607	6.19
".....	9-12	.283	3.19	Silver.....	0-3.2	.442	4.21
".....	12-15	.295	3.31	Copper.....	0-3.35	.331	3.72
".....	15-18	.302	3.39	".....	3.35-6.69	.347	3.90
Nickel.....	0-4	.252	2.95	".....	6.69-10.0	.328	3.66
".....	4-8	.347	4.07	".....	10.0-13.4	.313	3.52
Wrought iron..	0-3	.206	2.64	Nickel.....	0-4	.357	4.49
".....	3-6	.212	2.72	Wrought iron..	0-6.1	.296	3.79
".....	6-9	.235	3.01	Cast iron.....	0-10.1	.259	3.67
".....	9-12	.254	3.25	".....	10.1-20.1	.247	3.49
".....	12-15	.272	3.50	Sulphur.....	0-6.9	.0897	4.46
".....	15-18	.291	3.73	Aluminum....	0-6.83	.110	3.88
".....	18-30.5	.305	3.91				
Aluminum....	0-8.7	.0736	2.60	<i>Experiment 6.</i>			
Sulphur.....	0-6.9	.0774	3.81	Radium covered with 31 mm. lead.			
<i>Experiment 2.</i>				Silver.....	0-3.2	.435	4.14
Bismuth.....	0-3.25	.464	4.73	Copper.....	0-3.36	.281	3.16
Silver.....	0-3.2	.344	3.27	Wrought iron..	0-6.1	.252	3.23
Copper.....	0-2.15	.323	3.63				
Wrought iron..	0-6.1	.245	3.14	<i>Experiment 7.</i>			
".....	6.1-12.2	.246	3.15	Radium covered with 14 mm. lead.			
".....	12.2-18.3	.243	3.11	Bismuth.....	0-3.25	.533	5.44
".....	18.3-24.3	.249	3.19	Silver.....	0-3.2	.421	4.00
<i>Experiment 3.</i>				Copper.....	0-3.33	.333	3.74
Lead.....	0-3.36	.769	6.75	Wrought iron..	0-6.1	.285	3.65
".....	3.36-9.96	.640	5.61				
Silver.....	0-3.2	.508	4.84	<i>Experiment 8.</i>			
Copper.....	0-2.15	.448	5.03	Radium covered with 10 mm. iron.			
				Lead.....	0-3.36	.759	6.67

TABLE I. Concluded.

Substance.	Thickness, Mm.	λ	λ/d $\times 100$	Substance.	Thickness, Mm.	λ	λ/d $\times 100$
Radium covered with 20.1 mm. iron.				Mercury.....	0-2.04	.621	4.58
Lead.....	0-3.36	.801	7.02	".....	2.04-4.08	.609	4.48
Radium covered with 30 mm. lead.				".....	4.08-6.12	.574	4.22
Lead.....	0-3.36	.805	7.06	".....	6.12-12.24	.554	4.07
Bismuth.....	0-3.25	.656	6.69	Silver.....	0-3.2	.421	4.00
Silver.....	0-3.2	.471	4.48	Copper.....	0-3.33	.333	3.74
Copper.....	0-3.33	.350	3.93	".....	3.33-6.69	.334	3.75
Cast iron.....	0-10.0	.273	3.86	".....	6.69-10.1	.329	3.69
<i>Experiment 9.</i>				".....	10.1-13.4	.331	3.72
Radium covered with 6.7 mm. copper.				Wrought iron..	0-6.1	.278	3.58
Lead.....	0-3.36	.751	6.59	Cast iron.....	0-10.1	.263	3.72
Radium covered with 13.3 mm. copper.				Sulphur.....	0-6.9	.079	3.93
Lead.....	0-3.36	.781	6.89	<i>Experiment 12.</i>			
Bismuth.....	0-3.25	.679	6.93	Radium covered with 14 mm. lead.			
Cast iron.....	0-10.1	.261	3.69	Lead.....	0-3.36	.537	4.71
<i>Experiment 10.</i>				Radium covered with 28 mm. lead.			
Radium covered with 12.2 mm. mercury.				Lead.....	0-3.36	.484	4.24
Lead.....	0-3.36	.580	5.09	Radium covered with 49 mm. lead.			
Silver.....	0-3.2	.386	3.68	Lead.....	0-3.36	.398	3.49
Cast iron.....	0-10.1	.235	3.32	Copper.....	0-3.36	.175	1.97
<i>Experiment 11.</i>				<i>Experiment 13.</i>			
Radium covered with 30 mm. iron.				Radium covered with 30 mm. iron.			
Lead.....	0-3.36	.554	4.86	Lead.....	0-3.36	.562	4.93
Bismuth.....	0-3.25	.496	5.06	Radium covered with 60 mm. iron.			
				Lead.....	0-3.36	.539	4.72

mm. of lead, and the absorbing layer placed directly over this. The base of the electroscope was covered with only a thin sheet of cardboard. Consequently any secondary radiation which was produced from the sides of the brass box and from the surface of the absorbing material could enter the electroscope in considerable amount. The total ionization in the electroscope is thus due to several components and there is difficulty in deciding just what proportion is due to the γ rays.

A number of substances were chosen running in atomic weight from aluminum to platinum and bismuth and differing widely in density. A study of these results shows that for the high atomic weight elements the values of λ decrease with increase of thickness of absorbing layer, while for the low atomic weights the exact opposite is true. In the case of iron for example the value of λ increases from .206 to .305, while the thickness changes from 3 mm. to 30 mm. In the case of lead and mercury the value of λ seems to be approaching a more nearly constant value. Bismuth is an interesting case, since we have here an element of compara-

tively low density but high atomic weight. The very large value of λ/d here recorded is undoubtedly due to the atomic weight. It is therefore certain from these results that the stopping power is some function of the atomic weight, but does not show much change until an atomic weight of about 100 is reached. The increase in value of λ , shown for iron and copper, and which would apparently seem at first sight to indicate a softening of the rays, must be explained in some other way. It is to be noted that the absorbing layer in this experiment from which λ is calculated moves nearer and nearer to the electroscope, as the thickness is increased, thereby changing its position relative to the radium and the electroscope. This factor may have a large influence on the value of λ on account of the change in the secondary rays.

Experiment 2.—Here the radium is covered with 24.5 mm. of lead, so that the rays measured must be very hard. The values of λ for iron are now almost constant with increasing thickness. λ/d is also practically a constant for the low atomic weights, but still very large for high atomic weights.

Experiment 3.—The radium is first covered with 7 mm. of lead, and then by 15 mm. of iron. The values of λ obtained are now very large. For example copper has a value of .448, whereas in experiment 1 it only had a value of about .300.

The rays which were hardened by the lead are apparently softened by the iron.

Experiments 4 and 5.—A preliminary experiment showed that if the absorbing layer of constant thickness was placed in various positions between the radium and electroscope the values of λ did not change very much after the absorbing layer had reached a distance of about 4 cm. away from the radium. Therefore in these experiments a thin aluminum platform (*cd* in Fig. 1) was placed at a distance of 4.7 cm. from the radium. The absorbing material was placed on this platform, and any substance for hardening the rays was placed underneath.

Thus the thickness of the absorbing layer, and its relative position with regard to the radium and the electroscope, could be kept constant while the nature of the rays was changed by placing various materials underneath.

The results obtained show that for the high atomic weight elements the value of λ steadily decreases with the thickness and does not seem to be approaching a constant value. The low atomic weight atoms do not show now an increase of λ as in experiment 1, but only a slight decrease.

Experiments 6, 7, 8, 9, and 10.—These show the effect of placing various

thicknesses of different substances between the radium and the absorbing layer. It is seen that all the substances having an atomic weight over 100 harden the rays, which is shown by a decreased value of λ for all substances. Substances like iron, and copper, apparently under these experimental conditions, cause an increase in λ .

Experiment 11.—In this the base of the electroscope was covered by a plate of lead 6.6 mm. in thickness. Under this condition no secondary rays of a corpuscular nature could enter the electroscope.

The results obtained show a much more uniform character than in any of the preceding. Below an atomic weight of 100 there is not any great variation in λ/d . Above 100 the effect of the atomic weight can still be seen, though not to so great an extent as before. Below an atomic weight of 100 the variation of λ with thickness is very small, but above 100 it is still large.

Experiments 12 and 13.—These show the great hardening effect of lead and the very small hardening effect of iron.

TABLE II.

Substance.	Thickness, Mm.	Intensity, Per Cent.	λ	$\lambda/d \times 100$
Bismuth.....	3.00	80	.744	7.59
".....	5.40	70	.660	6.73
".....	8.80	60	.579	5.91
Lead.....	2.62	80	.851	7.53
".....	4.85	70	.735	6.50
".....	7.70	60	.663	5.87
Tin.....	6.50	80	.343	4.70
".....	10.90	70	.327	4.48
".....	16.60	60	.308	4.22
Cadmium.....	5.50	80.8	.388	4.56
Copper.....	6.00	80	.372	4.18
".....	9.90	70	.360	4.04
".....	14.5	60	.352	3.95
Iron (wrought).....	8.20	80	.272	3.49
".....	13.20	70	.270	3.46
".....	18.90	60	.270	3.46
Aluminum.....	8.90	91.8	.0975	3.61
Carbon (coke).....	36.0	80	.0620	4.13
".....	60.0	70	.0594	3.96

We thus see from a detailed study of the foregoing results, that the value of λ can vary within wide limits depending on the experimental conditions, and the nature of the substance. It can readily be seen why it was that former experimenters gave such different results.

In Table II. are shown a number of results for different metals taken

under the following conditions. The bottom of the electroscope was covered by a plate of iron 26 mm. thick, and the radium was uncovered except for the glass tube.

The thicknesses of absorbing material are much greater than in any of the other experiments. The experimental values obtained were plotted in curves and the thickness of each substance necessary to cause absorptions of 20 per cent., 30 per cent., and 40 per cent., was taken from these curves. The curves for lead and iron are shown plotted in Fig. 2. In each case there are drawn two curves; the upper one was taken with the thin-walled brass electroscope, and the lower one with the electroscope completely surrounded by a wall of lead.

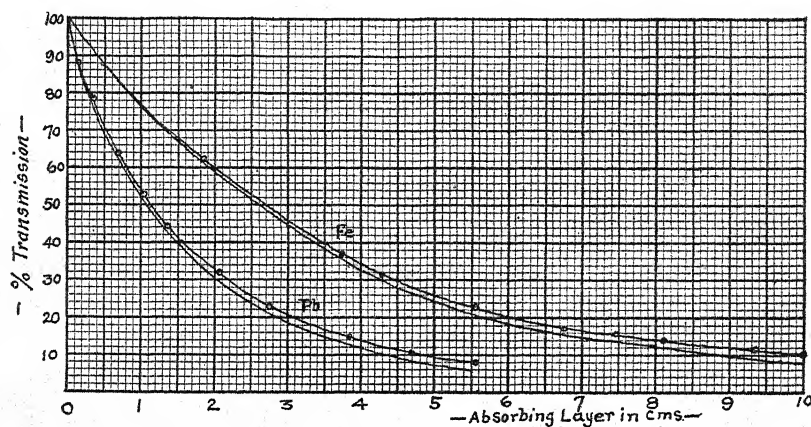


Fig. 2.

It is therefore apparent that the thickness of the electroscope has some effect on the value of the absorption coefficient; which agrees with the results of Soddy and Russell.¹ Further experiments showed that variation soon became constant and after a thickness of a half a centimeter had been reached there was no further change. This effect is ascribed by Soddy and Russell to secondary rays which are produced when the γ rays strike up surrounding objects.

An examination of Table II. illustrates very clearly the dependence of λ/d upon the atomic weight. The substances there are arranged in descending order of atomic weights and it is seen that λ/d decreases in the same order. After an atomic weight of about 100 is reached the values of λ/d do not change much.

In Fig. 3 an attempt has been made to plot the variation of λ/d with the atomic weight, taking equal absorptions of 10, 20, and 30 per cent.

¹ Loc. cit.

in each case. The curve is fairly smooth with the exception of wrought iron.

A comparison of the results in Table II. with those in Table I., when the electroscope base was covered with a thick slab of lead, shows that the values for the high atomic weight elements are very different while for atomic weights of less than 100 they are about the same. For example, the mean value of $\lambda/d \times 100$ for a lead base is 3.79, while for an iron base it is 3.83.

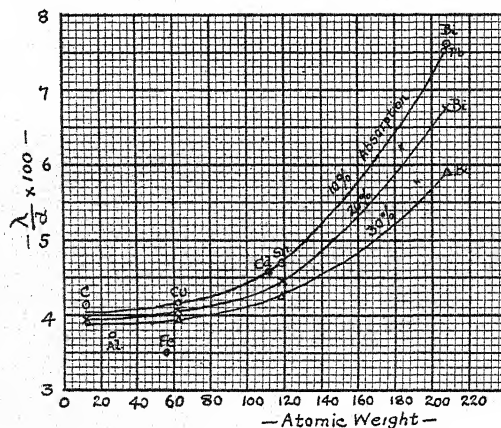


Fig. 3.

Soddy and Russell give for a number of substances below an atomic weight of 100 a value of $\lambda/d \times 100 = 3.83$, when the base of the electroscope was covered with a lead plate of 6.4 mm.

It therefore seems that the thickness or nature of electroscope base has very little effect upon the coefficient of absorption of elements of an atomic weight below 100, but above 100 the effect may be large.

HARDENING EFFECT OF HIGH ATOMIC WEIGHTS.

A series of experiments were made to test the hardening effect of various substances upon the primary γ rays. In all cases the absorbing layer was 3.54 mm. of lead, placed in a fixed position relative to the radium and the base of the electroscope. The hardening material was placed between the radium and the absorbing layer.

The results are tabulated in Table III. There are two series of results shown here; one when the electroscope base was covered with the plate of lead, and the other when it was closed by the iron plate.

The first column gives the hardening substance, the second the relative amount of rays which get through the hardener, the third the proportion

TABLE III.

Hardening Substance.	Transmission Through Hardener in Per Cent.		Transmission Through Absorber in Per Cent.		λ	
	Lead Base.	Iron Base.	Lead Base.	Iron Base.	Lead Base.	Iron Base.
Nothing.....	100	100	81	75.4	.596	.800
Lead.....	65.5	63.5	82.6	80.9	.542	.607
".....	45.9	43.3	83.6	82.2	.509	.553
".....	25.2	30.6	84.8	83.8	.469	.500
".....	—	16		86.7		.404
Bismuth.....	71.8	79.6	81.6	78.5	.576	.686
".....	49.0	52.7	84.3	82.5	.483	.556
".....	34.3	26.0	86.3	85.7	.418	.438
Mercury.....	68.3	65.0	82.2	80.0	.553	.633
".....	34.9	44.5	84.0	81.8	.491	.571
".....	19.5	25.0	85.1	85.3	.455	.452
Platinum.....	77.2	84.5	82.2	78.4	.554	.690
".....		75.0		79.3		.659
".....		66.6		80.8		.605
Tin.....		64.5		78.2		.698
".....		52.5		79.0		.667
Antimony.....		73.7		78.5		.687
Cadmium.....	75.5	72.0	81.5	77.5	.579	.726
".....	67.1	54.4	82.4	79.2	.548	.656
".....	48.1	43.8	83.0	80.2	.525	.628
Copper.....	40		81.0		.596	
Iron.....	40		81.4		.582	
Nickel.....		70		75.5		.794

of these which get through the lead absorber, and the final values of λ calculated from column three.

Three general groups are shown here; those having a high atomic weight, such as lead, bismuth, platinum and mercury; those having a medium atomic weight, such as tin, antimony, and cadmium; and those having a low atomic weight, such as copper, iron, and nickel.

It can readily be seen that the hardening effect is a function of the atomic weight, and is quite large for high atomic weights. Cadmium shows quite an appreciable effect, but very little effect can be detected

with copper or iron. That this hardening effect is not due to the absorption is easily seen, for copper after absorbing 60 per cent. of the rays, shows very little hardening, while a very small absorption by lead is sufficient to harden the rays.

For any one substance the hardening effect increases with the absorption.

That the rays have been actually changed in penetrating power after they have passed through a hardening substance is shown by the fact that the coefficient of absorption of a substance like iron, or copper, is changed when the rays have previously passed through lead, although iron and copper themselves show very little hardening effect. For example λ for copper is .343 when no hardener is present, but .302 when the rays have passed through a thickness of 3 cm. of lead.

Soddy and Russell¹ give some results on this hardening effect which are about the same as in this paper. In Table IV. are given some results taken from their paper.

TABLE IV.

Thickness of lead in cm.	0	.124	.249	.373	.64	1.24	2.50	3.75
λ for zinc.	.325	.322	.309	.300	.291	.272	.265	.258

THE ABSORPTION BY LIQUIDS.

A number of experiments were made on the absorption of the rays by liquids. The liquids were placed in a cylindrical glass vessel, and the rays passed through the glass bottom before entering the liquid.

In most of the cases the liquids were placed in a large glass vessel upon the aluminum platform, but for a few of the liquids the quantity available was not large enough to fill the vessel to a sufficient height to cause appreciable absorption. In these cases a narrower vessel was used, placed beneath the platform, and by using water under both conditions the values for any liquid could be corrected to what they would have been in the large vessel.

In Table V. the results are shown for a number of pure liquids and for three characteristic aqueous solutions. These were taken without any covering of the base of the electroscope.

The results show about the same characteristics as for the solid substances. The value of λ/d is practically a constant for all substances which do not contain any high atomic weight elements. The effect of high weight atoms is clearly shown by the lead nitrate solution in its large value of λ/d . The value of λ/d for water seems larger than one would expect, also for the alcohols.

¹ Loc. cit.

TABLE V.
Pure Liquids.

Substance.	Thickness, M m.	λ	$\lambda/d \times 100$
Water.....	0-20.4	.0462	4.62
Ethyl alcohol.....	0-20	.0352	4.40
Ether.....	0-20	.0326	4.46
Ester.....	0-20	.0417	4.50
Amyl bromide.....	0-20	.0344	4.50
Carbon tetra bromide.....	0-14.8	.1144	3.73
Benzene ".....	0-20	.0607	4.04
Ethelene ".....	0-18	.0870	3.95
Propyl ".....	0-9	.0617	4.54
Iso amyl ".....	0-19	.0685	4.10
Meta brom anilene.....	0-20	.0731	4.56
Ethyl iodide.....	0-12	.0860	4.52
Methyl ".....	0-8	.0907	4.12
Iod benzene.....	0-20	.0783	4.28

Aqueous Solutions.

Lead nitrate.....	0-20	.0754	5.57
Potassium iodide.....	0-20	.0760	4.81
Copper chloride.....	0-20	.0594	4.12

TABLE VI.
Pure Liquids.

Substance.	Thickness, Mm.	λ	$\lambda/d \times 100$
Water.....	0-18.4	.0465	4.65
".....	0-32.7	.0472	4.72
Amyl alcohol.....	0-18.3	.0355	4.37
Amyl bromide.....	0-18.3	.0497	4.11
Iod benzene.....	0- 6.5	.0740	4.05
Chloroform.....	0-18.4	.0603	4.06
Sulphuric acid.....	0-18.4	.0757	4.11

Aqueous Solutions.

Lead nitrate.....	0-18.4	.0652	4.82
" ".....	0-32.7	.0636	4.68
Potassium iodide.....	0-18.4	.0658	4.17
" ".....	0-26.5	.0653	4.13
Copper chloride.....	0-18.4	.0609	4.22
" ".....	0-32.7	.0612	4.25
Cadmium iodide.....	0-18.4	.0549	4.28
Nitric acid.....	0-18.3	.0497	4.19
Bismuth trichloride.....	0-18.4	.0465	4.33

Any particular molecular grouping has no effect upon the absorption, which seems to obey the additive law in the same manner as β rays.

The stopping power of a liquid per unit of mass depends only on the kind, and relative number, of the constituent atoms.

In Table VI. are shown a similar set of results when the base of the electroscope was covered by the lead plate.

The constancy of λ/d for low atomic weights is here even better than in Table V. The mean value of a number having atomic weights below 100 is .0412, which is a little higher than the same for the solids.

In Table VII. are shown results on two solutions at different concentrations.

TABLE VII.

Solution.	Concentration by Mass.	Density.	λ	$\lambda/d \times 100$
Water.....	0	1.00	.0459	4.59
Lead nitrate.....	.10	1.082	.0534	4.93
" ".....	.20	1.162	.0592	5.09
" ".....	.30	1.241	.0640	5.16
" ".....	.40	1.319	.0670	5.08
" ".....	.50	1.393	.0739	5.30
Potassium iodide.....	.20	1.133	.0510	4.50
" ".....	.40	1.257	.0606	4.82
" ".....	.60	1.364	.0640	4.70
" ".....	.80	1.461	.0681	4.66
" ".....	1.00	1.551	.0698	4.50
" ".....	1.20	1.621	.0718	4.43
" ".....	1.40	1.691	.0756	4.47

SUMMARY AND DISCUSSION OF RESULTS.

1. It has been shown that the coefficient of absorption of the γ rays, as determined in the ordinary way by the ionization method, depends greatly upon the experimental disposition of the various parts of the apparatus. The chief factors are: the position of absorbing layer relative to the radium and the electroscope; the thickness and nature of the electroscope base; and the previous screening of the rays. This to a great extent explains the diversity of results furnished by past experimenters on this subject. The results of the author in this respect are in general agreement with those of Soddy and Russell.

2. When the best arrangement suitable for the given case has been chosen, it can be shown that the "stopping power per unit mass" (λ/d) is a function of the atomic weight of the absorbing material, increasing very slowly with the atomic weight until an atomic weight of about 100 is reached, and then more rapidly.

3. When the rays first pass through any substance before reaching the absorbing layer they are hardened. This hardening is a function of the

atomic weight and is large for high atomic weight atoms. It is not proportional to the amount of absorption.

4. It has been shown that the absorption of the γ rays by liquids is a function of the atomic weights of the constituent atoms. In other words the absorption of the γ rays is an additive property of the atoms, and is independent of molecular grouping.

5. The values of λ/d obtained in this work are in general agreement with those of Soddy and Russell, wherever the experimental conditions are the same.

Excluding the initial soft rays, the value of λ/d for all substances below an atomic weight of 100 may be taken for approximate calculations as .0410. If there has been previous hardening, or if the absorbing material has an atomic weight above 100, the value of λ/d will depend on the conditions of the case.

At the present time there are two theories advanced to account for the phenomena which occur when γ or X rays penetrate an absorbing layer. One of these considers the rays to consist of a succession of ether pulses while the other considers them to be negative corpuscles which in some unknown manner have had their charge neutralized by equal quantities of positive electricity. These two theories differ materially in their consideration of the so-called secondary radiations. On the ether pulse theory the secondary rays derive their energy from the atoms from which they come; but on the corpuscular theory they have the same energy that they had as constituent parts of the primary beam.

Barkla¹ divides the secondary X radiation into two kinds: one which is merely the primary beam deflected and has the same properties; the other which is characteristic of the atom bombarded and is called the homogeneous fluorescent radiation. This latter kind increases in penetrating power with the atomic weight and can only be excited by a primary beam of greater penetrating power. Besides these two radiations there are at all times a large number of electrons liberated.

The ether pulse theory was devised to account for X ray phenomena originally, whereas the corpuscular theory was based on experiments dealing chiefly with γ rays.

At the present time most physicists are convinced of the similar nature of X and γ rays, so that any theory which explains one must of necessity include the other. Yet there is a possibility that they are not quite the same. Until one has examined X and γ rays under exactly the same conditions as to hardness one cannot finally decide the question. X rays have a number of peculiarities which γ rays have not exhibited as yet.

¹ Barkla, Phil. Mag., 1911.

For example in the case of X rays, some substances, notably silver, show selective absorption varying with the hardness of the ray. A similar phenomenon is not present in the case of γ rays. Unfortunately the hardest X rays yet examined are still a long way from the γ rays.

The author is at present engaged in trying to bridge over this gap, and the results will be incorporated in a paper to be published soon.

The hardening of the γ rays in the case of radium by successive layers of any material has been explained in the past by the theory that the rays are at the start heterogeneous in penetrating power and that the atoms of high atomic weight offer more rapid absorption to the soft rays than do the lighter atoms. The soft components are thus cut down very rapidly leaving the residue more and more penetrating.

Soddy is of the opinion that the γ rays of radium are homogeneous, but in his results gives several cases of hardening. If the rays are homogeneous it is difficult to see how they can be hardened, according to the corpuscular theory, where each is a separate entity unchangeable in nature.

My thanks are due to Mr. Edward Lorentz, of the Department of Physics, for his assistance during the course of this work.

UNIVERSITY OF CINCINNATI,
January, 1912.

PROCEEDINGS
OF THE
AMERICAN PHYSICAL SOCIETY.

VARIATION OF ELECTRICAL RESISTANCE WITH TEMPERATURE.¹ IV. OXIDES.

BY A. A. SOMERVILLE.

SINCE the effect of temperature upon the electrical resistance of many of the so-called conductors found in a laboratory has been studied, the writer has found an interesting field for work in the investigation of the behavior of the non-conductors at ordinary conditions, if those materials are subjected to higher temperatures.



Fig. 1.

Also as most metals and alloys oxidize when heated in the air, it seems just as natural to investigate the products of oxidation as to try to prevent oxidation by excluding oxygen.

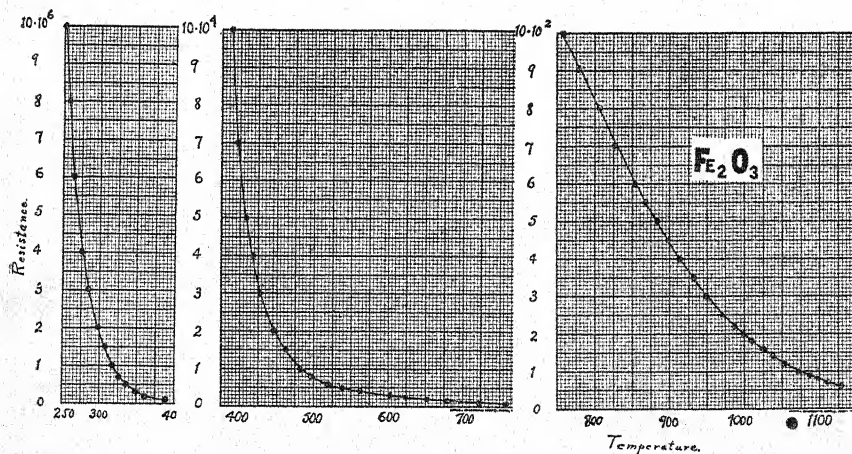


Fig. 2.

For these reasons the work has been begun of measuring resistance as a function of temperature of the oxides.

¹ Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1912.

As a rule, oxides at room temperature are non-conductors. As the temperature is increased the resistance of an oxide decreases until at 1000°C . it may be classed as a conductor.

The oxide in the form of a powder is placed in a porcelain or quartz tube (Fig. 1) 1 cm. in diameter and 10 cm. in length. Nickel rods are machined to fit into the ends of the tube against the ends of the column of oxide which is about 5 cm. long. Two to four pounds pressure is put on the ends of the nickel rods in order to effect constant contact with the oxide. The oxide in its containing tube is heated in a tubular resistance furnace. Temperature is measured with a platinum resistance thermometer. Electrical contact is made with the nickel rods for measuring the resistance of the oxide.

A column 1 cm. in diameter and 5 cm. in length, of iron oxide, Fe_2O_3 , has a resistance of 10,000,000 ohms at a temperature of 250° . As the temperature

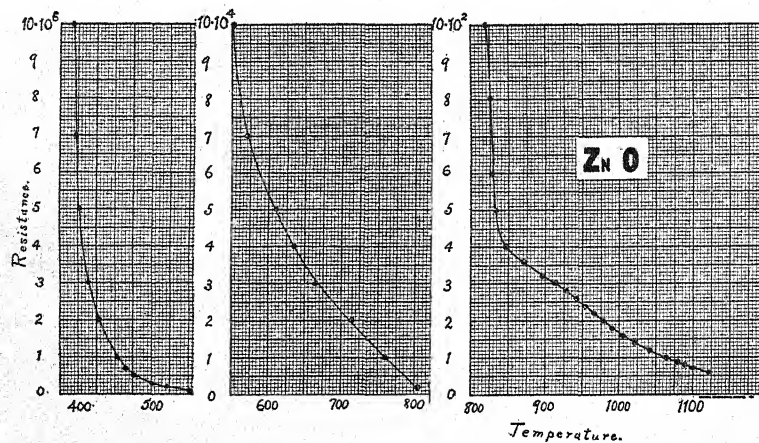


Fig. 3.

is increased the resistance decreases according to a relation similar to a parabolic curve. At a temperature of 1125°C . the resistance is 60 ohms.

A similar column of zinc oxide, ZnO , measures 10,000,000 ohms at 385°C . and the resistance varies in a body somewhat similar to that of iron oxide but not in such a regular manner. The resistance is 30 ohms at 1125°C .

CORNELL UNIVERSITY,
ITHACA, N. Y.

INTERNATIONAL CANDLE-POWER MEASUREMENTS.¹

BY CLAYTON H. SHARP.

IN 1903 the writer took a set of seasoned incandescent lamps to the National Physical Laboratory in London, to the Laboratoire Central d'Électricité in Paris and to the Physikalisch-Technische Reichsanstalt in Charlottenburg, in each of which laboratories the candle-power of these lamps was measured. The results were studied with a view of determining the status as to the standards of luminous intensity used in the countries in question. In this country the Bureau of Standards had adopted a ratio of 1 Hefner = 0.88 candle-power which value of the standard corresponded to that which had been in use in the electrical industry for a good many years previous and which had been maintained by the use of standardized incandescent lamps. It was found by the measurements referred to that the standard used in this country was in reasonably good conformity with the German standard, using this ratio. It appeared further that the value of the bougie décimale as used in France was very nearly equal to our candle, while in England the standard was not sufficiently well defined, although it apparently did not differ very much in value from the standard in this country.

Since 1903 numerous international comparisons using incandescent lamps have been very carefully carried out by the Bureau of Standards, which organization has also been able to bring order out of confusion through the adoption of the so-called "International Candle." The matter of standards therefore being presumably put on a satisfactory basis, it was deemed interesting to repeat the operation of carrying a set of seasoned lamps once more to these various laboratories and to extend the comparison somewhat in view of more recent developments in lamp manufacture. This the writer did during the past summer.

Four lamps were taken, of the drawn wire tungsten type. At each point where a spire of the filament passed a support, it was welded fast to the support, so that all loose or variable contacts in the lamp were avoided. These lamps, after having been seasoned, were standardized at two different voltages, (1) the voltage at which the color of the light was the same as the color of the carbon lamp as ordinarily operated, and (2) the voltage at which the color of the light was the same as that of the tungsten filament lamp as ordinarily operated. The second voltages were substantially 50 per cent. higher than the first. The reason for taking two voltages was that it was known that the color difference introduced in comparing a carbon with a tungsten lamp caused an error in photometry which differed with different individuals and which was by no means negligible. On account of this the Electrical Testing Laboratories had already adopted a series of tungsten lamp standards for regular work and it was desired to see how closely the value of these standards would be represented by the measurements obtained in foreign laboratories.

¹ Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1912.

The results of the measurements are given in the following table. The values given by the Reichsanstalt are reduced to candle-power by multiplying them by the present nominal ratio of the Hefner to the candle, namely, 0.9.

	Lamp No. 3,892 at 69.2 Volts.		Lamp No. 3,893 at 69.8 Volts.		Lamp No. 3,894 at 69.5 Volts.		Lamp No. 3,895 at 69.15 Volts.		Averages.	
	C.P.	Per Cent. Dev.	C.P.	Per Cent. Dev.	C.P.	Per Cent. Dev.	C.P.	Per Cent. Dev.	C.P.	Per Cent. Dev.
E.T.L.	6.0	-0.50	7.0	-0.28	8.5	-0.47	8.5	+0.47	7.5	-0.18
P.T.R.	6.075	+0.75	7.10	+1.14	8.65	+1.17	8.57	+1.18	7.60	+1.13
L.C.E.	6.0	-0.50	6.95	-1.00	8.5	-0.47	8.35	-1.30	7.45	-0.85
N.P.L.	6.06	+0.50	7.05	+0.43	8.565	+0.29	8.50	+0.47	7.54	+0.35
B.S.	6.0	-0.33	7.0	-0.28	8.5	-0.47	8.4	-0.71	7.48	-0.45
Mean	6.03		7.02		8.54		8.46		7.51	

	Lamp No. 3,892 at 106.8 Volts.		Lamp No. 3,893 at 100.4 Volts.		Lamp No. 3,894 at 105.75 Volts.		Lamp No. 3,895 at 105.2 Volts.		Averages.		Ratio.
	C.P.	Per Cent. Dev.	C.P.	Per Cent. Dev.	C.P.	Per Cent. Dev.	C.P.	Per Cent. Dev.	C.P.	Per Cent. Dev.	H.V.C.P. L.V.C.P.
E.T.L.	31.0	0	27.97	+0.25	42.0	+0.48	42.0	+0.48	35.74	+0.42	4.76
P.T.R.	31.6	+1.93	28.4	+1.79	42.9	+2.56	42.95	+2.75	36.45	+2.42	4.80
L.C.E.	30.5	-1.61	27.5	-1.43	40.7	-2.63	40.6	-2.86	34.82	-2.16	4.68
N.P.L.	30.9	-0.32	27.7	-0.72	41.75	-0.12	41.65	-0.36	35.50	-0.25	4.71
B.S.	30.8	-0.64	27.8	-0.36	41.6	-0.48	41.6	-0.48	35.45	-0.39	4.74
Mean	31.0		27.9		41.8		41.8		35.59		4.74

It will be seen that the various laboratories are in close agreement in their evaluation of these lamps as far as their determination at the ordinary color of carbon lamps is concerned. The greatest deviation from the mean is shown by the Reichsanstalt, namely, 1.1 per cent., which is not a surprising deviation in view of the fact that the Reichsanstalt measurements are all carried back to the Hefner lamp itself, whereas in the case of the other laboratories, measurements are carried back to a series of standardized incandescent lamps. Therefore the 1.1 per cent. may represent a variation of the Hefner lamp from its mean value at the particular time it was used in connection with the measurements in question. With respect to the other deviations it may be said that they are so small that they illustrate the extreme closeness with which the standards are kept and the great care with which the photometric measurements were carried out. Such precision in international candle-power determinations was a thing not to be thought of only a few years ago.

With respect to the measurements of the lamps at the whiter color, the situation is not so good. Here considerable deviations come in which are evidently due to different individual evaluations of lights of different color. The variations run from + 2.4 per cent. to - 2.2 per cent. However, it is to be noted that in only three laboratories, namely, the National Physical Labo-

ratory, the Bureau of Standards and the Electrical Testing Laboratories, had special attention been given to the establishment of a well defined line of white light standards and that among these laboratories the variations are less than the ordinary error of precision photometric work.

The above contribution therefore goes to show that international precision as well as precision in a single laboratory has been established in photometric measurements, and to emphasize the fact that a series of standards of only one color will not do. Different series of standards are required in precision work in order that the color of the standard may not differ too much from the color of the source of light which is under measurement.

ELECTRICAL TESTING LABORATORIES,
NEW YORK.

MAGNETIC REACTIONS PRODUCED BY A COPPER DISC ROTATING BETWEEN THE POLES OF A MAGNET.¹

BY F. G. BENEDICT AND W. G. CADY.

THESE observations were made on the revolving disc of a "bicycle ergometer" belonging to the Nutrition Laboratory of the Carnegie Institution of Washington, but the results are generally applicable to all conductors moving in the neighborhood of magnet poles. The machine experimented upon is a stationary bicycle having a disc of copper about 40 cm. in diameter and 6 mm. thick mounted in place of the rear wheel. A calorimetric method is used to determine the amount of energy expended (chiefly in the form of eddy currents in the disc) when the latter is rotated between the poles of a fixed electro-magnet. These calibrations have shown that the rate of production of heat does not increase with increasing angular velocity as rapidly as would be expected from elementary considerations. When, however, the self-inductance of the current paths in the disc is taken into account, the peculiar form of the calibration curves is fully explained.

Starting with the assumptions that the angle of lag of the currents is directly proportional to the angular velocity, and that the consequent demagnetizing effect of the currents is proportional to this angle and to the intensity of the currents, the equation

$$\frac{\sigma}{k} = \omega^2 \frac{\varphi}{\varphi'}$$

is deduced, in which σ is the specific resistance of the disc, ω the angular velocity, φ the resultant magnetic flux, and φ' the counter-flux due to the eddy currents; k is a constant of proportionality.

Some preliminary observations were made with a ballistic galvanometer, in order to learn the extent to which the total flux was reduced when the disc was set in rotation. A magnetic survey of the whole region between and near

¹ Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1912.

the magnet poles was then made, using a bismuth spiral to measure the magnetic induction at different points. The results when exhibited in the form of curves show that the magnetic distortion and the weakening of the magnetic field increase very greatly with increasing angular velocity, and that the equation above is approximately satisfied for all speeds up to 364 revolutions per minute of the disc.

A simple computation shows that the total intensity of the currents in the disc is of the order of 2,000 amperes.

THE ELECTRON THEORY OF PHOSPHORESCENCE.¹

BY CHESTER A. BUTMAN.

A THEORY of phosphorescence has been more completely developed, which explains in a consistent manner the usual phenomena observed. The principal points in the theory are as follows:

1. There is a direct relation between the photoelectric effect and phosphorescence.
2. The component which gives the photoelectric effect is *not* the one which emits the light.
3. The spectra of the phosphorescent material are due to a *selective emission* of light.
4. If the selective emission is caused by the incident light alone, fluorescence occurs. However, if the fluorescence is maintained by *collisions* of the particles which have received electrical charges due to the electrons ejected by the photoelectric material, the phenomenon of phosphorescence is observed.
5. In the case of either fluorescence or phosphorescence the electronic systems of the atom emitting the light must be brought to their *critical values* of oscillating energy before an emission of light will occur.
6. It is necessary that phosphorescent substances be good insulators so that the charges produced by the photoelectric component will not disappear too quickly.
7. The effect of red and infra-red light is to dampen the oscillations of the electronic systems emitting the light, causing them to fall below the critical value, thus extinguishing the phosphorescent light.
8. The different phosphorescent bands can be generated separately, because the electronic systems in the atom are most easily brought to their critical value by particular wave-lengths.

According to the theory ($\text{CaO} + \text{BiO} + \text{S} + \text{Flux}$) is phosphorescent because BiO is capable of giving a selective emission of light in the visible spectrum. The BiO is diluted with CaO, which is an insulator and is transparent for the visible spectrum. The sulphur is photoelectric with visible light and furnishes the supply of electrons. The oscillating electronic systems in the Bi,

¹ Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1912.

which cause the selective emission bands, having been brought to their critical value by the incident light, are maintained in that state by the collision of the electrified particles with the Bi. Inasmuch as the material has high insulating qualities the charges (ejected photoelectrically from the sulphur) remain for some time, hence the luminosity also remains. However, the duration of the luminosity is also dependent on the ease with which the oscillating systems are maintained at their critical value.

Feb. 21, 1912.

ON THE EXPANSION OF ALTERNATING CURRENT E. M. F.'s IN A FOURIER'S SERIES.¹

BY GEO. R. OLSHAUSEN.

IN many text-books and treatises on alternating currents the statement is made that practically all alternating current machines generate E.M.F.'s which, when plotted in rectangular coördinates as a function of the time, present a certain symmetry with respect to the axis of abscissæ when it is taken as the time axis. The further statement is then made that, owing to this symmetry, an expansion of the E.M.F. function in a Fourier's series contains only the odd harmonics. As I do not know of any general proof of the last statement, the following one may be of interest.

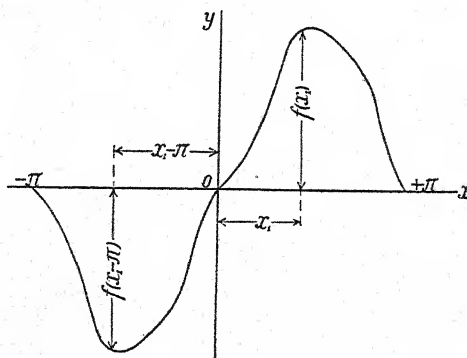


Fig. 1.

The symmetry referred to is of the type shown in Fig. 1, in which the E.M.F.'s are the ordinates and the products $2\pi ft$ the abscissæ, t being the time and f the frequency. We have therefore

$$y = f(x) = \text{E.M.F.}, \\ x = 2\pi ft.$$

If we develop

$$y = f(x)$$

¹ Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1912.

in a Fourier's series, we have in general

$$f(x) = a_0 + a_1 \cos x + a_2 \cos 2x + \cdots + a_n \cos nx + \cdots \\ + b_1 \sin x + b_2 \sin 2x + \cdots + b_n \sin nx + \cdots,$$

where the coefficients, as is well known, have the following values, viz.:

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{+\pi} f(x) dx, \\ a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos nx dx, \\ b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin nx dx.$$

We shall now show that a_0 and all coefficients for which n is even vanish on account of the assumed symmetry.

If we let x_1 be a particular value of x , such that

$$0 < x_1 < \pi,$$

we have, as seen from the figure,

$$f(x_1) = -f(x_1 - \pi).$$

It is easily seen that for each positive value x_1 of x there exists a corresponding negative value $x_1 - \pi$ for which $f(x)$ has the same absolute value.

Consider now the constant term

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{+\pi} f(x) dx.$$

The elements of this integral can be divided into pairs, such that

$$f(x_1) dx = -f(x_1 - \pi) dx,$$

dx being always positive since $f(x)$ is a single-valued function. The elements of this integral therefore annul each other in pairs and a_0 is consequently zero.

Next, consider the elements

$$f(x_1) \cos nx_1 dx \quad \text{and} \quad f(x_1 - \pi) \cos (x_1 - \pi) dx$$

of the integral in

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos nx dx.$$

$f(x_1)$ and $f(x_1 - \pi)$ have the same absolute value but have opposite signs. $\cos nx_1$ and $\cos n(x_1 - \pi)$ have the same value when n is even, but are of equal magnitude and of opposite sign when n is odd. The elements under consideration have therefore the same absolute value for all values of n , but are of opposite sign when n is even. In the latter case the sum of each pair of such elements is therefore zero, and consequently a_n vanishes.

In precisely the same manner it can be shown that the coefficients b_n are zero only when n is even.

For curves of the assumed symmetry the development of $f(x)$ into a Fourier's series therefore contains only the sine and cosine terms of the odd multiples of x , so that we have

$$f(x) = a_1 \cos x + a_3 \cos 3x + a_5 \cos 5x + \dots \\ + b_1 \sin x + b_3 \sin 3x + b_5 \sin 5x + \dots$$

The form of the expansion of $f(x)$ for other forms of symmetry may be determined in a similar manner. For example, in Fig. 2 we have a curve which

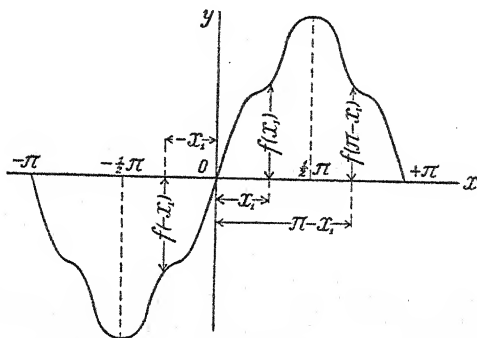


Fig. 2.

is symmetrical with respect to the origin and at the same time the two halves of the curve are also symmetrical to ordinates passing through the points

$$x = -\frac{\pi}{2} \quad y = 0 \quad \text{and} \quad x = \frac{\pi}{2}, \quad y = 0$$

respectively. We have therefore

$$f(x_1) = -f(-x_1) = f(\pi - x_1).$$

It can easily be shown that on account of these two conditions the expansion of $f(x)$ will only contain the sines of the odd multiples of x , so that

$$f(x) = b_1 \sin x + b_3 \sin 3x + b_5 \sin 5x + \dots$$

If $f(x)$ is symmetrical with respect to the y axis and each half of the curve is also symmetrical as above, so that

$$f(x_1) = f(-x_1) = f(\pi - x_1),$$

the expansion will in general contain a constant term and the cosines of the even multiples of x . We have then

$$f(x) = a_0 + a_2 \cos 2x + a_4 \cos 4x + \dots$$

NEW BOOKS.

- The Measurement of High Temperatures.* Third Edition. By G. K. BURGESS and H. LECHATELIER. New York: John Wiley and Sons, 1912. Pp. xviii + 510. Price, \$4.00.
- A Laboratory Manual of Physics and Applied Electricity.* Vol. I. Revised. By EDWARD L. NICHOLS and ERNEST BLAKER. New York: Macmillan Company, 1912. Pp. xiii + 417. Price, \$3.00.
- Outlines of Applied Optics.* By P. G. NUTTING. Philadelphia: P. Blakiston's Son and Co., 1912. Pp. ix + 234. Price, \$2.00.
- The Chemistry of the Radio-Elements.* By FREDERICK SODDY. New York: Longmans, Green and Co., 1911. Pp. 1 + 92. Price, \$.90.
- Calcul et Construction des Alternateurs Mono- et Polyphases.* By HENRI BIRVEN. Translated from the German by P. DUFOUR. Paris: Gauthier-Villars, 1911. Pp. 1 + 179. Price, 6 Fr.
- Text Book of Physics.* By C. E. LINEBARGER. Boston: D. C. Heath and Company, 1911. Pp. viii + 471.
- Théorie de la Couche Capillaire Plane des Corps Purs.* By GERRIT BAKKER. Paris: Gauthier-Villars, 1911. Pp. 1 + 93. Price, 2 Fr.

THE PHYSICAL REVIEW.

THE THEORETICAL AND EXPERIMENTAL DETERMINATION OF REFLECTION COEFFICIENTS.

BY J. T. TATE.

THE optical constants of absorbing media have been studied by two general methods, according as the observations have been made on transmitted or on reflected light. The transmission determinations comprise notably Kundt's¹ metallic prism method for refractive indices and Wernicke's² direct photometric method for absorption coefficients. Owing, however, to the large experimental error introduced in measuring the thickness of the necessarily very thin films used, the "reflection" method, which obviates this, has in recent years been applied almost exclusively. The "reflection" method consists in a polarimetric analysis of the ellipticity and rotation produced in plane polarized light by reflection from the surface of the medium under investigation. Drude,³ who was the first to apply this to an extensive study of the optical constants of absorbing media, has conclusively shown from a consideration of the experimental errors involved that the values obtained by polarimetric measurements are decidedly more accurate than those by any other known method.

It must be noted, however, that the values for the optical constants obtained by these two entirely different methods (the reflection and the transmission) have not in general agreed within the errors of observation and that an explanation of this variation must take into account the essential difference between them. The one deals with the light which has been affected by the surface of the body only, while the other deals with that which has been affected by the medium as a whole. The discrepancies in the observed magnitudes indicate, therefore, that the media

¹ Kundt, A., *Wied. Ann.*, 34, p. 469, 1888.

² Wernicke, W., *Pogg. Ann. Ergbd.*, 8, p. 65, 1878.

³ Drude, P., *Ann. d. Physik*, 39, p. 481, 1890.

dealt with are not strictly speaking homogeneous but possess characteristics at their surfaces differing from those within.

This condition of affairs has been emphasized in the more recent investigations by the reflection method. Much uncertainty as to the reliability of the results obtained by it has arisen on account of the inability of different observers, and even of the same observer, to obtain consistent results from any one substance, it being found that changing the method of polishing, or even repolishing by the same method, changes decidedly the characteristics of the surface. Drude¹ first, and more recently Minor² and Tool,³ have investigated this fact and from their results the following conclusions may be drawn:

1. The mere process of polishing modifies the properties of the surface considerably, increasing the absorption and reflection coefficients and decreasing the refractive indices. Drude¹ showed this conclusively by making observations on surfaces of cleavage of lead sulphid first in their natural condition, then after polishing. From the polished surface he obtained an absorption coefficient 35 per cent. larger, and a refractive index proportionately smaller, than the same surface gave in its natural condition.

2. Impurities worked into the surface during the polishing process augment the above changes.

3. An incomplete polish or matt surface usually tends to decrease the values of the constants.

4. Films formed on the surface by oxidation or other chemical change ordinarily have the same effect as a matt surface.

5. With but few exceptions, however, the *form* of the curves is not affected by the polishing process or by impurities.

6. The efforts which have been made by the different observers to obtain a so-called "standard" method of preparing the surfaces have met with only doubtful success on account of the very large effect produced by even very slight changes in the method of polishing.

From the above conclusions it is evident that the reflection method does not give true values for the optical constants of the *body* of the medium but only those which are characteristic of the particular *surface* under investigation. Nothing is to be gained from a comparison of the results obtained by different observers except the *form* of the dispersion curves and rough average values for the constants. Despite these facts, the reflection method, with any one particular surface, does give con-

¹ Drude, P., Ann. d. Physik, 36, p. 532, 1889.

² Minor, R. S., Ann. d. Physik, 10, p. 581, 1903.

³ Tool, A. Q., PHYS. REV., 31, p. 1, 1910.]

sistent results for polarimetric measurements taken at varying angles of incidence. It is important therefore to determine whether all of the constants obtainable from any one surface by reflection measurements are consistent with each other according to the accepted theories.

The object of the present investigation was to compare the reflection coefficients computed from the optical constants obtained polarimetrically with the reflection coefficients actually observed on the same surface. The comparisons which have been made by Drude,¹ Minor,² Tool,³ and others between the theoretical and observed magnitudes of the reflection coefficients are all open to the same weakness in that the theoretical values were calculated for the particular mirror which happened to be under investigation and compared with those which had been experimentally determined by other observers on other mirrors. From the facts brought out above it is evident that any close agreement would be largely accidental and hence of little or no value as a rigid test of the theory. As might be expected they found the agreement to be far from satisfactory in almost every case. On the other hand the comparison made in the present investigation, based on observations taken on one and the same surface and therefore free from the above weakness, shows excellent agreement for all surfaces tested.

APPARATUS.

For Polarimetric Measurements.—To measure the ellipticity and rotation produced in plane polarized light by reflection from the surface of the absorbing medium the analyzing system devised by Tool was used, in which, for the sake of accuracy, the Brace Half Shade is introduced as a modification of the well-known Stokes' analyzer. With sunlight (analyzed by a spectral system) this instrument is capable of measuring an ellipticity of approximately unity to .05 per cent. and of 0.2 to 1 per cent., while the rotation is accurately obtained to about .05 per cent.

For Photometric Measurements.—A Brace spectro-photometer, in connection with a special reflecting device, was used for the photometric measurements. With this instrument an accuracy in the reflection coefficients of about .5 per cent. (under best conditions) to 1 per cent.—depending on the color and the surface dealt with—was obtained. An acetylene jet which gave a steady flame and whose illumination was uniform over a sufficient area served as a light source. Light from a very small area on one side of the flame was reflected into the collimator C_1

¹ Drude, P., loc. cit., 1 preceding page.

² Minor, R. S., loc. cit.

³ Tool, A. Q., loc. cit., 4.

(see Fig. 1) by the total reflecting prisms at P . Light from the same part of the flame on the other side was directed through the reflecting instrument, finally entering the other collimator C_2 . To obtain the relative

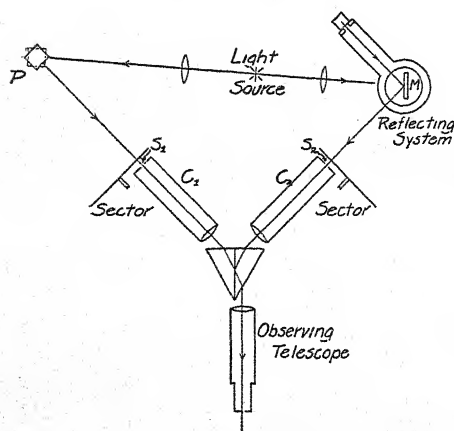


Fig. 1.

intensities of reflected and unreflected light it was then only necessary to direct the light into the collimator C_2 successively with and without reflection at the mirror surface, and measure the intensity in each case with the light entering the collimator C_1 as a standard of comparison. It is of course essential that the optical path and the equivalent focus of the light source be the same for both unreflected and reflected rays. The reflection instrument (shown diagrammat-

ically in vertical section in Fig. 2) was designed to secure these conditions. The light enters the instrument through the fixed total reflecting prism A , so adjusted as to reflect it up along the axis of rotation of an arm which carries other total reflecting prisms B , C and D . After reflection at their surfaces (as indicated in the figure) the light finally reaches the mirror M whose surface lies in the axis of rotation of the system of prisms. By M it is reflected directly into the col-

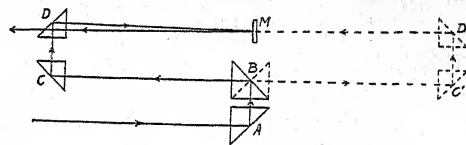


Fig. 2.

limator C_2 . With the mirror M removed and the prisms rotated into the position indicated by the broken lines, the same light which was previously reflected by M into the collimator, now passes directly into the collimator. It is readily seen that the optical path of the ray in both cases is the same and, provided the mirror M is perfectly plane, the equivalent focus of the light source is also the same. With this instrument, therefore, it is possible to readily adjust for measuring the reflection coefficient at any angle of incidence insuring at the same time the identity of optical path.

Comparisons of intensity were made in the usual way by means of the graduated sectors and the adjustable slits S_1 and S_2 .

FORMULÆ.

For calculating the refractive indices and absorption coefficients the following customary equations were used.

$$\begin{aligned} m_{\theta} &= -\frac{\sin \theta \operatorname{tg} \theta \cos 2\psi \cos 2\gamma}{1 - \cos 2\psi \sin 2\gamma}, \\ m_{\theta} \kappa_{\theta} &= \frac{\sin \theta \operatorname{tg} \theta \sin 2\psi}{1 - \cos 2\psi \sin 2\gamma}, \\ \kappa_{\theta} &= -\frac{\operatorname{tg} 2\psi}{\cos 2\gamma}, \end{aligned} \quad (1)$$

in which m_{θ} is defined by the equation

$$m_{\theta} = v_{\theta} \cos \rho,$$

where v_{θ} is the refractive index of the medium at the angle of refraction ρ and the corresponding angle of incidence θ . The quantity m_{θ} may therefore be defined as the ratio of the velocity of the incident light to the velocity of the refracted light measured along the normal to the planes of equal amplitude. The quantity 2ψ is a measure of the ellipticity of the reflected light defined by the relation

$$E = \operatorname{tg} \psi,$$

where E is the ratio of the minor to the major axis of the ellipse. The quantity 2γ is the so-called azimuth of restored polarization of the elliptic vibration measured with the plane of polarization of the incident light as the plane of reference. The quantity κ_{θ} is the absorption coefficient of the medium for the angle of incidence θ such that $e^{-2\pi\kappa_{\theta}}$ gives the ratio of the amplitudes of the vibration at two successive points of equal phase on any normal to the planes of equal amplitude. In all cases where the light enters from a transparent medium these planes of equal amplitude are parallel to the dividing surface.

These quantities, m_{θ} , $m_{\theta} \kappa_{\theta}$ and κ_{θ} , which are strictly speaking properties of the medium only for the particular angle of incidence θ , are related to the corresponding constants of the medium at normal incidence by the following equations:

$$\begin{aligned} 2v_0^2 &= \sqrt{(m_{\theta}^2 - m_{\theta}^2 \kappa_{\theta}^2 + \sin^2 \theta)^2 + 4m_{\theta}^4 \kappa_{\theta}^2} + (m_{\theta}^2 - m_{\theta}^2 \kappa_{\theta}^2 + \sin^2 \theta), \\ 2v_0^2 \kappa_0^2 &= \sqrt{(m_{\theta}^2 - m_{\theta}^2 \kappa_{\theta}^2 + \sin^2 \theta)^2 + 4m_{\theta}^4 \kappa_{\theta}^2} - (m_{\theta}^2 - m_{\theta}^2 \kappa_{\theta}^2 + \sin^2 \theta), \end{aligned} \quad (2)$$

where v_0 and κ_0 are respectively the refractive index and absorption coefficient at normal incidence. For convenience in computation where $\sin^2 \theta$ is small in comparison with v_0^2 and κ_0^2 these equations have been expanded in series involving powers of $\sin^2 \theta$ as follows:

$$\begin{aligned}
 \nu_0 &= m_\theta \left[1 + \frac{\sin^2 \theta}{2m_\theta^2(1 + \kappa_\theta^2)} \right. \\
 &\quad \left. + \frac{1}{2} \left(\frac{m_\theta^2 \kappa_\theta^2}{m_\theta^2(1 + \kappa_\theta^2)} - \frac{1}{4} \right) \frac{\sin^4 \theta}{m_\theta^4(1 + \kappa_\theta^2)^2} - \dots \right], \\
 \nu_0 \kappa_0 &= m_\theta \kappa_\theta \left[1 - \frac{\sin^2 \theta}{2m_\theta^2(1 + \kappa_\theta^2)} \right. \\
 &\quad \left. + \frac{1}{2} \left(\frac{m_\theta^2}{m_\theta^2(1 + \kappa_\theta^2)} - \frac{1}{4} \right) \frac{\sin^4 \theta}{m_\theta^4(1 + \kappa_\theta^2)^2} - \dots \right].
 \end{aligned} \tag{3}$$

From ν_0 and κ_0 the reflection coefficient at normal incidence J_0 is given by the relation

$$J_0 = \frac{\nu_0^2(1 + \kappa_0^2) + 1 - 2\nu_0}{\nu_0^2(1 + \kappa_0^2) + 1 + 2\nu_0}.$$

From the high accuracy possible in measuring the ellipticity and rotation, it is interesting to note that the probable error in J_0 as calculated from the polarimetric measurements was found to be only about .1 per cent. while that in the photometrically determined value was from .5 to 1 per cent. If the polarimetrically determined values of J_0 are reliable it must therefore be concluded that the values thus obtained are much more accurate than those obtainable by any direct photometric determination.

EXPERIMENTAL RESULTS.

Silver.—The silver was chemically deposited on a plate of glass perfectly free from double refraction, and observations made both on the air-silver and on the glass-silver surface. Unfortunately it was impossible to secure a naturally brilliant air-silver surface without recourse to polishing with chamois skin. For polarimetric measurements on the glass-silver surface a 60° prism of the same optical glass as the plate upon which the silver was deposited was cemented to it with Canada balsam (Fig. 3). This



Fig. 3.

allowed the light to enter and leave the glass normal to its surface thus obviating any difficulties arising from rotation of the azimuth of the incident plane polarized light or of change in the angle of incidence by refraction. To obtain photometric measurements at normal incidence on the same surface this prism was removed.

The results obtained from both surfaces are given in Tables I. and II. To obtain a correct value for the "observed" reflection from the silver-glass surface the theoretical coefficient (.04) was used for the glass-air boundary. By calculation this surface was found to produce an inappreci-

TABLE I.
Silver.
(Light incident in Air.)

Wave-length ($\mu\mu$).	ν_0	κ_0	$\nu_0\kappa_0$	J_0 (calc.).	J_0 (obs.).
460	.270	12.02	3.245	.911	.905
500	.273	13.03	3.557	.923	.927
540	.279	13.95	3.892	.933	.930
580	.284	14.77	4.195	.941	.948
620	.291	15.53	4.519	.947	.950
660	.299	16.28	4.868	.953	.952
700	.308	16.87	5.196	.957	.953

TABLE II.
Silver.
(Light incident in Glass.)

Wave-length ($\mu\mu$).	ν_0	κ_0	$\nu_0\kappa_0$	J_0 (calc.).	J_0 (obs.).
460	.339	6.90	2.340	.817	.810
500	.343	7.72	2.648	.844	.840
540	.348	8.51	2.962	.869	.862
580	.356	9.26	3.301	.889	.885
620	.365	9.93	3.625	.900	.902
660	.375	10.46	3.921	.913	.919
700	.386	10.96	4.230	.922	.930

able effect. A comparison of these tables shows a striking difference in the properties of the silver at the two surfaces. The absorption coefficients κ_0 on the side next the glass have about one half the magnitude of those next the air. Similarly the calculated value of the refractive index ν_0 of the silver on the side next the glass is nearly double that on the other. It is impossible to calculate this accurately because, while the actual refractive index of the glass was about 1.51, a polarimetric measurement of this constant on a part of the surface not covered by the silver gave a value of 1.30. Using, however, the refractive index as 1.51 that of the silver for a wave-length of $580\mu\mu$ would be $1.51 \times .356 = .538$, while the value 1.30 gives .463. For the same wave-length on the air side a distinctly smaller value than either of these, namely, .284 was obtained. The ratio of both absorption coefficients and refractive indices (for the two silver surfaces) is about the same as that obtained by Drude¹ on lead sulphid before and after polishing. It is interesting to note here, as shown by Tool in a number of cases, that the product of refractive index and absorption coefficient is practically the same for both surfaces. Another interesting difference between the two surfaces is that relatively less blue light is reflected from the glass-silver surface than from the other.

¹ Drude, P., loc. cit., 4.

A comparison of the calculated and observed values for the reflection coefficients shows in general a very close agreement (see Fig. 4). That the small differences—the maximum being less than 1 per cent., the average less than .5 per cent.—arise from experimental errors is apparent from the fact that the observed values lie on both sides of the curve for the calculated values.

Gold and Copper.—The large variation in the visible spectrum shown by the reflection coefficients of gold and copper made these very desirable substances for the present investigation. As was stated in the paragraph on spectro-photometric adjustments it is essential that the mirror surfaces be very nearly plane and reasonably free from scratches. In attempting to surface these two metals great difficulty was experienced in realizing these requirements, owing to their softness and the consequent difficulties in securing a good polish. The following electrolytic process was finally adopted and proved to be a very satisfactory solution of the problem. A large mirror of speculum metal whose surface was plane and free from scratches was cut into small mirrors and upon these the copper and gold were deposited electrolytically. The resulting surfaces were exact duplicates of the speculum metal surface in respect to both planeness and polish—the plating solutions and current density being so chosen that perfect mirrors were produced without any subsequent polish being necessary. Although the individual mirrors obtained in this way were very satisfactory it was found impossible to duplicate, within one per cent., the results with different samples, even though the conditions of deposit were carefully reproduced.

The gold mirror was prepared by electrolytic deposition from a solution made up as follows. Sodium phosphate 5 gr., potassium hydrate 3 gr., potassium cyanide 15 gr., gold chloride 1 gr., water 1 liter. This solution was warmed to 50° C. and the current regulated to about .005 ampere per square centimeter.

The results (Table III., Fig. 4), compared with those obtained by other investigators on specimens which had been polished, again show quite strikingly the effect of the polishing process in changing the characteristics of the surface. The absorption coefficients of the untreated surface are from 20 to 50 per cent. smaller than those observed by Tool on one which had to be polished, while the refractive indices are correspondingly higher, so that the value of n_0k_0 does not differ greatly from his.

As with silver, the agreement between calculated and observed values of the reflection coefficients is very close, the fluctuation being on both sides of the smooth curve—the maximum difference being 2.5 per cent. on the steepest slope of the curve, and the mean 1 per cent. The character

of this curve is slightly different from that obtained by Tool—relatively less red light being reflected, the mirror used here appearing more greenish-yellow than his.

TABLE III.

Gold.

Wave-length ($\mu\mu$).	ν_0	κ_0	$\nu_0\kappa_0$	J_0 (calc.).	J_0 (obs.).
460	1.662	1.098	1.825	.362	.370
480	1.533	1.219	1.865	.381	.380
500	1.330	1.510	2.008	.438	.430
520	1.104	1.981	2.187	.521	.530
540	.937	2.717	2.545	.634	.618
560	.805	3.503	2.819	.703	.708
580	.729	4.144	3.023	.760	.758
600	.666	4.864	3.239	.800	.792
620	.645	5.294	3.414	.820	.812
640	.630	5.687	3.581	.837	.840
660	.636	5.862	3.727	.847	.852
680	.617	6.255	3.859	.859	.853

The copper mirror was deposited from a solution made up as follows: Copper acetate 25 gr., sodium carbonate 25 gr., sodium bisulphite 20 gr., potassium cyanide 50 gr., water 1 liter. The solution was used at room temperature and the current density regulated to .005 ampere per square centimeter. The same differences in the constants ν_0 and κ_0 (Table IV., Fig. 4) of the surface thus prepared as compared with those obtained on a polished surface by Tool is again noticed here—the magnitude and direction of the change being about the same as with gold. Expecially striking therefore is the close agreement between our values of $\nu_0\kappa_0$ —the difference not exceeding 2 per cent., although the differences in ν_0 and κ_0 amount to about 40 per cent.

TABLE IV.

Copper.

Wave-length ($\mu\mu$).	ν_0	κ_0	$\nu_0\kappa_0$	J_0 (calc.).	J_0 (obs.).
460	1.570	1.440	2.261	.463	.460
480	1.531	1.503	2.301	.477	.480
500	1.500	1.580	2.370	.494	.490
520	1.434	1.675	2.402	.510	.512
540	1.378	1.788	2.464	.530	.540
560	1.269	2.018	2.561	.566	.575
580	1.067	2.659	2.837	.653	.655
600	.980	3.250	3.185	.721	.710
640	.958	3.685	3.530	.764	.765
660	.996	3.716	3.701	.775	.777
700	1.035	3.688	3.817	.778	.786

The agreement between the observed and calculated values of the reflection coefficients is again very close, the largest variation for any one wave-length being about 2 per cent. and the mean 1 per cent. That these differences are also due to photometric errors is shown by the fact that the observed values fluctuate about equally on both sides of the curve of calculated values. The reflection coefficients for this surface are about 8 per cent. lower than those given by Tool but the form of the curve is the same.

Steel.—The steel mirror was one from a König tuning fork possessing a very perfect surface. It was thoroughly cleansed and rubbed with chamois skin just before the observations were made. The results (Table V., Fig. 4) agree very closely with those obtained by other observers on polished steel—differing from those given by Tool by less than 2 per cent.

TABLE V.

Steel.

Wave-length ($\mu\mu$).	ν_0	κ_0	$\nu_0\kappa_0$	J_0 (calc.).	J_0 (obs.).
460	1.930	1.535	2.965	.556	.555
500	2.130	1.460	3.110	.562	.560
540	2.300	1.405	3.230	.569	.560
580	2.435	1.355	3.300	.571	.573
620	2.536	1.340	3.395	.578	.573
660	2.635	1.310	3.450	.581	.580
700	2.695	1.305	3.515	.587	.580

The calculated and observed reflection coefficients show close agreement—the fluctuation being on both sides of the curve and in no case exceeding 1.5 per cent. It is important to note one thing shown by all the reflection coefficient curves and most strikingly by the curve for steel, namely, that far more consistent and accurate results are evidently obtained by polarimetric measurements than by the more direct photometric method—the points representing the calculated values always lying closer to the smooth curve than the values directly observed.

A comparison of the magnitudes obtained here with those given by Hagen and Rubens¹ for the same metals shows in some cases excellent agreement, in others a marked difference—as is to be expected.

Fuchsin.—In order that the test for the agreement between calculated and observed reflection coefficients might be as comprehensive as possible it was thought advisable to test a mirror of fuchsin, this being a semi-absorbing medium, whose reflection coefficient is low compared with the more strongly absorbing metals. The mirror was produced by flowing

¹ E. Hagen and H. Rubens, *Ann. d. Physik*, 1, p. 352, 1900.

an alcoholic solution of fuchsin over a glass plate and allowing it to dry evenly. After a few trials a satisfactorily brilliant and plane surface was obtained upon which the observations were made. The results from this mirror (Table VI., Fig. 4), to obtain which equations (2) instead of (3) were necessarily used, agree very closely with those given by Cartmel¹

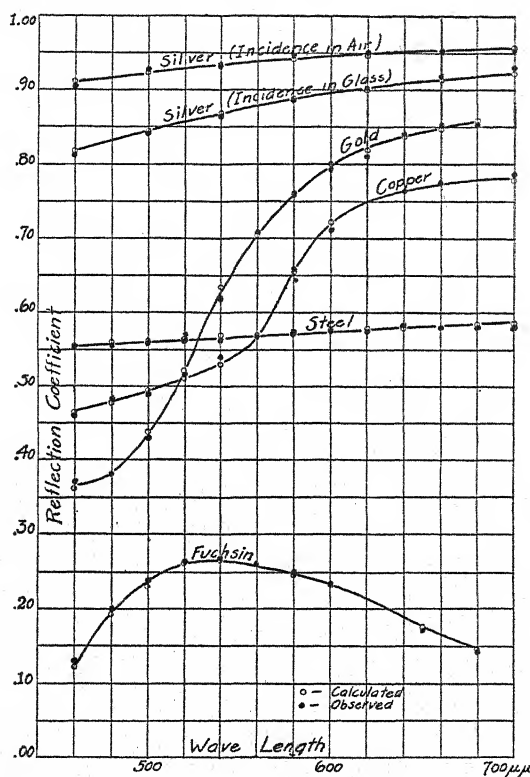


Fig. 4.

TABLE VI.

Fuchsin.

Wave-length ($\mu\mu$).	ν_0	κ_0	$\nu_0\kappa_0$	J_0 (calc.).	J_0 (obs.).
460	1.056	.720	.760	.121	.130
480	1.204	.876	1.055	.193	.200
500	1.545	.825	1.275	.230	.238
520	1.860	.739	1.375	.261	.265
540	2.150	.626	1.345	.267	.270
560	2.400	.498	1.195	.261	.260
580	2.555	.378	.966	.247	.250
600	2.660	.252	.683	.232	.235
650	2.438	.055	.135	.176	.170
680	2.231	.025	.056	.146	.142

¹ Cartmel, W. B., Phil. Mag., 6, p. 213, 1903.

whose observations were made on films from the same sample of fuchsin as here used. The curve shows the effect of an absorption region whose maximum lies at about $540\mu\mu$.

There is general agreement between the calculated and observed values of the reflection coefficients though not quite so satisfactory as with the metals. Owing to the small intensity of the reflected light, particularly in the violet, there was great difficulty in making observations and the agreement in that part of the spectrum is not very close.

Conclusions.—Observations have been made on a number of different substances, with both polished and unpolished surfaces, the magnitude of whose reflection coefficients varied between wide limits (.12 to .96). The reflection coefficients, photometrically measured, agree in all cases within the experimental errors with those theoretically determined from polarimetric measurements on the same surface, thus showing the sufficiency of the present theory to account for these relations. The greatest deviations usually occurred in the violet where the small intensity made accurate observations difficult, and on the steepest slopes of the curves.

Owing to the fact that the observed values vary more widely from the smooth curve than do the calculated values and since a consideration of the experimental errors involved in the measurements indicates that this is to be expected, the reflection coefficients calculated from polarimetric observations are distinctly more accurate than those observed by any photometric method.

In conclusion the writer wishes to express his appreciation for the many helpful suggestions and aid given him throughout the course of this investigation by Professors Skinner and Tuckerman.

THE BRACE LABORATORY OF PHYSICS,
UNIVERSITY OF NEBRASKA,
February, 1912.

RADIANT EFFICIENCY OF INCANDESCENT FILAMENTS.

BY W. E. FORSYTHE.

OF all the ways in which "efficiency" might be defined, there are two ways which suggest themselves first, the one being purely physical and the other physiological. The first is what has been called "Radiant Efficiency" and is defined as the ratio L/R , where L is the visible radiant energy and R the total radiant energy. The second is the one used by the engineer, that of candle power per watt, which means how much "light" in some arbitrary unit (candle-power) is obtained for each watt of power consumed. This efficiency, to be sure, will depend to a certain extent on the method used to measure the candle power. There has been a great deal of recent discussion concerning the "efficiency" of some of the new metallic filament incandescent lamps, some attributing the increased efficiency to increase in the operating temperature, while others say that it is due to selective radiation in the visible region. The object of the present paper is a determination of the efficiencies defined above as a function of the temperature.

Radiant efficiency as here defined, is a definite physical quantity, but in the various determinations of it, many errors have been introduced due to the methods used to measure either L or R . In the first attempts to measure radiant efficiency of incandescent lamps, the total energy was assumed equal to the wattage input and the energy in the infra-red region measured by putting the lamp in a calorimeter with transparent sides and measuring the rise in temperature of the water in the calorimeter. As this assumes that the energy in the visible region passes out of the calorimeter, a correction must be made for the amount of light absorbed by the water. Merritt¹ with this method obtained from 3.6 per cent. to 7 per cent. as the radiant efficiency of the carbon lamp. However, it is known that the water cell does not absorb all the infra-red radiation, so that his values would on this account be too high. Later Russner² attempted to measure the radiant efficiency of incandescent lamps by this method, using in place of a water cell a solution of a double sulphate of iron and ammonia which he says absorbs all the infra-red radiation and transmits all the visible radiation. He obtained as the

¹ American Journal of Science, Vol. 37, p. 167, 1889.

² Phys. Zeitschr., 8, p. 120, 1907.

radiant efficiency 0.6 per cent. for carbon and 2.3 per cent. for tantalum, which are much lower than other values. There seems to be some question¹ as to whether his lamps were at normal brilliancy. However, in no case does this method give the radiant efficiency above as defined, as the total energy measured would not be equal to R , the energy radiated from the filament, but would contain also the energy lost by conduction, etc.

Radiant efficiency can be measured by determining an energy curve of the filament studied and taking the ratio of the area under the curve in the visible region to the total area under the curve. This method would require a great number of readings to obtain the curve and also a troublesome correction, due to the absorption bands in the infra-red. Again, radiant efficiency may be determined by measuring the total energy radiated by one of several methods and then measuring the energy in the visible region from corrected observations of the amount of energy transmitted by a water cell. Houston² made use of this method, measuring the visible portion of the energy transmitted by the water cell by a method which will be described later.

Another method is that due to Ångström³ which involves dispersing the total radiation, screening off the infra-red portion, then recombining the visible portion and photometrically balancing it directly against the direct radiation from a similar source. If now the photometer screen be replaced by a thermopile, the energy in the two sources can be compared. A modification of this method has been used⁴ to measure the radiant efficiency of the carbon filament as a function of the wattage input. As the temperature was not measured, a comparison cannot be made with the results obtained in this paper, except for normal brilliancy, for which point the value of the radiant efficiency as obtained was 2.6 per cent.

Houston (loc. cit.) measured the radiant efficiency of the tungsten, tantalum and carbon, by measuring the total radiation (apparently through the walls of the incandescent bulb) with and without a water filter, thus determining the percentage of the total radiation transmitted by the filter. Then with a spectrometer having a glass prism and a Rubens linear thermopile in place of cross-hairs, he determined what fraction of the energy, which was transmitted by the water cell, was in the visible spectrum. He assumed that the visible spectrum ends at 0.76μ . By measuring the fraction of the light transmitted by the water cell, he was able to compute L/R , the radiant efficiency. He

¹ W. Voëge, *Phys. Zeitschr.*, 8, p. 306, May, 1907.

² P. R. S. of Edinburgh, Vol. XXX., Part 7, 41.

³ *Astrophys. Jour.*, XV., 223, 1902.

⁴ Mendenhall, *Phys. Rev.*, XVII., 1903.

obtained as the radiant efficiency of carbon 2.9 per cent., of tantalum 6.5 per cent. and of tungsten 7.5 per cent. This method seems to be liable to two errors, both of which would give too high values for the efficiency. In the first place, the glass walls of the lamp bulb would surely cut down the total radiation, thus making R too small, and the efficiency too high. Again, according to Coblentz,¹ the glass prism and lenses would not transmit visible and infra-red radiation in the same proportion, but would transmit less in the infra-red region which would result in making the observed efficiency too high. Thus we should expect his results to be too high, the error increasing with the temperature of the source.

Nyswander² attempted to determine the radiant efficiency of tungsten from measurements of the areas of a normal energy curve and obtained from 1.5 per cent. to 2 per cent. for this efficiency. He stated that his results were too low due to several errors, but made no attempt to correct for these errors.

As was pointed out above, radiant efficiency is a purely physical efficiency and has but little relation to the other efficiency of "candle power per watt." The question is often asked whether it is worth while to measure radiant efficiency as here defined. This is a purely physical quantity which shows the relative distribution of energy in what is spoken of as the visible and the infra-red spectrum, and if for no other reason, would be of interest. The location of the dividing line between the visible and the infra-red radiation is somewhat artificial but it has been generally agreed to take this at wave-length 0.76μ . It might well be that this could be set at a smaller wave-length and still not very much affect the amount of "light" sent out, still the visible spectrum certainly extends to the A line, if not beyond, and when we speak of the energy in the visible spectrum it should extend as far as the average eye is capable of being affected. Then too, for comparison with other work, the limits should be chosen at a definite position. To say that this should be taken at a smaller wave-length, as was done by Gage,³ because such a change does not alter the amount of "light" radiated by an appreciable amount as compared to the amount that the energy is altered, seems to rob this efficiency of its meaning. As there is no reason why merely changing the position of the screen with a straight edge should give values that would correspond to the physiological definition of efficiency, we might just as well, if that be

¹ *Infra Red Spectrum*, Carnegie Publication, Vol. 3-4.

² *PHYS. REV.*, Vol. 28, p. 438, 1907.

³ *PHYS. REV.*, Vol. 33, 2, p. 111, 1911.

the object in view, adopt some such method as that suggested by Fery,¹ which was to use a screen, not with a straight edge, but having a curved edge and the curve being so chosen as to cut down the radiation at each wave-length in inverse proportion to the effect of the radiation at this wave-length on the average eye. This screen would have zero height between 0.5μ and 0.6μ depending upon the intensity of the source and would cut off all the infra-red and ultra-violet radiation, its shape being determined by the visibility curves as found experimentally. Fery did not use this screen but used instead a solution of copper acetate which he found to have a transmission curve similar in form to the visibility curve of the eye. The relative candle powers of different sources as determined by Fery with this method, agree well with values as determined by photometry. A lower limit should also be chosen, but the amount of energy in the ultra-violet region is so small that it can be safely neglected.

To obtain the radiant efficiencies given in this paper, a modification of Ångström's method was used. To avoid the difficulty of maintaining two sources exactly alike the set up was so arranged, by using a bent-arm spectroscope, that the light after passing through the spectroscope could be balanced photometrically against the original source. The photometer screen was then replaced by a Rubens thermopile, connected with a Broca galvanometer, so adjusted as to have a sensibility of 150 scale divisions on a scale one meter away for a candle at one meter. As the total radiation contained on the average about twenty times as much energy as that in the visible, the amount of energy falling on the thermopile on the "total" side was cut down by placing between the source and the thermopile a rotating sector, the size of the openings in the sector being chosen such that the deflection of the galvanometer was about the same as that obtained with the visible radiation.

There are several sources of errors, some of which can be corrected for, while others must be guarded against. The slit in the spectroscope must not be too wide or there will be an overlapping of the infra-red radiation in the visible, thus making the radiant efficiency too high. It was assumed that with the width of the slit which permitted one to observe the principal Fraunhofer lines in the solar spectrum, this error would be negligible. The slit width as thus determined was about 0.5 mm. The greatest source of error would be due to the screen used to cut off the infra-red radiation not being exactly in position. This screen was set before each run by reflecting sunlight through the spectroscope. It is thought that most of the variations in the final results are due to

¹ Jour. d. Phys., Vol. 7, p. 632, 1908.

this error and to an error in bringing the filament to the same temperature each time. Precautions were also taken to prevent radiation from the walls of the lamp or other lights in the room from falling on the thermopile. In addition a correction¹ of 1.7 per cent. of the efficiency must be added due to the selective reflection of the silver mirror used in the bent-arm spectroscope, and also there is a correction of 7.8 per cent.² to be added due to the fact that the fluorite window used on the lamp to transmit the total radiation was not equally transparent for visible and total radiation. Thus we have a total correction of 9.5 per cent. of the efficiency to be added.

The filaments whose radiant efficiencies were to be measured, were mounted in a special water-cooled brass case with two windows at right angles to each other for making the observations. To obtain the vacuum a Boltwood mercury pump was used, which was allowed to run for several hours. To remove the last traces of the gases another filament, mounted in the base of the can, was burned for a half an hour before the other was started. With this arrangement, if the current through the filament was kept constant, both the temperature of the filament as observed with the optical pyrometer and the voltage across the filament would remain constant. This was taken to indicate good working conditions. In making the observations, the light in the visible and the total spectrum was first balanced photometrically, the photometer being then replaced by the thermopile, and the energy measured in both the visible and the total radiation as pointed out above. This was repeated for different temperatures. From these readings, applying the corrections mentioned above, the radiant efficiency could be calculated.

The temperatures of the filaments were measured with a Holbourn-Kurlbaum optical pyrometer sighted through the fluorite window, the pyrometer lamp being set for the desired temperature and the current through the filament varied until the two appeared to be at the same temperature. This, to be sure, gave black body temperatures, which for the tantalum, tungsten and carbon were corrected to actual temperatures by the wedge method described by Professor Mendenhall.³ As the temperatures of the filaments were measured through the fluorite window, a correction was necessary for the light that was absorbed and reflected by this window. This correction was easily determined by measuring the temperature of some steady source with the pyrometer, and then the apparent temperature of the same source when sighted through the

¹ Ingersoll, *Phys. Rev.*, Vol. 17, 1903.

² Mendenhall, *loc. cit.*

³ *Astrophys. Jour.*, Vol. 33, 2, March, 1911.

fluorite window. The Nernst glower was used for the steady source, the correction amounting to about 12° C. at 1500° C. and about 25° C. at 2000° C. In Table I. are given the results for the efficiencies together with the black body and true temperatures.

TABLE I.
RADIANT EFFICIENCY.

Temperature.								Mean.	Corr. Mean.
Black Body. $\lambda=0.653\mu$	True.								
<i>Tungsten.</i>									
1558	1646	2.9	2.86	2.06	2.55		2.72	2.64	2.89
1698	1805	3.82	3.6	3.29	3.85		3.45	3.60	3.94
1852	1968	3.9	4.8	3.87	4.2	4.22	4.2	4.19	4.59
2000	2135	5.07	5.08	5.23	5.4	5.5	5.25	5.26	5.76
2164	2314	6.35	5.8				6.77	6.31	6.91
<i>Tantalum.</i>									
1558	1662	1.62	1.54	1.38	1.56	1.56		1.53	1.68
1698	1828	2.80	2.27	2.67	3.06	2.92		2.74	3.00
1852	2012				4.85	4.67		4.76	5.22
<i>Gem.</i>									
1558		1.8	1.8			1.5		1.7	1.86
1698		2.1	2.2	2.0	2.1	2.1		2.1	2.3
1852		2.7	2.6	3.1	3.0	2.7		2.8	3.1
2000					4.3	4.5		4.4	4.8
<i>Carbon.</i>									
1515	1550	1.3	1.0	1.1	1.0			1.1	1.2
1720	1762	2.0	1.7	1.6	2.1			1.8	2.0
1815	1860	2.2	1.9	3.1	2.7			2.5	2.7
1945	1995	3.3	2.8	3.1	4.0			3.3	3.6
2025	2082			4.0	4.6			4.3	4.7
2122	2185			5.3	5.9			5.6	6.2

The tungsten filaments, obtained from the General Electric Co., were from a series lamp intended to take about 7.5 amperes for normal brilliancy. The diameter of the filaments was about 0.32 mm. The tantalum filaments were strips 1 mm. wide and about 0.3 mm. thick, rolled from sheets of tantalum obtained from Siemens and Halske. The carbon filaments were taken from a 190-watt Peerless lamp and were about 0.3 mm. in diameter. The metalized filaments furnished by the General Electric Co. were about a square millimeter in cross section. All the filaments were about 2.5 cm. in length.

Temperature of the Filaments in the Lamp Bulbs.—There have been many attempts to determine the temperature of the different incandescent filaments at normal brilliancy, there being quite a wide variation in the different results. For the most part, these determinations are based upon empirical laws which attempt to state the relation between the temperature and either the resulting candle power, resistance of the filament, or the voltage drop or current through the filament. In Table II. are given the different results together with the method used. To

measure directly the temperature of a small filament either with a thermocouple or an optical pyrometer is very apt to lead to wrong results. In either case the results would be apt to be too low, due in the first case to the temperature being lowered by the conduction of the thermocouple and in the second case to the refraction of the lamp bulb and also to the very small surface that one must compare with the pyrometer lamp filament.

The black-body temperature of the different filaments given in this paper were measured in a manner similar to that used by Waidner and Burgess,¹ excepting that the lamp filaments being tested were balanced photometrically against a Nernst glower which permitted the temperature measurements to be carried much higher than with the carbon strip. In measuring the temperatures the image of the pyrometer lamp filament and the image of the lamp filament being tested were both projected on the same spot of the glower and the black-body temperature of the glower taken when the black-body temperature of the glower was the same as that of the lamp. To obtain monochromatic radiation the same kind of red glass was used before both eye pieces. This method requires a correction for the lens used to project the image of the Nernst glower on the lamp filament as well as for the front wall of the lamp bulb. These corrections were determined in the same manner as the corrections for the fluorite window. The black-body temperatures thus obtained were reduced to true temperatures by the method mentioned above. In Table III. are given results for the temperature as obtained from the different lamps when operated at the voltage or watts marked as normal.

When measuring temperatures above 1500°C. , the limit so far attained with the gas thermometer, one must either use a thermocouple or radiation pyrometer. Either of these must be calibrated in terms of some fixed point, the melting point of palladium or platinum being usually chosen. Now as there is a disagreement as to the exact value of these melting points, one should always mention the value taken. Again, as the black-

¹ Bulletin of the Bureau of Standards, Vol. 2.

TABLE II.

Author.	Year.	Filament.	Temperature (Centigrade).	Watts per Candle Power.	Method.	Reference.
Weber.	1892	Carbon	1565 to 1580	3.5	Radiation law.	Ver. d. D. Phys. Ges., Vol. 1, 1899.
Lummer & Pringsheim ..	1899	Carbon	1620 to 1827		Bolometer. ¹	Comptes Rendus, V., 126, p. 734, 1898.
Janet P.	1898	Carbon	1610 to 1720		Relation between power consumed and temperature.	Bulletin Bureau Standards, V. 2.
Waidner & Burgess ²	1906	Carbon Tantalum	1860	3.1	Photometric balance against carbon strip.	
		Tungsten	1865	2.0		
			2135	1.0		
Grau ³	1907	Carbon	1660	3.5	Photometric balance against iridium	Elektrotechnik und Masche- nebau, April, '07.
		Tungsten	1850		strip.	
Morris, Stroud & Ellis ⁴ ..	1907	Tungsten	1727	1.14	Stefan's law and matching carbon at 3.5 w./c.p.	The Electrician, Vol. 59, Aug., 1907.
		Tantalum	1577			
Le Chatelier & Bourdouard		Carbon	1800		Absorbing pyrometer.	High Temp. Meas., p. 171.
Coblentz, W. W.	1909	Carbon met. Tantalum	1570		Energy curve and platinum con- stant.	Bulletin Bureau of Standards, Vol. 5, p. 339.
		Tungsten	1670			
			1810			
Jolley ⁵	1909	Carbon	1575 to 1657	3.5	Empirical relations.	The Electrician, Vol. 63, 1909.
		Gem	1632 to 1717	3.1		
		Tantalum	1427 to 1737	1.66		
		Tungsten	1517 to 1807	1.25		
Fery and Cheneveau ⁶	1909	Carbon	1780	3.4	Absorbing pyrometer.	Bull. Soc. Int. Electricians, '09.
Pirani ⁷	1910	Tungsten	1875	1.21	Thermo-couple between twisted fila- ments and extrapolating relation between temperature and power.	Ber. d. D. Phys. Ges., No. 7, p. 301, 1910.
		Tantalum	1975	1.43		
		Tungsten	2060	1.25		
This work ⁸		Carbon	1820	3.04	Photometric balance against	
		Carbon	1845	2.72	Nernst glower for black body	
		Gem	1865	2.32	temperature and true tempera- ture from wedge.	
		Tantalum	1947	1.52		
		Tungsten	2032	1.12		

TABLE III.

No.	Lamp.	Candle Power per Watt.	Temperature.		Style of Lamp.
			Black Body. $\lambda=0.658\mu$	True.	
1	Tungsten	0.811	1863	1980	Old style 25 watt.
1	Tungsten	0.821	1865	1982	Wire filament 25 watt.
1	Tungsten	0.850	1887	2008	Old style 40 watt.
1	Tungsten	0.890	1895	2020	Wire filament 40 watt.
3	Tungsten	0.914	1900	2025	Wire filament 60 watt.
3	Tungsten	0.895	1915	2035	Old style 100 watt.
1	Tungsten	0.944	1920	2040	Wire filament 100 watt.
2	Tantalum	0.513	1727	1862	25 watt.
2	Tantalum	0.561	1750	1890	40 watt.
2	Tantalum	0.655	1795	1945	100 watt.
2	Carbon	0.329	1775	1820	60 watt.
3	Carbon	0.369	1802	1847	120 watt.
3	Carbon	0.368	1798	1843	190 watt.
2	Gem	0.430	1865		50 watt.

body temperature of any substance depends upon the wave-length used the wave-length should be specified along with the black-body temperature. In very few of the values given in Table II. was there any mention of either the temperature scale or the wave-length used. For this work the pyrometer was calibrated by the sector method,⁹ taking the melting point of palladium as 1549° C. and that of platinum as 1755° C. c_2 of Wien's equation was taken as 14,500, while the wave-length used with the optical pyrometer was 0.658 μ .

¹ The wave-length for maximum energy obtained from energy curve and temperature obtained from Wien's law — $\lambda_m T = \text{constant}$. Assumed that the constant for carbon would lie between that for black body and for platinum.

² Black-body temperatures — $\lambda = 0.66\mu$. No mention is made as to whether they correct for the absorption of the lens and the walls of the lamp bulb.

³ Black-body temperatures. Does not give wave-length used. No mention is made of corrections for absorption of lens or walls of lamp bulb.

⁴ Assumed that the temperature of the carbon at 3.5 watts per candle power was 1727° C. Obtained the constant for Stefan's law by bringing other lamps to color match with the carbon.

⁵ Measured candle power and watts and used empirical laws connecting one or the other of these with the temperature. Obtained constants of equations by references to other work.

⁶ Black-body temperature for carbon, wave-length not given. Black-body temperature of tungsten measured and reduced to true temperature by assuming that tungsten behaved the same as platinum. This would give them a black-body temperature of the tungsten of about 1650° C. which is much lower than other values. As was pointed out above, one would expect a low value for this temperature as measured directly with the pyrometer.

⁷ These values have been reduced to a scale with the melting point of palladium at 1549° C.

⁸ Black-body temperature for the gem.

⁹ Mendenhall, *PHYS. REV.*, Vol. 33, 1, 1911.

The candle power per watt as a function of the temperature was measured for a hundred and ninety watt carbon, a hundred watt tungsten an eighty watt tantalum and a fifty watt gem lamp. These temperatures were first measured as described above, and then the candle power measured with a Lummer-Brodhun photometer, in terms of a standard sixteen candle power carbon lamp obtained from the Leeds and Northrup Co. No very great accuracy can be claimed for the candle power at the high temperatures, due to the very great difference in color between the standard and the lamps tested. These lamps did not appreciably deteriorate when used at this high voltage for the time necessary to make the measurements, as they gave the same candle power per watt when again tested at normal brilliancy. In Table IV. are given the results for this work.

TABLE IV.

Carbon Lamp.

Volts.	Current.	Temperature.		Candle Power per Watt.
		Black Body. $\lambda = 0.658\mu.$	True.	
70	1.13	1435	1465	0.057
80	1.29	1526	1560	0.118
90	1.44	1610	1646	0.176
100	1.61	1692	1743	0.278
110	1.76	1780	1824	0.360
120	1.92	1862	1910	0.477
130	2.10	1942	1992	0.662
140	2.29	2012	2067	0.837

Gem Lamp.

80	0.335	1578		0.138
90	0.368	1600		0.206
100	0.400	1742		0.296
110	0.430	1829		0.392
120	0.460	1925		0.508
130	0.491	1995		0.653
140	0.521	2080		0.806

Tantalum Lamp.

70	0.530	1522	1622	0.195
80	0.585	1600	1711	0.289
90	0.635	1672	1798	0.382
100	0.689	1740	1879	0.502
110	0.735	1805	1955	0.637
120	0.785	1868	2031	0.746
130	0.820	1929	2101	0.902
140	0.884	1988	2173	1.020

Tungsten Lamp.

70	0.705	1612	1708	0.303
80	0.765	1712	1818	0.425
90	0.826	1783	1895	0.563
100	0.880	1840	1955	0.720
110	0.930	1906	2025	0.885
120	0.978	1970	2096	1.100
130	1.025	2026	2160	1.210
140	1.070	2075	2215	1.390

In the curves in Fig. 1 are shown graphically the variation of the radiant efficiency as a function of the temperature and also the candle power per watt as a function of the temperature for the different lamps. For comparison the radiant efficiency of a black body computed from Wien's equation, has been plotted along with the other radiant efficiencies. It will be noticed that the curve showing the efficiency of the carbon runs

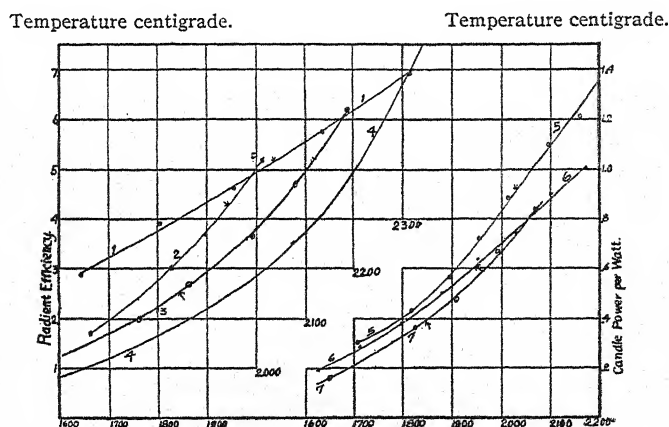


Fig. 1.

Radiant efficiency, (1) tungsten, (2) tantalum, (3) carbon, (4) black body. Candle power per watt, (5) tungsten, (6) tantalum, (7) carbon. (Arrows indicate normal brilliancy.)

almost parallel to that for the black body, being always above it, while the efficiency curve for the tungsten crosses that for the black body at about 2200° C. The tantalum curve at the lower temperatures lies between the tungsten and carbon but crosses the tungsten curve at about 2000° C. The curves showing the variation of the candle power per watt as a function of the temperature lie in a different manner, the tantalum curve in this case lying between the other two for the greater part of the range. A comparison of the two sets of curves would indicate that the distribution of energy in the visible region for the three filaments was entirely different, the tantalum being much richer in the long wave-

lengths as compared with the tungsten. Thus the increased efficiency of the tungsten lamp over that of the tantalum is seen to be due both to its increase in temperature of operation and also to selective radiation in the visible region. Owing to the different arrangements of the filaments in the carbon and the metal filament lamps it is unfair to the carbon lamp to compare horizontal candle power. The value of the radiant efficiency for carbon agrees well with the value found by Professor Mendenhall (*loc. cit.*), his value being 2.6 per cent.

TABLE V.

Lamp.	Normal Temperature.		Radiant Efficiency.	Candle Power per Watt.
	Black Body. $\lambda = 0.658\mu$.	True.		
Tungsten	1912	2032	5.2%	0.890
Tantalum	1795	1945	4.3	0.655
Gem	1865		3.2	0.430
Carbon	1800	1845	2.5	0.368

In Table V. is given a summary of the different results. It will be noticed that the value for the black-body temperature of carbon at normal brilliancy agrees well with the value obtained by Fery and Cheneveau and also that of Waidner and Burgess, if it be assumed that neither of them made corrections for the glass that was in the path. The temperature of tantalum is a little lower than that given by Pirani, the black-body temperature of tantalum agrees well with that of Waidner and Burgess, if we again assume that no correction for glass absorption was made. The value of the normal temperature of tungsten is very much lower than that obtained by Waidner and Burgess, and about 1 per cent. lower than the value obtained by Pirani.

SUMMARY.

The radiant efficiency, black-body temperature and true temperature at normal brilliancy have been determined for carbon, tantalum and tungsten, and the radiant efficiency and black-body temperature of the gem lamp at normal temperature have also been determined. In addition to this, the radiant efficiency and the candle power per watt as a function of the temperature, have been determined for the above lamps.

UNIVERSITY OF WISCONSIN,
DEPARTMENT OF PHYSICS,
February, 1912.

ONE-WAVENESS IN WIRELESS TELEGRAPHY; PSEUDO-IMPACT EXCITATION.¹

BY SHUNKICHI KIMURA.

I.

THE characteristics of impact excitation are the fulfillment of one-waveness, at the original wave-length of the secondary system and the absence of the swing of energy between the secondary and primary systems, so that oscillations continue alone in the secondary system, with its own low decrement. In pseudo-impact excitation, the latter characteristic is the result of the former, since at one-waveness there may not be any beat phenomena which may cause the swing of energy. The large value of the primary decrement is enough to account for rapid quenching of primary oscillations.²

Of different processes investigated for impact excitation and excitations taken as impact, we may enumerate the short quench sparks,³ quench-tubes,⁴ mercury vapor sparks,⁵ hydrogen sparks,^{6, 7} ohmic resistance,^{5, 6} and partial sparks,^{8, 9} all of which are devices to increase the decrement of the primary system. Interest in this impact excitation and its investigation seems to center very legitimately upon the improvement of coupling, hitherto attained in most cases with a limitation in spark potential in the primary system.

The fundamental condition of impact excitation, namely the fulfillment of one-waveness, is also attained, employed and appreciated¹⁰ in ordinary

¹ Excitation akin to the impact excitation.

² If δ_1 and ∂_1 are the damping factor and logarithmic decrement of the primary system, the time in which the initial amplitude falls to $1/e$, and the number of oscillations during the same time are, respectively,

$$\tau = \frac{1}{\delta_1}, \quad m = \frac{1}{\partial_1}.$$

In the history of wireless telegraphy, when we required a great number of m , m was called "Resonanzgrade," and when we require a short time for τ , τ is called "Abklingzeit."

³ Wien, Jahrb., I, 469, 1908.

⁴ Wien, Jahrb., IV., 135, 1910.

⁵ Glatzel, Jahrb., II., 65, 1909.

⁶ Espinosa de los Monteros, Jahrb., I., 480, 1908. Rau, Jahrb., IV., 52, 1910.

⁷ Glatzel, Ann. Phys., 34, 711, 1911.

⁸ Rohmann, Phys. Zt., 12, 649, 1911.

⁹ Galletti, The Electr., Jan. 20, 1911.

¹⁰ One-waveness is found to give the greatest range, though at a great reduction in coupling and efficiency.

excitation, with many kinds of dischargers,¹ though at lower primary decrement, and a looser coupling. The essentials of impact excitation are here almost realized.

Since the same name is sometimes given to different quantities, I write here the formula in full,²

$$K'^2 = K^2 - \left(\frac{\partial_1 - \partial_2}{2\pi} \right)^2,$$

where K' is the *degree of coupling*, ∂_1 , ∂_2 are the original logarithmic decrements per complete period of primary and secondary systems, and K is the *coefficient of coupling*, defined by

$$K^2 = \frac{L_{12}^2}{L_1 L_2}.$$

The condition of one-waveness may be written in many ways, namely³

$$K'^2 = 0; \text{ or } K^2 = \frac{1}{4\pi^2} (\partial_1 - \partial_2)^2; \text{ or } L_{12} = \frac{1}{2\pi} \sqrt{L_1 L_2} \cdot (\partial_1 - \partial_2);$$

or

$$\partial_1 = \partial_2 + 2\pi K.$$

Here it is assumed from practical cases that the decrement in the primary with a spark gap is larger than the decrement in the secondary which consists of ohmic and radiation decrements. Of the above condition, the first comes from the theory of coupled systems; the second may be reached from practical trials.⁴ Of the third form, we may say that the measurement of wave-length with a wavemeter employs this or less amount of mutual induction at perfect synchronism, and one wave is produced in the instrument, the wave-length of which is read and made the wave-length of exciting circuit. The fourth form is known from the experimental fact that the primary decrement must be much larger than the secondary decrement to effect a good impact excitation; the difference

¹ Fessenden's synchronous gap in U. S. N. Experiment; Austin, Bull. Bur. Stand., 7, No. 3, 1911. Marconi's rotating gap, The Electrician, July 14, 1911. Gaps in series, gaps with air blast have the same function.

² The expression of the degree of coupling and its connection with the two coupling waves seem to appear originally in Drude's paper, Ann. Phys., 13, 1904, p. 544.

³ Pseudo-impact excitation occurs at the value of K'^2 very little less than zero. The value of K here expressed will be called the *critical coupling* and will be understood as the limiting value above which the pseudo-impact excitation is impossible.

⁴ For instance, Wien, Jahrb., I., 471. The theory neglects the presence of a spark in the primary system, and hence we are to expect some correction in practice, and this correction is very little. For example, in Wien's measurement (Wien, Ann. Phys., 29, 1909, p. 711, Tables 29, 30) values of K'^2 for $K = 0.013$, were -0.0003 and $+0.000098$, respectively, and in these cases maximum effects in the third circuit or resonator were observed.

between the two decrements being here $2\pi K$, depending on the amount of coupling. The theory of this one-waveness is very simple; we revert to the case of two systems at an extremely loose coupling,¹ and take the condition of isochronism² in theory, expecting a certain correction in case of resonance in practice.³

The resulting oscillation in the secondary system is the difference between two oscillations with the same frequency but different decrements and gives rise to an amplitude curve tapering at the beginning. The maximum current amplitude attainable in the secondary system depends on the fundamental quantities of the primary and secondary systems and is represented in practical units by

$$I_{\max}(\text{amp.}) = \frac{1}{2} \cdot \frac{\partial_1 - \partial_2}{\partial_2} \left(\frac{\partial_2}{\partial_1} \right)^{\frac{\partial_1}{\partial_1 - \partial_2}} \sqrt{\frac{C_1(\text{mf.})}{L_2(\text{mh.})}} \cdot V_f(\text{volt}),^4$$

where C_1 , V_f are the capacity in microfarads and spark potential in volts in the primary system, and L_2 is the self-inductance in microhenries in the secondary system. Taking this formula for the pseudo-impact excitation and provisionally also for the impact excitation, let us try some calculations with some practical data.⁵

Example I.—One-waveness by large spark ordinary excitation. Given

$$\partial_2 = 0.004; \quad C_1 = 0.01 \text{ mf.}; \quad V_f = 6 \cdot 10^4 \text{ volts}; \quad L_2 = 100 \text{ mh.}$$

Pseudo-impact excitation is attained at $K = 4.5$ per cent.⁶ for instance. Then we obtain

¹ Bjerknes, Wied. Ann., 44, 1891, 74; 55, 1895, 121; Zenneck, E. M. S., 584, 1905, L. D. T., 79, 1909. It is interesting to note that the theoretical and practical investigations were directed for some time to the reduction of primary decrement and to the production of maximum potential and current in the secondary system with two coupling-waves at close coupling, and now we have changed the course.

² In impact excitation, this condition must also be observed in order to obtain the most powerful oscillations in the secondary system.

³ In practical trials with ordinary and impact excitations at the condition of one-waveness with maximum effect in the secondary or aerial system, isochronism and resonance (Zenneck, E. M. S., 556) often, though not always, become identical, within the reading-error of a wave meter. Perfect adjustment to one-waveness at maximum effect is not a very easy process, nor of a very definite delineation. Frequency and wave-length are to take account of the correction due to the larger values of decrements; the wave-meter indicates the corrected quantities.

⁴ In case of $\partial_1 = \partial_2$ at isochronism, it requires a special treatment in theory, leading to quite different formulæ; as to the production of one-waveness, it is also a door case. This case is excluded in this paper.

⁵ The theory of impact excitation will perhaps require the primary decrement ∂_1 to be treated as a function of time, practically to reach an infinitely large value at the end of first half beat time, and not as a constant as in case of ordinary excitation.

⁶ Glatzel, Ann. Phys., 34, 711, 1911; Fleming and Dyke, The Electr., March 3, 1911; Wien, Jahrb., 4, 135, 1911.

$$\partial_1 = 0.2867;^1 \quad I_{\max} = 273 \text{ amp.}$$

Example II.—One-waveness by short spark impact excitation. Given

$$\partial_2 = 0.004; \quad C_1 = 0.01 \text{ mf.}; \quad V_f = 3 \cdot 10^4 \text{ volts}; \quad L_2 = 100 \text{ mh.}$$

Impact excitation is attained at $K = 13$ per cent.² for instance. Then we obtain

$$\partial_1 = 0.821; \quad I_{\max} = 146 \text{ amp.}$$

Since the above formula, $\partial_1 - \partial_2$ is nearly equal to ∂_1 , the factor consisting of decrements is almost unity, more so as ∂_1 is larger than ∂_2 , and with no respect to coupling; therefore, under given constants of primary and secondary systems, the maximum current amplitude is almost proportional to the spark potential in the primary system. If ∂_2 is made larger, the same factor may differ more from unity. Now, with the condition of one-waveness, we may write,

$$D = \frac{\partial_1 - \partial_2}{\partial_2} \cdot \left(\frac{\partial_2}{\partial_1} \right)^{\frac{\partial_1}{\partial_1 - \partial_2}} = \frac{x^x}{(1+x)^{1+x}},^3$$

where

$$x = \frac{\partial_2}{2\pi K} = \frac{\partial_2}{\partial_1 - \partial_2}.$$

This factor is larger as x is smaller, or as ∂_2 is smaller at constant ∂_1 , or as ∂_1 is larger at constant ∂_2 . The value of ∂_2 can not be smaller than the radiation decrement; hence ∂_1 must be made larger and then the coupling is improved.⁴

Considering the other factor of I_{\max} , namely $\sqrt{C_1} \cdot V_f$, pseudo-impact excitation with larger coupling and smaller energy per spark and the same

¹ Measured with Marconi's decimeter, series gaps with 2 mm. spark each, gave $\partial_1 = 0.41$ almost constant, single-blasted gap gave $\partial_1 = 0.30$, larger for shorter spark, smaller for longer. Coupling must be adjusted every time for one-waveness when sparks or dischargers are changed.

² Glatzel, Ann. Phys., 34, 711, 1911; Fleming and Dyke, The Electr., March 3, 1911.

³ A table of this quantity for values of x which occur in practice is given below.

⁴ In similar manner, the time at which the secondary current reaches the maximum value I_{\max} may be expressed in terms of x , namely,

$$t = \frac{2}{n} \cdot \frac{\log \partial_1 - \log \partial_2}{\partial_1 - \partial_2} = \frac{2}{n\partial_2} x [\log (1+x) - \log x] \\ = \frac{2}{n\partial_2} \log \frac{1}{(1+x)^D}.$$

Hence this time is a function not only of ∂_1 , ∂_2 , or K and ∂_2 , but it changes with the working wave-length. For example:

$$\begin{array}{llllll} 1. & \partial_1 = 0.4; & \partial_2 = 0.08; & n = 10^8; & \lambda = 600m; & t = 10 \cdot 10^{-6}; \\ 2. & \partial_1 = 0.8; & \partial_2 = 0.08; & n = 10^8; & \lambda = 600m; & t = 6 \cdot 10^{-6}. \end{array}$$

In the first case, the maximum amplitude occurs after 5 complete oscillations; in the second after 3.

excitation with smaller coupling and larger energy per spark are something like a balance between two above factors, and it can not be decided at once which factor is more important, best result being expected from the satisfaction of two factors, namely the closer coupling and larger energy per spark in the primary system.¹

Example III.—Fleming and Dykes' Experiment.²

	Given Secondary Decrement.	One-wave- ness Ob- served. Per Cent.	Calculated Primary Decrement.	Number of Primary Oscilla- tions till Amplitude Falls to 1/e.	$x = \partial_2/2\pi K.$	D	$I_{\max}/\sqrt{\partial_2}$ Ratios.
Ordinary . .	$\partial_2 = 0.02$	$K = 4.5$	$\partial_1 = 0.3027$	3.3	0.07074	0.7706	1
Excitation.	$\partial_2 = 0.18$	$K = 6.7$	$\partial_1 = 0.6049$	1.6	0.4235	0.4204	0.2
Impact . . .	$\partial_2 = 0.02$	$K = 11$	$\partial_1 = 0.7111$	1.4	0.0289	0.8787	1
Excitation.	$\partial_2 = 0.18$	$K = 13$	$\partial_1 = 0.9968$	1.0	0.2203	0.5620	0.2

In the next to last column, the factor of I_{\max} depending only on the decrements is calculated. In the last column, these numbers are taken proportional to I_{\max} . Anticipating the indication of a heat-working current measurement given below, currents in the secondary system are calculated in ratios, each for two excitations, and these ratios are about the same as those given in Fig. 4 in the authors' paper.

II.

The indication of a heat-working instrument in the secondary system takes a different form from that in case of damped trains of oscillations with one decrement and maximum initial amplitude. In the calculation I take the assumptions which are plausible in practical cases, namely,

$$(\partial_1 + \partial_2)^2 \ll 4\pi^2, \quad \partial_2 \ll \partial_1.$$

The initial value of current amplitude which is equal for the two synchronous oscillations in the secondary system, but which is not a measurable quantity, is

$$\begin{aligned} I_0 &= \pi \frac{L_{12}}{L_2(\partial_1 - \partial_2)} I_{10} = \frac{1}{2} \sqrt{\frac{L_1}{L_2}} \cdot I_{10} = \frac{1}{2} \sqrt{\frac{C_1}{L_2}} \cdot V_f; \\ &= \frac{\partial_2}{\partial_1 - \partial_2} \left(\frac{\partial_1}{\partial_2} \right)^{\frac{\partial_1}{\partial_1 - \partial_2}} \cdot I_{\max}; \end{aligned}$$

¹ So far as theory goes, the advantage resulting from the coupling reaches a limit, when $x = 0$; $D = 1$, max. and limiting value. In practice, values 0.7 to 0.8 are already attained in case of impact excitation.

² Fleming and Dyke, *The Electrician*, March 3, 1911. As I take the data from the abstract, calculation is only half way and regret not having access to their original paper.

where I_{10} , V_f are the initial and maximum current amplitude and the spark potential in the primary system and I_{\max} is the maximum current amplitude attainable in the secondary system. The current in the secondary system takes the form

$$I = I_0(e^{-\delta_1 t} - e^{-\delta_2 t}) \sin \pi n t,$$

where δ_1 , δ_2 are the original factors of damping in the two systems, and n is twice the isochronous frequency. Therefore the heat produced in the instrument with a low resistance R in one second of time and at ζ sparks per second is represented by

$$\begin{aligned} Q &= \zeta R \int_0^\infty I^2 dt \\ &= \frac{\zeta R I_0^2}{2n} \left[\frac{1}{\delta_1 \left[1 + \left(\frac{\partial_1}{2\pi} \right)^2 \right]} + \frac{1}{\delta_2 \left[1 + \left(\frac{\partial_2}{2\pi} \right)^2 \right]} \right. \\ &\quad \left. - \frac{2}{(\delta_1 + \delta_2) \left[1 + \left(\frac{\partial_1 + \partial_2}{2\pi} \right)^2 \right]} \right]. \end{aligned}$$

Whence we obtain by the above assumptions

$$\begin{aligned} Q &= \frac{\zeta R}{8n} \cdot \frac{1}{\delta_2} \cdot \frac{L_1}{L_2} \cdot I_{10}^2 \\ &= \frac{\zeta R}{8n} \cdot \frac{1}{\delta_2} \cdot \frac{C_1}{L_2} V_f^2 = \frac{\pi^2}{4} \zeta R n \frac{C_2}{\delta_2} \cdot W \\ &= \frac{\zeta R}{2n} \cdot \frac{1}{\delta_2} \cdot I_{\max}^2. \end{aligned}$$

Reducing this to the indication of a hot wire ammeter, we have

$$\begin{aligned} I &= \frac{\pi}{2} \sqrt{\zeta n} \sqrt{\frac{C_2}{\delta_2}} w \\ &= \sqrt{\frac{\zeta}{2n\delta_2}} \cdot I_{\max}. \end{aligned}$$

This is the usual expression for one group of damped trains, except that I_{\max} takes the place of the initial amplitude.

III.

In order to study the results of coupling when K'^2 is positive, negative or zero, I start from the biquadratic Z -equation of Drude,¹ which is

$$Z^4 + Z^2[\tau_1^2 + \tau_2^2 - \frac{1}{2}(\vartheta_1 - \vartheta_2)^2] - Z(\tau_1^2 - \tau_2^2)(\vartheta_1 - \vartheta_2)$$

¹ Drude, Ann. Phys., 13, 1904, p. 534.

$$+ \left[\left(\frac{\vartheta_1 - \vartheta_2}{2} \right)^2 + \tau_1^2 \right] \left[\left(\frac{\vartheta_1 - \vartheta_2}{2} \right)^2 + \tau_2^2 \right] = K^2 (\tau_1^2 + \vartheta_1^2) (\tau_2^2 + \vartheta_2^2),$$

with

$$y = -\vartheta + i\tau = -\frac{\vartheta_1 + \vartheta_2}{2} + Z.$$

Reducing to the notation previously used and at isochronism of the two systems and with the valid assumption, ϑ_1^2 and $\vartheta_2^2 \ll 4\pi^2$,

$$\tau_1 = \tau_2 = \frac{1}{\pi n}; \quad \vartheta_1 = \frac{\partial_1}{2\pi^2 n}; \quad \vartheta_2 = \frac{\partial_2}{2\pi^2 n};$$

$$\tau_1^2 + \vartheta_1^2 = \tau_2^2 + \vartheta_2^2 = \frac{1}{\pi^2 n^2}.$$

The above equations then take the form

$$\left\{ Z^2 + \frac{1}{\pi^2 n^2} \left[1 - \frac{1}{4} \left(\frac{\partial_1 - \partial_2}{2\pi} \right)^2 \right] \right\}^2 = \frac{1}{\pi^4 n^4} \left\{ K^2 - \left(\frac{\partial_1 - \partial_2}{2\pi} \right)^2 \right\} = \frac{1}{\pi^4 n^4} K'^2,$$

with

$$y = -\frac{\partial_1 + \partial_2}{4\pi^2 n} + z = -\frac{\Delta}{2\pi^2 N} + i\frac{1}{\pi N},$$

where Δ 's and N 's are the decrements and double frequencies after the coupling, to be found from the solution of above equation.

1. K'^2 positive, with any magnitude. In this case we have

$$Z = \pm i\frac{1}{\pi n} \sqrt{1 - \frac{1}{4} \left(\frac{\partial_1 - \partial_2}{2\pi} \right)^2} = K',$$

and the required quantities are

$$N = \frac{n}{\sqrt{1 - \frac{1}{4} \left(\frac{\partial_1 - \partial_2}{2\pi} \right)^2}} = \frac{n}{\sqrt{1 \pm K'^2}},$$

$$\Delta = \frac{N}{n} \cdot \frac{\partial_1 + \partial_2}{2} = \frac{1}{\sqrt{1 \pm K'^2}} \cdot \frac{1}{2} (\partial_1 + \partial_2),$$

the two coupling waves, each with different decrement.

2. K'^2 zero. Here we have

$$Z = \pm i\frac{1}{\pi n} \sqrt{1 - \frac{1}{4} \left(\frac{\partial_1 - \partial_2}{2\pi} \right)^2}$$

and

$$N = \frac{n}{\sqrt{1 - \frac{1}{4} \left(\frac{\partial_1 - \partial_2}{2\pi} \right)^2}} = n, \quad \Delta = \frac{N}{n} \cdot \frac{\partial_1 + \partial_2}{2} = \frac{1}{2} (\partial_1 + \partial_2).$$

Here only one wave is produced with nearly the original isochronous frequency, and the decrement is also unique but its value is nearly equal to the arithmetical mean of those of the two systems before coupling.

3. K'^2 negative, small in absolute value. In this case we use the expression of K^2 and we have

$$Z^2 = -\frac{1}{\pi^2 n^2} \left[1 - \frac{1}{4} \left(\frac{\partial_1 - \partial_2}{2\pi} \right)^2 \pm i \sqrt{\left(\frac{\partial_1 - \partial_2}{2\pi} \right)^2 - K^2} \right];$$

whence, for small absolute value of K'^2 ,

$$Z = \pm \frac{1}{2\pi n} \sqrt{\frac{\left(\frac{\partial_1 - \partial_2}{2\pi} \right)^2 - K^2}{1 - \frac{1}{4} \left(\frac{\partial_1 - \partial_2}{2\pi} \right)^2}} \pm \frac{i}{\pi n} \sqrt{1 - \frac{1}{4} \left(\frac{\partial_1 - \partial_2}{2\pi} \right)^2}.$$

Therefore the required quantities are

$$\begin{aligned} N &= \frac{n}{\sqrt{1 - \frac{1}{4} \left(\frac{\partial_1 - \partial_2}{2\pi} \right)^2}} = n, \\ \Delta &= \frac{N}{n} \left\{ \frac{\partial_1 + \partial_2}{2} \pm \pi \sqrt{\frac{\left(\frac{\partial_1 - \partial_2}{2\pi} \right)^2 - K^2}{1 - \frac{1}{4} \left(\frac{\partial_1 - \partial_2}{2\pi} \right)^2}} \right\} \\ &= \frac{1}{2}(\partial_1 + \partial_2) \pm \frac{1}{2}(\partial_1 - \partial_2) \sqrt{1 - \frac{4\pi^2 K^2}{(\partial_1 - \partial_2)^2}}. \end{aligned}$$

Thus there are produced two coincident waves with the same frequency which is nearly the same with those original isochronous frequencies, but these two waves have different decrements. When we treat for small quantities, especially for K^2 , these decrements become

$$\Delta_1 = \partial_1 - \frac{\pi^2 K^2}{\partial_1 - \partial_2}; \quad \Delta_2 = \partial_2 + \frac{\pi^2 K^2}{\partial_1 - \partial_2}.$$

Thus the decrement in the primary system is very little less than its original value ∂_1 , and that in the secondary system very little larger than its original value ∂_2 . Such is the pseudo-impact excitation, improved in the resulting decrements by making the primary decrement larger and the secondary decrement smaller.

4. The nature of *impact excitation* may be conceived in the following way as the result of the change in primary decrement. Starting with an inherently large value of primary decrement at a positive value of K'^2 , hence with two coupling waves producing beats, the same decrement in-

creases at a rapid rate until it reaches a final large value at the end of one half beat time. Consequently K'^2 passes from its positive phase, through zero to a negative phase, and at the time when the primary decrements reach a final value, the two systems are at the condition of an extremely loose coupling with large ∂_1 and negative K'^2 , with the resulting decrements and double frequency

$$\Delta_1 = \frac{N}{n} \partial_1; \quad \Delta_2 = \frac{N}{n} \partial_2; \quad N = \frac{n}{\sqrt{1 - \frac{1}{4} \left(\frac{\partial_1}{2\pi} \right)^2}};$$

where ∂_1 is the final value.

Every such transition takes place within a very short time, and the law of change of primary decrement as a function of time is not yet known. As a consequence, the degree of coupling represented by

$$K'^2 = K^2 - \left(\frac{\partial_1 - \partial_2}{2\pi} \right)^2$$

and the primary double-frequency represented by

$$n_1 = \frac{1}{\pi} \frac{1}{\sqrt{C_1 L_1}} \sqrt{1 - \left(\frac{\partial_1}{2\pi} \right)^2}$$

are not constant, but simply decrease to the final values with a law not yet known, and also the time of half beat can not be written in the usual formula in terms of K' , assumed constant. The value of ∂_1 calculated in part I. for impact excitation is only the *equivalent value*, reduced to the case of constant decrement and at zero phase of K'^2 . Meanwhile, if we limit ourselves to the final phase at which the primary oscillation ceases, the condition of impact excitation is the same with that of pseudo-impact excitation and therefore we may take the expression I_{\max} for the secondary maximum amplitude in part I. to be the same for the impact as well as for the pseudo-impact excitations, with ∂_1 as the equivalent value in the former and as a constant in the latter. Also the indication of a heat-working instrument, containing ∂_2 only in part II., may be taken, as it is, in case of impact excitation, and on the assumption $\partial_1 \gg \partial_2$ in case of pseudo-impact excitation. Distinction between these two excitations may be made only by observing the oscillation curves in the primary and secondary oscillations, but probably not in any other way hitherto known. These two excitations may also be a continuation with a difference only in degree, the phenomena not being fundamentally different, if we can prove the variation of the primary decrement existing also in the ordinary excitation.

TABLE I.

$$\text{Values of } D = \frac{\partial_1 - \partial_2}{\partial_2} \left(\frac{\partial_2}{\partial_1} \right)^{\frac{\partial_1}{\partial_1 - \partial_2}} = \frac{x^x}{(1+x)^{1+x}}$$

$$\text{for the Values of } x = \frac{\partial_2}{\partial_1 - \partial_2} = \frac{\partial_2}{2\pi K}$$

x	D	x	D
0.005	0.969	0.055	0.805
0.010	0.945	0.060	0.794
0.015	0.925	0.065	0.783
0.020	0.905	0.070	0.770
0.025	0.889	0.075	0.762
0.030	0.873	0.080	0.750
0.035	0.857	0.085	0.741
0.040	0.843	0.090	0.731
0.045	0.831	0.095	0.723
0.050	0.816	0.100	0.715

TABLE II.

Values of Critical Coupling, K in Per Cent.

∂_2	∂_1 0.1	∂_1 0.2	∂_1 0.3	∂_1 0.4	∂_1 0.5	∂_1 0.6	∂_1 0.7	∂_1 0.8	∂_1 0.9	∂_1 1.0
	Mean 0.76 Per Cent.	Mean 2.35 Per Cent.	Mean 3.94 Per Cent.	Mean 5.53 Per Cent.	Mean 7.12 Per Cent.	Mean 8.71 Per Cent.	Mean 10.31 Per Cent.	Mean 11.90 Per Cent.	Mean 13.49 Per Cent.	Mean 15.08 Per Cent.
0.005	1.51	3.10	4.70	6.29	7.88	9.47	11.06	12.65	14.24	15.84
0.010	1.43	3.02	4.62	6.21	7.80	9.37	10.98	12.57	14.17	15.76
0.015	1.35	2.94	4.54	6.13	7.72	9.31	10.90	12.49	14.09	15.68
0.020	1.27	2.86	4.46	6.05	7.64	9.23	10.82	12.41	14.01	15.60
0.025	1.19	2.79	4.38	5.97	7.56	9.15	10.74	12.33	13.93	15.52
0.030	1.11	2.71	4.30	5.89	7.48	9.07	10.66	12.26	13.85	15.44
0.035	1.03	2.63	4.22	5.81	7.40	8.99	10.58	12.18	13.77	15.36
0.040	0.95	2.55	4.14	5.73	7.30	8.91	10.50	12.10	13.69	15.28
0.045	0.88	2.47	4.06	5.65	7.24	8.83	10.42	12.02	13.61	15.20
0.050	0.80	2.39	3.98	5.57	7.16	8.75	10.35	11.93	13.53	15.12
0.055	0.72	2.31	3.90	5.49	7.08	8.67	10.27	11.86	13.45	15.04
0.060	0.64	2.23	3.82	5.41	7.00	8.59	10.19	11.78	13.37	14.96
0.065	0.56	2.15	3.74	5.33	6.92	8.52	10.11	11.70	13.29	14.88
0.070	0.48	2.07	3.66	5.25	6.84	8.44	10.02	11.62	13.21	14.80
0.075	0.40	1.99	3.58	5.17	6.76	8.36	9.95	11.54	13.13	14.72
0.080	0.32	1.91	3.50	5.09	6.68	8.28	9.87	11.46	13.05	14.64
0.085	0.24	1.83	3.42	5.01	6.61	8.20	9.79	11.38	12.97	14.56
0.090	0.16	1.75	3.34	4.93	6.51	8.12	9.71	11.30	12.89	14.48
0.095	0.08	1.67	3.26	4.85	6.45	8.04	9.63	11.22	12.81	14.40
0.100	—	1.59	3.18	4.77	6.37	7.96	9.55	11.14	12.73	14.32

The secondary decrement, when the secondary system is an aerial circuit, consists of ohmic and radiation decrements and takes the form¹

$$\partial_2 = \frac{\lambda(m)}{6 \cdot 10^2 L_2(\text{mh.})} \cdot \left\{ R(\text{ohm}) + K \frac{h^2}{\lambda^2} \right\},$$

where the quantity within the bracket is the total equivalent resistance of the same circuit. The radiation decrement increases as the wave-length is shorter for the same effective height h , while the ohmic resistance may be almost constant and may be kept low. In practical cases this radiation decrement alone may reach as much as 0.1. Since the value of ∂_2 changes with the working wave-length, coupling for one-waveness must be adjusted every time the working wave-length is changed. Table II. gives K in per cent. for one-waveness for various combinations of ∂_1 and ∂_2 .² We see that for each value of ∂_1 , K is less as ∂_2 is increased; but for the values of ∂_1 greater than 0.3, and $0 < \partial_2 < 0.1$, K is almost determined from the values of ∂_1 alone. For a constant secondary system, everything which increases ∂_1 increases the coupling for one-waveness and hence the efficiency is improved, and such may be the same with the improvement of "Löschwirkung"³ in impact excitation.

IV.

ONE-WAVENESS AT THE RECEIVER.

One-waveness is also important in the receiver as in the transmitter. In order to consider this question we will take one sample diagram shown in Fig. 1, by which couplings and connections will be made manifest and definite.

When the trains of waves arrive at the place of an isochronous receiving aerial circuit and there produce the field represented by $E_{ao}e^{-\delta t} \cos \pi n t$, there are set up oscillations in the aerial circuit, of which energy, one part is transferred to the inner circuit coupled to it, but the other part is partly lost as heat and partly radiated. This radiated energy has the same frequency as the field, and superpose another field on it with a sort of coupling, the result being likely to produce the coupled frequencies both in the aerial circuit and the surrounding field. This can

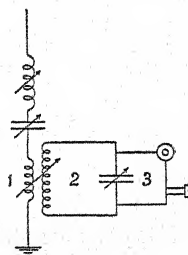


Fig.1.

¹ Rudenberg, Ann. Phys., 25, 1908, 451. The law $k \frac{h^2}{\lambda^2}$ appears to be confirmed experimentally by Austin, in case of $k = \text{constant}$; Bull. Bur. Stand., 7, No. 3, 1911.

² The relation (∂_1 , K) with parameter ∂_2 are all parallel straight lines, easily extended for larger values of ∂_1 and ∂_2 .

³ Wien, Jahrb., 4, 135, 1911.

be avoided by making the aerial circuit less radiative by inserting an inductance without increasing the resistance in a proportionate amount. By this device the damping of the aerial circuit becomes much smaller than that of the arrived waves, and the resulting current oscillations in this circuit may be represented by

$$I_1 = I_{10}(e^{-\delta t} - e^{-\delta_1 t}) \sin \pi n t,$$

where δ_1 is the damping of the circuit and the maximum amplitude is represented by¹

$$I_{10} = \frac{E_{ao}}{2\pi n L_1(\delta - \delta_1)}.$$

This is one-waveness and takes the maximum amplitude at the time

$$t = \frac{1}{\delta - \delta_1} \log n \frac{\delta}{\delta_1}$$

and

$$I_{1\max} = I_{10} \frac{\delta}{\delta_1} \left(e^{-\frac{\delta_1}{\delta - \delta_1}} - e^{-\frac{\delta}{\delta - \delta_1}} \right).$$

After this time we may write the equation for the aerial circuit in the form

$$I_1 = I_{1\max} e^{-\delta_1 t} \sin \pi n t.$$

In the next circuit 2, which is a closed condenser circuit made to have a very much smaller damping δ_2 than δ_1 , and coupled to a one-waveness to the aerial circuit,² we may write the potential equation in the form

$$V_2 = V_{2\max} e^{-\delta_2 t} \sin \pi n t,$$

where

$$V_{2\max} = \frac{1}{4\pi n} \cdot \frac{\delta_1 - \delta_2}{\delta_2} \left(\frac{\delta_2}{\delta_1} \right)^{\frac{\delta_1}{\delta_1 - \delta_2}} \frac{1}{C_1} \sqrt{L_1 L_2} \cdot I_{1\max}.$$

And this oscillating potential works upon the next circuit. The circuit 3 consists of the detector and telephone in which the resistance and self-inductance are very large compared to the previous circuit. In this circuit we must make distinction between wave frequency and group frequency. With respect to the wave frequency, circuit 3 is entirely out of tune or aperiodic and hence there is produced only the forced oscillations. The detector here works by rectification³ and, if this rectification

¹ Bjerknes, Wied. Ann., 55, 1895, 121.

² At close coupling, presence of two coupling waves may be observed, see also L. W. Austin, Bull. Bur. St., Vol. 7, No. 2, 1911.

³ This is one explanation, and in mineral detectors, some much more complicated action is suspected, though it takes no definite shape. So far as known, all forms of present detectors possess the property of polarity, whether electric rectification or magnetic hysteresis. The sense of rectification is constant while the potential working on the mineral detectors is

is perfect, current which passes through the telephone coil may be represented by

$$I_3 = I_{3\max} \left[\int_0^{1/2n} + \int_{3/2n}^{5/2n} + \int_{7/2n}^{9/2n} + \int_{11/2n}^{13/2n} + \dots \right] e^{-\delta_2 t} \cos \pi n t dt,$$

where

$$\phi(I_3, V_2) = \frac{\pi}{2},$$

$$I_{3\max} = \frac{V_{2\max}}{\sqrt{R_3^2 + (\pi n L_3)^2}} = \frac{1}{\pi n L_3} \cdot V_{2\max}.$$

This current is taken as a simple summation for this reason. The frequency of the oscillatory current is of the order 10^6 and that of the telephone diaphragm of the order 10^3 ; hence the most part of the oscillations in one train of currents passes through the telephone coil while the diaphragm is making one quarter vibration, from its normal position to the extreme amplitude. When the above integrations are made, the current through the telephone coil is represented by

$$\begin{aligned} I_3 &= \frac{I}{\pi n} I_{3\max} \left[\frac{1}{2\pi} \partial_2 + \frac{e^{\frac{1}{2}\partial_2}}{e^{\frac{1}{2}\partial_2} - 1} \right] \\ &= \frac{1}{\pi n} I_{3\max} \left(\frac{2}{\partial_2} + \frac{1}{2} + \frac{1}{2\pi} \partial_2 \right) \\ &= \frac{2}{\pi n \partial_2} \cdot I_{3\max}, \end{aligned}$$

where ∂_2 is the decrement of the circuit 2. Thus there is apparently an accumulative action of current wave-trains in the receiver, but this is due to the mechanical slowness of the telephone diaphragm and does not extend its integrating action to the group frequency as in a heat-working instrument. It is the above current which is indicated in a moving-coil galvanometer placed in the circuit 3, and it is this current which produces the amplitude of vibration of the diaphragm, and which may be smaller and still keep up audibility, according as the group frequency approaches the natural frequency of the vibrations of the diaphragm. As regards the group frequency ζ , the intermittent current I_3 flows through the telephone coils, ζ times per second and gives rise to the note of ζ . For this frequency we have mechanical resonance and a certain kind of coupling. For the former, the damping of the diaphragm is very great; consequently it vibrates with forced group frequency. If the lowest natural frequency

large, but when it is small we observe reversals, the galvanometer deflecting in opposite senses irregularly, and the tone audible in the telephone while the galvanometer shows no deflection at all.

of the diaphragm is p , and the group frequency ζ , the intensity of tone produced will be proportional to

$$\frac{1}{(p^2 - \zeta^2)^2},$$

which expression of course excludes the idea of damping.¹ As to the kind of coupling, transfer of energy from the diaphragm back to the circuit is in the form of an opposite current in the same double frequency ζ due to the vibration of an iron plate in front of the pair of permanent magnets, and there is need of something like one-waveness, by the adjustment of coupling. Such is already attained by Pickard's adjustable telephone, in which the magnet and coils are adjusted to a proper distance from the diaphragm, and the effect of this adjustment on the intensity of tone is something marvellous, while in some other telephone such adjustment is made at the manufacturing place. Thus, in the telephone for use in wireless telegraphy, mechanical resonance of the diaphragm to the group or spark frequency and the proper coupling or distance between it and the magnet, are important items of manufacture, while its resistance is immaterial within a wide limit, since it is negligible in comparison with its inductance.

¹ M. Wien, *Ann. Phys.*, 18, 1049, 1905.

AN INTERPRETATION OF CURL AND DIVERGENCE IN
TERMS OF VECTOR-GROWTH.

BY FREDERICK SLATE.

THE essential steps preliminary to the classification and description of stationary vector fields are now usually presented substantially as follows:

1. The idea of gradient for a given scalar field is introduced and brought into relation with potential.
2. The test is formulated whether or not a scalar field can be specified, of which a given vector field would be the gradient. This yields the criterion for the occurrence of a single-valued scalar potential.
3. The curl of a vector field is then definitely set up as a measure of departure from that criterion.
4. This measure of curl is made to appear in two forms, whose relation leads to the possible equivalence of integrations round a bounding curve and over the enclosed area (Theorem of Stokes). The theorems of Gauss and Green are then supplementary in their idea. They and the measure of divergence can be put first, however, if made a part of the discussion of potential, as they often are.

The reasons for adopting such a sequence lie obvious in the history of the subject. Curl is made, so to speak, an offshoot of potential because the latter type of field was earlier prominent; and because the equality of $\partial V_y / \partial x$ and $\partial V_x / \partial y$ ¹ gave a natural starting-point for the general expression of their difference. But the actual free use of curl and divergence in electro-magnetism has been leading toward revision that may exhibit the kernel of these matters differently; and this general aim guides the present paper, whose special intention is to emphasize another possible logic in the order of ideas, and in their foundations. The discussion is limited to steady fields.

If we approached without bias the question of classifying vector fields we might begin with the simple case—the uniform field, in which the resultant vector is constant in direction and magnitude. For such fields, too, as we know, curl and divergence are everywhere zero. It is known, moreover, that two pure types of modified field can be specified: one

¹ With some apology for the hybrid notation. But it seemed desirable to accentuate the central vector quantity, even where the equations are algebraic.

exhibiting curl alone, and the other divergence alone; while, further, any steady vector field can be analyzed, for purposes of quantitative description, as a combination of the two pure types. If our thought follows this sequence, less artificial than some others, it is plain that we coördinate curl and divergence on equal footing in their origin; each represents a particular condition of departure from the same original uniformity. At the same time, when taken in superposition, they describe the local arrangement of vector throughout any field adequately, and with simplifications due to the mode of segregation. The question is approached as here indicated.

In any uniform field the projection of the vector in any given direction is equal at all points. If then any base-point be chosen within such a field, and any definite radius-vector be drawn, all along that line there will be equal radial projections of the field-vector; and, of course, equal projections perpendicular to that radius-vector. But we must note that the conventions attached to the radius-vector will result in opposite sign for both radial and perpendicular projections, if we prolong the radius-vector backward through the origin; that is, in effect, use a pair of radius-vectors at an angle of 180° . Any non-uniform field will in general show change in the radial component or in its perpendicular, or in both, as we pass from point to point along any given radius-vector; and this brings the immediate suggestion that we should decipher in the character and the rapidity of such changes the special phases of non-uniformity occurring in any particular case. By following this clew we are in fact led to find in curl and divergence an appropriate and simple measure of the important changes in vector-arrangement.

Assume in the vector field an origin and a right-handed rectangular set of axes. Then at any point of the field the component of the field-vector parallel to any chosen radius-vector is

$$V_r = V_x \cos \alpha + V_y \cos \beta + V_z \cos \gamma. \quad (1)$$

Write the rate of change in this component, for the special case of leaving the origin along the particular radius-vector; it is

$$\left(\frac{dV_r}{dr}\right)_o = \left(\frac{\partial V_x}{\partial x}\right)_o \cos^2 \alpha + \left(\frac{\partial V_y}{\partial y}\right)_o \cos^2 \beta + \left(\frac{\partial V_z}{\partial z}\right)_o \cos^2 \gamma + \left\{ \begin{array}{l} \text{terms containing} \\ \text{products of cosines} \end{array} \right\}, \quad (2)$$

the derivatives being all particular values at the origin. Form now the mean for all orientations of the radius-vector, which gives as a well-known result,

$$\left(\frac{dV_r}{dr}\right)_o = \frac{1}{3} \left\{ \left(\frac{\partial V_x}{\partial x}\right)_o + \left(\frac{\partial V_y}{\partial y}\right)_o + \left(\frac{\partial V_z}{\partial z}\right)_o \right\}. \quad (3)$$

In a uniform field eq. (2) and consequently eq. (3) become zero identically. For the general case, therefore, a finite value of eq. (3) measures non-uniformity; and it does this in terms of divergence, whose known local value occurs in the second member. Divergence then is three times the mean rate of change in the radial component of the field-vector, for all directions of departure from a given point. Its invariant property is apparent in both members of eq. (3), since XYZ are assumed arbitrarily; and the proportional factor $(1/3)$ can be removed by giving a modified expression to this invariance:

$$\left(\frac{dV_r}{dr}\right)_1 + \left(\frac{dV_r}{dr}\right)_2 + \left(\frac{dV_r}{dr}\right)_3 = \left(\frac{\partial V_x}{\partial x}\right)_o + \left(\frac{\partial V_y}{\partial y}\right)_o + \left(\frac{\partial V_z}{\partial z}\right)_o. \quad (4)$$

The subscripts denote radius-vectors forming a shifting set of permanently rectangular axes, the expression of taking mean values being now omitted as superfluous. But the most significant feature of eq. (3) is the necessary occurrence of mean value in generalized connection with divergence. This occurs both here and in other relations, as will be shown later. The usual mean-value theorems credited to Boussinesq are stated for potential fields only; and though the extension to any steady vector-field is obvious enough, it seems worth while to take the step explicitly; especially when a parallel procedure, which is perhaps a little more novel, is found applicable to curl, as we now proceed to make plain.

For the component of field-vector perpendicular to a given radius-vector we have the expression, within the XY plane,

$$V_P = V_y \cos \alpha - V_x \sin \alpha. \quad (5)$$

Then for all orientations round the chosen origin and lying in the XY plane we obtain, essentially by repeating the previous process, the mean value of the rate of change in this perpendicular component as we leave the origin,

$$\left(\overline{\frac{dV_P}{dr}}\right)_o = \frac{1}{2} \left\{ \left(\frac{\partial V_y}{\partial x}\right)_o - \left(\frac{\partial V_x}{\partial y}\right)_o \right\} (XY). \quad (6)$$

Similarly for the two other reference planes,

$$\begin{aligned} \left(\overline{\frac{dV_P}{dr}}\right)_o &= \frac{1}{2} \left\{ \left(\frac{\partial V_z}{\partial y}\right)_o - \left(\frac{\partial V_y}{\partial z}\right)_o \right\} (YZ); \\ \left(\overline{\frac{dV_P}{dr}}\right)_o &= \frac{1}{2} \left\{ \left(\frac{\partial V_x}{\partial z}\right)_o - \left(\frac{\partial V_z}{\partial x}\right)_o \right\} (ZX). \end{aligned} \quad (7)$$

Here again the rates of change, and consequently the mean values that occur, become identically zero in a uniform field; while finite values for equations (6) and (7) may be used to measure this second aspect of non-uniformity. We have in fact introduced for each reference plane a quantity equally proportional to the curl-component normal to that plane; and the form of the equations exhibits the invariant nature of the quantities in the second members; since the axes XYZ are selected at random; and offers another definition of curl through the first members. With each reference plane we can associate curl whose local measure is twice the mean rate of change in the component of the field-vector perpendicular to the radius-vector, for all directions of departure in that plane, as we leave a given origin. One method of removing the proportional factor (1/2) is by using the statement of invariance, as was done in writing eq. (4). For any pair of permanently rectangular radius-vectors congruous with X and Y

$$\left(\frac{dV_P}{dr_1}\right)_o + \left(\frac{dV_P}{dr}\right)_o = \left(\frac{\partial V_y}{\partial x}\right)_o - \left(\frac{\partial V_x}{\partial y}\right)_o, \quad (8)$$

the difference of sign in the two members harmonizing with the usual conventions.

The results reached above do not introduce explicitly the idea that curl is of vector nature. But that link may be supplied by transferring consideration from the vector-component (V_P) to its vector-moment $[\mathbf{rV}]_{xy}$ which is axial on Z . We write, for the XY plane,

$$\mathbf{M}(z) = [\mathbf{rV}]_{xy} = [\mathbf{rV}_P] = rV_P. \quad (9)$$

And for the rate of growth in proceeding along a given radius-vector,

$$\frac{d\mathbf{M}(z)}{dr} = [\mathbf{r}_1V_P] + \left[\mathbf{r} \frac{dV_P}{dr} \right] = V_P + r \frac{dV_P}{dr}, \quad (10)$$

where \mathbf{r}_1 is a unit vector, and the third members of the equations are meant to indicate that they can be used for calculation. This rate and its companions for the X and Y axes have not the general value zero for a uniform field; in which, for example, the third member of eq. (10) becomes V_P , constant along any particular radius, but changing value with orientation. The mean rate for all orientations, however, does vanish for each reference plane in a uniform field; and that fact yields a starting-point for measuring the departure in this respect of a given vector field from uniformity. For the general case dV_P/dr of eq. (10) has a finite value; and assuming linear relation near the limit ($r = 0$), the proportional factor depending on orientation,

$$V_P = (V_P)_o + r \frac{dV_P}{dr}, \quad (11)$$

which reduces eq. (10) to the form,

$$\frac{d\mathbf{M}(z)}{dr} = (V_P)_o + 2r \frac{dV_P}{dr}. \quad (12)$$

Next imagine the process of finding the mean rate of change of this vector-moment round Z expressed for a circumference of unit radius. The first term in the second member evidently contributes on the whole nothing to the mean. The result then reduces to

$$\left(\frac{d\mathbf{M}(z)}{dr} \right) = 2 \left(\frac{dV_P}{dr} \right)_{r=1} = 2 \left(\frac{dV_P}{dr} \right)_o = \text{curl } (z). \quad (13)$$

Curl is measured for any axis, therefore, by a mean rate of change of vector-moment round that axis. The application at unit distance of the value of the derivative for the origin is based on the ordinary double aspect of a limiting rate.

The conditions supposed to lie behind the result of eq. (13) give an easy transition to the specification of curl as the limiting ratio of a line-integral to an area. Consider the particular case of a circle in the XY plane and centered on the origin. Assuming linear relation in preparation for the limit ($r = 0$), we have

$$\begin{aligned} \text{curl } (z) &= \lim_{r=0} \left[\frac{\int_0^{2\pi} \left\{ (V_P)_o + r \frac{dV_P}{dr} \right\} r d\gamma}{\frac{1}{2} \int_0^{2\pi} r^2 d\gamma} \right] \\ &= \frac{2 \int_0^{2\pi} \frac{dV_P}{dr} \cdot d\gamma}{2\pi} = 2 \left(\frac{dV_P}{dr} \right)_o; \end{aligned} \quad (14)$$

the first term in the integral reducing to zero as before; and the ratio is independent of the radius. But if $r = 1$ the area $= \pi$; and we are thus made aware of a reason for the proportional factor of equations (6) and (7) of different type from that assigned previously. The means found for one complete revolution (2π) are half the quotient when the divisor is (π). The circumstances for divergence are so nearly parallel that it seems necessary only to indicate the result briefly. In terms of surface-integral and bounded volume we can write, for a sphere of any radius round the origin, and again with assumed linear relation,

$$\text{Div.} = \lim_{r=0} \left[\frac{\int_0^{4\pi r^2} \left\{ (V_r)_o + r \frac{dV_r}{dr} \right\} dS}{\frac{4}{3} \pi r^3} \right] = 3 \left(\frac{dV_r}{dr} \right)_o. \quad (15)$$

The third member here is seen to verify eq. (3). The radius of the sphere does not appear in the result; and from the special choice $r = 1$ we can trace the origin of the proportional factor to the simple fact that the volume ($4\pi/3$) is then one third of the total solid angle (4π) for which the mean value was formed previously. And finally these interpretations of the connected processes furnish a plain reason for what is not otherwise so obvious: Why these limit-ratios for evaluating curl and divergence are independent of the *form* of the bounded surface and volume in the corresponding finite terms of the ratios. For any such approach toward the limit, the linear relation becomes a valid substitute at the vanishing-point; while curl and divergence involve in effect only the mean values of the respective *linear coefficients* at the origin.

The aspects of curl and divergence here developed seem to recommend themselves as offering a plan of straightforward description and measurement, applicable immediately to the vector distribution and arrangement, without the intervention of Ampère meshes, incompressible fluids, and the like; indispensable perhaps as historic scaffolding, but not perpetually to be retained after thought has clarified. We may note by way of conclusion one point about the factors ($1/2$) and ($1/3$). Their unconstrained occurrence seems to open a question of final choice between two standard expressions for curl, at least. It will doubtless always be felt as awkward that *twice* the angular velocity measures curl in cases of rotation, to mention only one item among several. On this issue the applications might prove to fall into two classes, each with its own best convenience. A group of such related matters is suggested—including the bearing of this point of view on vector potential—to which I propose to return in a second paper.

UNIVERSITY OF CALIFORNIA.

A COMPENSATING, LINEAR SCALE, HOT-WIRE
AMMETER.

BY A. H. TAYLOR.

THE objectional features of the usual hot-wire ammeters are too well known to deserve more than a brief enumeration. A shifting zero, a non-linear scale and a shunt ratio which depends on temperature, current strength and frequency seem to be the most obnoxious features of these instruments. At the same time they are valuable instruments for work with variable currents, especially where the frequency is very high.

In connection with the equipment of a radio-electric station at the University of North Dakota, the writer had need of an ammeter to put in the antennæ circuit, and not finding on the market a suitable instrument at a low price, he had one built by the departmental mechanic. This instrument has so satisfactorily met all objections, including the item of expense, that it is hoped that this brief description of it will be of value.

The wire used is No. 34 "Therlo," furnished by the Driver Harris Wire Co., and up to 0.6 ampere per strand seems to suffer very little elastic after effect when the heating current is removed. The sagging

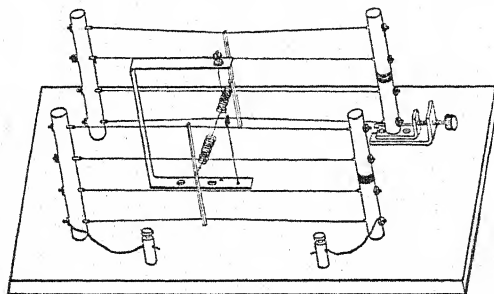


Fig. 1.

of a wire spanned between two supports is utilized to rotate a mirror or deflect a needle as the case may be. The instrument also has the differential feature, as has the Leeds and Northrup A.C.-D.C. comparator, but unlike the latter, the currents to be compared are carried through two widely separated and highly insulated circuits, enabling one to use the ammeter on high tension and high frequency circuits.

Fig. 1 shows one view of the instrument with the cover removed. There are two independent systems of four Therlo wires each, stretched between fiber posts set on an insulating base. Only two of these wires on either side take part in the indication of the instrument, the others being connected in at will as shunts, to alter the sensitiveness of the ammeter. These shunts carry practically the same currents as the indicating wires, hence have the same temperature, and thus give a shunt ratio independent of the current strength.

Owing to the small diameter of indicating and shunt wires there is but little skin effect. What little skin effect remains produces no effect on the shunt ratio, although it does undoubtedly slightly affect the sensibility of the ammeter as a whole. The two indicating wires on each side of the instrument are connected by light fiber bridges which pass inside of the shunt wires.

The right and left sides of the ammeter are made as symmetrical as possible and the initial tension on all wires can be adjusted by means of lock nuts on the terminal lugs which pass through the corner posts. After adjustment is once attained the lugs can be held fast by the set screws in these posts.

Referring to Fig. 2, one of the pair of active wires on each side is represented by W_1 and W_2 ; the vertical bridges by B_1 and B_2 ; the initial length of the wires by L ; the expanded length by $L + \Delta L$; the needle by N ; the mirror by M , placed on the suspension or pivot, and the corner posts by $P_1, \dots P_4$. The last post, P_4 , is mounted on a sliding base operated by a screw with an insulating head which passes outside of the case of the instrument. This movement of the post provides for a simultaneous slackening of the pair of wires W_2 , thus allowing the sensitiveness of the right-hand side of the instrument to be altered, as the deflection, X , depends on the amount of initial tension in the wires.

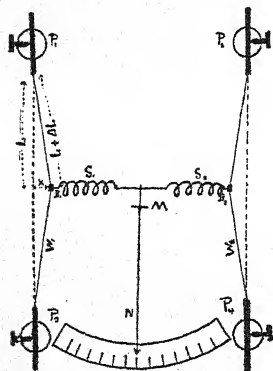


Fig. 2.

The differential feature is provided for by the two similar springs, S_1 and S_2 , which connect the two bridges with each other and with the end of the needle.

Further insulation of the right from the left-hand side of the instrument is attained by separating the springs S by an inch of silk thread. With this arrangement only half of the deflection X is reproduced at the end of the needle, but this disadvantage is more than offset by the advantage

of having a zero which is not fluctuating with the temperature. The temperature within the case of the ammeter after continued use runs up some 20 or 30 degrees, but the zero shift is scarcely noticeable, owing to the compensating action of the two sides of the instrument.

The ammeter can be used as a direct reading instrument, or as a comparator when both sides have been adjusted to the same sensitiveness. In the latter case the unknown current is sent through one side and the deflection reduced to zero by passing a known current through the other side. The pivot, or rather the suspension at M , is a coarse silk thread, held very taut. The readings given here were taken with a mirror and a scale at 50 cm. distance.

The following is an approximate calculation of the value of the deflection X in terms of current, with the slightly erroneous assumption that the initial position of the indicating wires is in a straight line connecting the corner posts. The temperature of the wires is estimated not to exceed 400 degrees, so that the heat is gotten rid of mainly by convection and conduction, both of which would depend on the temperature difference between the wires and the surrounding space. Equilibrium is then expressed by the equation

$$(1) \quad I^2 R = K(T - T_0).$$

From the geometry of Fig. 2 we see that

$$(2) \quad X^2 + L^2 = L^2 + 2L(\Delta L) + (\Delta L)^2,$$

or very approximately,

$$X^2 = 2L \cdot \Delta L.$$

Since

$$(3) \quad \Delta L = L(T - T_0)a,$$

where " a " is the expansion coefficient of the wire, we have from (1) and (3)

$$(4) \quad I^2 R = \frac{K \cdot \Delta L}{aL}.$$

Finally, from (4) and (2)

$$I^2 = \frac{K}{2L^2 \cdot a} \cdot \frac{X^2}{R},$$

or, $I = CX$, where $C^2 = K/2aL^2R$, a constant provided that R and a are constant. Now for Therlo wire R is very nearly constant, but so far as I am aware a is undetermined.

A series of measurements showed that the average resistance per wire between 0.1 amp. and 0.6 amp. lay between 3.2 and 3.13 ohms, a variation of about 2 per cent.

We see then that for measurements not requiring too high a degree of precision the ammeter could be conveniently used with an external constant resistance multiplier on low frequency circuits.

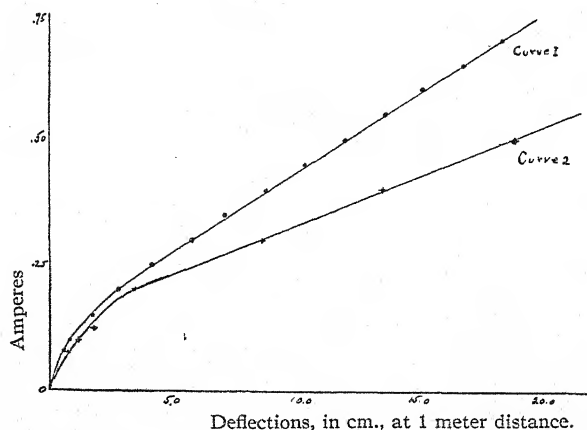


Fig. 3.

An inspection of curves 1 and 2 for two different tensions of the wires on the right-hand side of the instrument, will show not only how the sensibility is changed with the initial tension, but how the theoretical linear relationship between current and deflection is, after the first tenth of an ampere or so, almost exactly fulfilled. Curve 2 is for the higher tension.

The reason for the steeper rise of the first part of these curves is not apparent from the geometry of the instrument, but is rather to be deduced from the more rapid relaxation of the tension during the smaller expansions. It is also possible that the coefficient " a " is not constant, although this does not seem likely. This point is open to further investigation.

To calibrate the ammeter the maximum safe current is passed through both sides in series and the tension adjusted on the right-hand side until the deflection is zero. Then three sets of observations are taken: one for the right hand, one for the left-hand side, and one for the two, *i. e.*, a "difference" set. Table I. shows a set taken by a student in electrical measurements, curve 3 representing the results. A Weston mil-ammeter was used as a standard.

Curve 3 represents the data for both sides of the ammeter, right-hand side values being located by the dots, and left-hand by the crosses. For this test the two active wires on each side of the ammeter were connected in series. Placed in parallel, a far more efficient method of using the instrument, it would have one quarter the series resistance provided

TABLE I.

TABLE II.

Right Side.		Left Side.		Left Side.	
Deflection.	Amperes.	Deflection.	Amperes.	Deflection.	Amperes.
.70 cm.	.082	.70 cm.	.079	.25 cm.	.10
.90	.090	.90	.091	1.05	.20
1.10	.099	1.05	.100	3.65	.40
1.25	.109	1.20	.109	7.25	.60
1.50	.119	1.50	.121	11.60	.82
1.85	.133	1.90	.139	16.40	1.02
2.30	.151	2.60	.160	21.20	1.20
3.40	.189	3.70	.199		
5.40	.242	5.20	.229		
8.70	.321	9.10	.327		
16.40	.480	11.40	.378		
21.40	.579	16.90	.489		
23.60	.600	21.90	.584		

that the two active wires were of the same length. In this case doubling the currents of Table I. should give the same deflections. Table II. gives the data for this connection, for one side of the ammeter. The circles above curve 3 represent these currents of Table II. divided by two. It is seen then that it takes a little more than twice the current to get the same deflection with this connection.

This is due to the fact that the two heating wires are not quite equal in length; thus their parallel resistance is somewhat less than one quarter of their series resistance, requiring therefore more than double the current to obtain the requisite heating effect. This error in construction will be easy to remedy.

Dividing the scale of the ammeter into equal parts, it is seen that for most practical purposes, neglecting the first 12 per cent. of the range, the calibration curve is sufficiently represented by the formula $D = mI + b$. Thus two constants m and b determine the calibration. The cost of construction, including labor, is about eight dollars.

APPLICATION OF THE SELENIUM CELL TO PHOTOMETRY.

BY A. H. PFUND.

THE remarkable property possessed by selenium of changing its electrical resistance upon illumination has led many investigators¹ to construct photometric devices involving the use of a selenium cell. In spite of all past efforts, however, the selenium cell has failed to establish itself in the field of exact photometry—a state of affairs directly attributable to the complex peculiarities displayed by selenium. Recently the writer was asked to devise a method of measuring the energy in the visible spectrum carried by a beam of monochromatic light which had suffered diffuse reflection from a plaster of paris surface. In view of the fact that bolometers, etc., are entirely too insensitive for work of this nature, the selenium cell was chosen. Not only is the latter tremendously sensitive, but, in addition, its range of sensitiveness extends through the entire visible spectrum. It is the object of this paper to define the conditions under which accurate results may be obtained and also to call attention to the difficulties which present themselves when the selenium cell is applied to heterochromatic photometry.

GENERAL OUTLINE OF THE WORK.

It is first essential to establish a sensibility curve, *i. e.*, a curve plotted between changes in the resistance of the selenium cell and the wavelengths of the exciting light. In all cases, the energy carried by the exciting beam of monochromatic light is kept constant. With a sensibility curve at hand, the method of measuring energy is obvious. If the energy carried by an intense beam of monochromatic light is to be measured, the intensity is cut down by a *known* amount by means of a rotating-sector or some other geometrical device until the galvanometer deflection is the same as that indicated on the sensibility curve; if, conversely, a faint beam of known energy content is desired, again a fairly intense beam is cut down to the desired amount—only in this case the rotating sector is kept running instead of being removed, as in the previous case. This course of procedure is by far the safest as the relation between galvanometer deflections and energy has been found to change in a com-

¹ See "Die Elektrischen Eigenschaften u. die Bedeutung des Selens in der Elektrotechnik," Dr. Chr. Ries, 1908.

plicated manner as one passes through the spectrum. It has therefore seemed most advisable to refer all measurements to the initial sensibility curve and thus keep clear of the difficulties which are unavoidable in any other course of procedure. Next, the relation between the incident energy and the resulting change in resistance of the selenium cell is investigated and a law connecting these two quantities is established. The important result follows that, under the adopted method of procedure, the sensibility curve is not fixed but varies in a remarkable manner with the intensity of illumination. Finally, the applicability of Talbot's law, and hence the justifiability of using a rotating sector is established.

APPARATUS AND METHOD OF MEASUREMENT.

The light from a Nernst lamp L (Fig. 1), after having been passed through the mirror spectrometer $S_1M_1PM_2$, was resolved into a spectrum

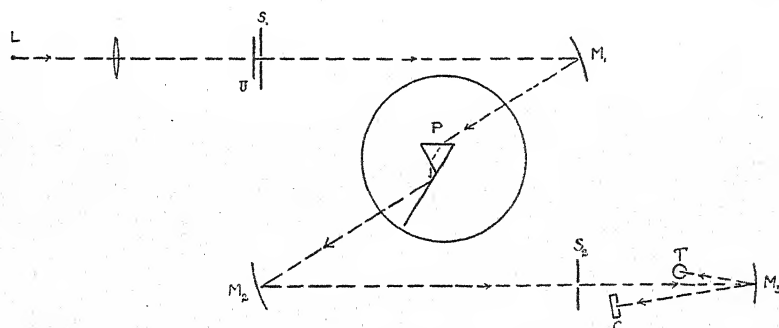


Fig. 1.

which was brought to a focus in the plane of the slit S_2 . This slit transmitted a bundle of quasi-homogeneous rays which fell upon the concave mirror M_3 and, after reflection, were projected either upon the thermal couple T or the selenium cell C . The latter was connected in the circuit of a simple potentiometer—as shown in Fig. 2. The object of the auxiliary circuit $E_1R_1R_2$ was to introduce a slight counter-E.M.F. at the galvanometer terminals and thus annul the initial "dark" deflection. It has been pointed out in one of my previous papers¹ that a selenium cell will furnish reliable results only when galvanometer deflections resulting from comparatively short exposures are taken. The device for procuring such exposures consisted of a circular disc of heavy paper from which a 75° sector had been cut out and which was mounted on the prolongation of the "second-hand" axis of a small alarm clock. This arrangement, which yielded exposures of 12.5 seconds, was mounted at U

¹ A. H. Pfund, *Phil. Mag.*, Jan., 1904.

in such a manner that the axis of rotation of the paper disc passed through the prolongation of the slit and was perpendicular to the plane of the slit. Under these conditions the light was admitted and cut off abruptly and results agreeing within less than 2 per cent. were obtained.

Two Giltay cells were studied and each exposed an area of about 1 square mc. and had a resistance of 3×10^6 ohms. Their behavior was so nearly the same that the results obtained from but one will be presented. Both were extremely sensitive and, under the comparatively faint illumination to which they were exposed, showed no observable hysteresis due to successive exposures to differently colored beams. Follow-

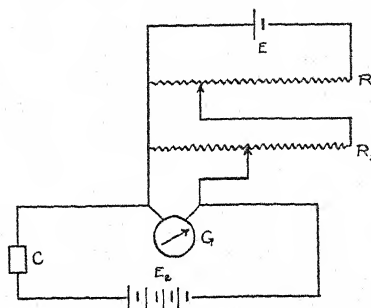


Fig. 2.

ing in part the course of procedure laid down by Stebbins,¹ the full current was permitted to pass through the cell for several hours before a series of measurements was taken and the temperature of the room was brought to about 21°C . after which it was kept constant to within 1°C . When, finally, a steady condition had been established, the selenium cell was exposed to the white light of an incandescent lamp until about twice the average deflection to be expected in later measurements was obtained. After waiting 3-4 minutes, this procedure was repeated and after a subsequent interval of 4 minutes the resistances in box R_1 were readjusted in order to bring the galvanometer back to zero. All this means that the "dark" resistance, to which all subsequent measurements are referred, is not that acquired by the cell after having been in darkness for a long time, but that acquired by the cell subsequent to an exposure to white light. Such a course of procedure was found necessary for the reason that a cell which had been kept in darkness for a long time and which was then exposed to light of the intensity used, required more than 10 minutes to return to its initial "dark" resistance—an interval of time entirely too long to be practicable. Under the actual conditions of the experiment the time interval rarely exceeded 4 minutes.

Both resistance boxes R_1 and R_2 contained 10,000 ohms. The D'Arsonval galvanometer G had a resistance of 469 ohms, a working sensibility of 280 megohms and a period such that 8.0 seconds were required to reach maximum deflection from rest. The other D'Arsonval galvanometer, which was connected to the thermopile, had but 7 ohms resistance

¹ J. Stebbins, *Astrophys. Jnl.*, XXXII., p. 185, 1911.

and a working sensibility of 2×10^{-9} amperes. The two-junction-vacuum thermopile T as well as the charcoal evacuator (used to produce and maintain high vacua) are of new design and have already been described elsewhere.¹ The sensibility was such that the total radiation from a meter-candle falling upon the blackened junctions (of 2 sq. mm. area) through a rock-salt window, gave rise to a deflection of 500 mm. In the present experiments, the thermal junctions were protected by a glass window—a similar window having been placed over the selenium cell in order to compensate for any selective absorption of the glass.

SENSIBILITY CURVE.

Having already defined the conditions under which sensibility curves are obtained, it only remains to present a typical curve obtained under very faint illumination. In order to obtain deflections of sufficient magnitude, a storage-battery of 20 volts was inserted at E_2 (Fig. 2). Successive galvanometer deflections resulting from exposures of the selenium cell under identical conditions, rarely differed by more than 1 per cent. In the following table λ is the wave-length of light used; E is the energy as measured by the deflections of the galvanometer connected to the thermal-junctions; D_0 is the deflection of the galvanometer connected to the selenium cell and D_c is the deflection corrected for the constant energy of 22.0 mm. in column E .

TABLE I.

λ $\mu\mu$	E Mm.	D_0 Mm.	D_c
453	22.5	109.5	108.2
494	22.0	113.5	113.5
521	22.0	118.0	118.0
556	21.2	119.9	122.0
601	22.2	118.0	117.5
625	22.0	107.4	107.4
651	21.7	86.1	87.0
663	22.0	80.0	80.0
675	21.6	75.8	78.0
686	21.6	75.0	77.5
711	22.5	58.0	56.2
738	21.3	35.0	37.5
785	22.0	5.0	5.0

These results are presented graphically in Fig. 3. It may be added that it was found possible, after or during a series of measurements, to

¹ PHYS. REVIEW, April, 1912. Abstract of paper presented at Washington meeting of Am. Phys. Soc., Dec., 1911.

expose the cell to light of any desired color and to repeat previously obtained observations very closely. These results establish, I think, the fact that the selenium cell may be used with success to measure the intensity of monochromatic light accurately

TALBOT'S LAW.

In view of the proposed use of a rotating-sector, a careful study of the applicability of Talbot's law to the selenium cell seemed imperative. No matter what type of instrument is used to measure radiant energy, the use of a rotating sector must first be justified,—for it is by no means evident that intermittent flashes of intense light will produce the same effect as a continuous beam of the same total energy content. In order to put these matters to test, the following course of procedure was followed out: a rotating sector containing angular openings of 180° , 60° and 20° was so mounted in front of the slit of the spectrometer, that any one of the three apertures could be used at will. First the thermal-junction was tested by means of the rotating-sector and it was found that over the entire range of the 50 cm. scale, the deflections were rigorously proportional to the energy. Then the work on the selenium cell was begun and it may be noted that the cell was exposed always for 12.5 seconds. The cell was first exposed to the uninterrupted beam and the resulting galvanometer deflection was noted—likewise, the energy carried by this beam was measured; then the 180° portion of the sector was made effective, the selenium cell was again exposed and the galvanometer deflection was noted. Next, the rotating sector was removed and the beam, which was concentrated on the thermocouple, was cut down by means of an optical wedge to 50 per cent.—as measured by the deflections of the galvanometer connected to the thermocouple; again the selenium cell was inserted. If, now, the deflections due to the selenium cell, excited by a continuous beam reduced to 50 per cent. of its initial intensity, is the same as the deflection obtained when the 180° sector was used, then Talbot's law holds. The long table of results bearing on this question is omitted. Suffice it to say that Talbot's law holds to within 2 per cent. (the limit of accuracy) through the spectral region $\lambda = 450\text{--}785\mu\mu$ for an intensity range of 1 : 18.

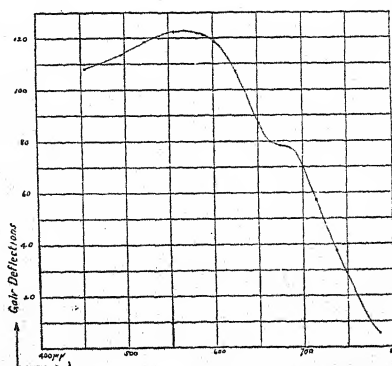


Fig. 3.

The sector was of the usual symmetrical type having equal angular openings on opposite sides of the center of rotation. Usually the disc was driven at high speed—about 30 revolutions per second, *i. e.*, 60 interruptions. It was found, however, that the “inertia” of the galvanometer as well as that of the selenium cell made it permissible to use so low a frequency of interruption (about 10 per sec.) that violent flicker was perceptible to the eye. These facts are brought out in the following table where galvanometer deflections for different frequencies of interruption are recorded.

TABLE II.

$\lambda \text{ } \mu\mu$	60 Interruptions per Second. Mm.	10 Interruptions per Second. Mm.
494	85.1	85.0
711	145.0	145.8
750	116.5	115.9

LAW CONNECTING E AND D .

A number of investigators¹ have formulated laws connecting the intensity or energy E of the exciting light and the resulting galvanometer deflection D which is a measure of the change in conductivity of the selenium cell. The variety of results obtained would seem to indicate that the experimental conditions in the several cases must have differed widely. As the final law will differ with the character of the cell, the time of exposure and the absolute intensity of light, it is to be stated definitely that the cell studied in this investigation was made by Giltay, the time of exposure was 12.5 seconds and the mean intensity of light corresponding to $E = 60$ mm. was 2.1 lux (the last result was obtained by balancing, photometrically, the total radiation from a Hefner lamp against the monochromatic radiation $\lambda = 590 \text{ } \mu\mu$). Due to experimental difficulties, no attempt was made to establish a law for very intense and very faint illuminations. From an inspection of a table of preliminary results it was evident that the law connecting E and D is very approximately of the form:

$$D = KE^B,$$

where K and B are constant as long as the wave-length of the exciting light remains unchanged. To evaluate B it is simply necessary to note the deflections D_1 and D_2 corresponding to the energies E_1 and E_2 . Thus, from the above relation it is found that

$$D_1 = KE_1^B, \quad D_2 = KE_2^B,$$

¹ See Ries, *l. c.*

whence

$$B = \frac{\log \frac{D_1}{D_2}}{\log \frac{E_1}{E_2}}.$$

To give some idea as to the constancy of B the following data (Table III.), taken from a preliminary series of results, are presented. Equally consistent results were obtained throughout the spectrum.

TABLE III.

Wave-length 460 μ m.

E Mm.	D Mm.	B
100.0	254.0	
74.1	220.0	0.515
42.5	164.6	.501
21.0	115.1	.508
7.9	68.0	.518
5.6	58.3	.508
		Average 0.510

All calculations of B are referred to the highest values of E and D .

To show how the value of B varies with change of wave-length a separate series of measurements was carried out. The results are given in Table IV., where E_0 is the energy of the beam and D_0 , the galvanometer deflection resulting from the increased conductivity of the selenium cell; D_1 is the deflection obtained upon cutting the light down to 50 per cent. of its initial intensity by means of the rotating sector and B is the value of the exponent.

TABLE IV.

λ μ m	E_0 Mm.	D_0	D_1	B
450	20.0	88.5	62.2	0.510
494	38.0	125.0	87.0	0.513
556	61.5	167.0	115.5	0.525
601	59.8	160.0	108.0	0.535
651	59.2	117.0	79.0	0.544
666	58.0	120.0	80.2	0.582
681	58.2	154.0	93.5	0.712
696	61.4	202.0	109.0	0.877
711	59.5	194.5	97.3	1.00
745	82.0	84.2	41.9	1.01
785	164.0	30.6	15.2	1.01
830	180.0	9.0	4.5	1.00

It will be noted that for wave-lengths shorter than $650\mu\mu$ the deflections are approximately proportional to the square-root of the energy ($B = 0.5$) while for wave-lengths greater than $\lambda = 700\mu\mu$ the deflections are directly proportional to the energy ($B = 1.0$).

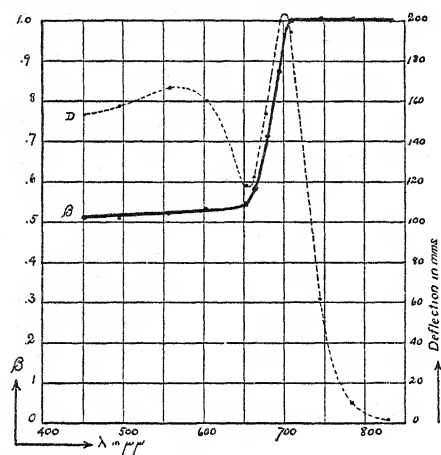


Fig. 4.

The results are presented graphically in Fig. 4 where the heavy curve represents the various values of B and the dotted curve, the sensibility curve corresponding to $E = 60$ mm.

These results offer, I think, a clue as to the difference between the so-called "hard" and "soft" Ruhmer cells. They are characterized by the fact that the former are comparatively sensitive to variations of intense sources of light and comparatively insensitive to faint sources, while the

latter behave in the opposite manner. Ruhmer, it appears, has developed a method of sensitizing cells so that the position of the sensibility maximum is under control. Let it be assumed that the law between E and D , previously deduced, also applies to Ruhmer cells and that we have two cells whose maxima lie, respectively, in the green and red. Let these two cells be placed side by side and exposed to the same source of white light at a distance of 1 meter and let the respective circuits be so adjusted that the galvanometer deflections, resulting from an exposure of the cells to light, are the same. If now the source be removed to a distance of 3 meters the intensity will drop to one ninth of its original value and, as a result, the deflections due to the "red" cell will likewise decrease to one ninth of its original value, while the deflection of the "green" cell will decrease only to about one third, since its maximum lies in that region of the spectrum in which the square-root law applies. Thus, under this weak illumination, the "green" cell is approximately three times as sensitive as the "red." If, on the other hand, the source be approached to within one third of a meter, the "red" cell will be found to be approximately three times as sensitive as the "green." Thus the "red" or "hard" cell is relatively the more sensitive to intense illumination, while the "green" or "soft" cell is relatively the more sensitive under faint illumination. Cells of these two types are not at my disposal. It would be most interesting to put the above deductions to an experimental test.

CHANGE OF SENSIBILITY CURVE WITH INTENSITY OF ILLUMINATION.

Since it has been demonstrated that the exponent B is not constant throughout the spectrum, it is evident that the form of the sensibility curve must vary with the intensity of illumination. To show how marked these variations are, the curves, shown in Fig. 5 have been pre-

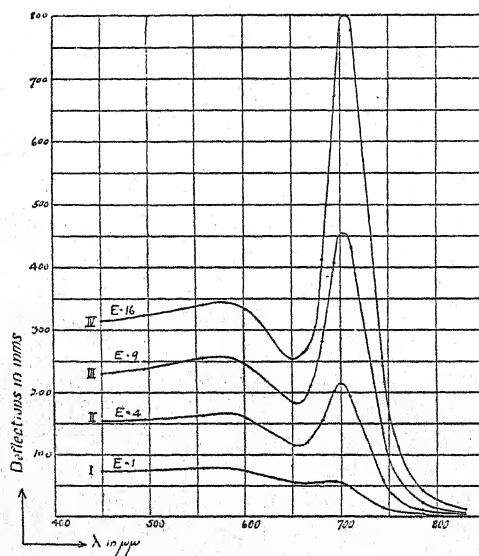


Fig. 5.

pared. The intensities or energies corresponding to Curves I., II., III. and IV. are in the ratios 1:4:9:16—the Curve II., which has already been presented in Fig. 4, serving as a basis for calculation. This procedure is entirely permissible since B has been found to remain sensibly constant over an intensity range even greater than 1:16. As a matter of fact, curves of all the types shown have been actually found and the deductions, based on this work, are fully borne out by the curve already presented in Fig. 3 (obtained under very feeble illumination), as well as by the sensibility curve presented in my earlier papers.¹ Referring once more to Fig. 5, it is to be observed that Curve I., obtained under the faintest illumination, has its maximum in the greenish-yellow and indicates but faintly the existence of a maximum in the red. As the energy is increased 4 times, Curve II. is obtained and from this it is seen that, while the deflections in the region of shorter wave-lengths increase only about two times, those in the red increase four times. The explanation of the remaining curves and the reason for the tremendous increase of the maximum near $700\mu\mu$ follows in a similar manner.

¹ A. H. Pfund, Phil. Mag., Jan., 1904; Phys. Zeitschrift., 10, p. 341; Phys. Review, May, 1909.

With these results as a basis, I venture the attempt to reconcile the discordant results obtained by various observers concerning the position of maximum sensibility in the spectrum. While¹ Minchin, Mercadier and Adams found that the greenish-yellow and yellow rays produced the greatest effect, Sale, Siemens, Forsmann, Marc, Pfund, Schrott and Athanasiadis found that the red rays were most effective. The curves in Fig. 5 would seem to indicate that both contentions may be correct—depending on the intensity of illumination. Under very faint illumination the greenish-yellow rays are most effective, while under very intense illumination the red rays produce by far the greatest effect. It is to be distinctly understood that these deductions apply only under the condition that the illumination of the cell takes place for a comparatively short interval of time—entirely insufficient to permit the cell to assume a steady resistance under continued illumination. As to whether or not relations, similar to those here presented, also obtain when the cell is exposed until an equilibrium condition has been established, is immaterial for the present, since such a procedure would exclude the practical applications of the selenium cell to photometric measurements.

Referring once more to Fig. 5, it is evident that very great caution must be exercised in using a selenium cell to measure the intensity of white (undispersed) light. Assuming that the selenium integrates the effects of the various colors according to the sensibility curves presented, it is clear that the galvanometer deflections obtained will vary neither directly as the energy or its square-root but according to some power determined jointly by the values of B (Fig. 4) and by the distribution of energy in the spectrum of the source. Without showing in detail how erroneous results might be obtained, it will suffice to say that only that method of procedure is free from objections which involves a calibration of the selenium cell against the same source whose variations in intensity are to be measured subsequently.

HYSTERESIS EFFECTS AND CONSTANCY OF BEHAVIOR.

A discussion of the reliability of the results obtained by means of the selenium cell and of its constancy of behavior with time is, naturally, of vital importance in this connection. While undoubtedly hysteresis, *i. e.*, the effect due to previous exposure, is noticeable when the illumination is strong, no such effect was ever observed in the previous experiments—all of which were carried out under faint and only moderate intensities. In fact, while a sensibility curve was being determined it was found that any reading could be repeated no matter to what colored light the cell had last been exposed. As for constancy of behavior with time it may

¹ See Ries, 1. c.

be stated that while every well-made cell experiences an increase in resistance after having been made, it eventually reaches a steady state. While my experience with the Giltay cells does not cover a sufficiently great interval of time to make the drawing of conclusions permissible, I have found that cells made and tested by me in 1904 had retained all of their characteristics in 1909 when they were last tested. These cells had carbon electrodes and were preserved in an evacuated tube. It is hence to be concluded that a seasoned and well-made selenium cell is a reliable instrument.

SUMMARY.

The results obtained may be summarized as follows:

1. It has been established that the selenium cell, when used as a photometer, will yield accurate results if the following conditions are fulfilled:

- (a) Monochromatic light must be used.
- (b) An accurate sensibility curve must have been established.
- (c) Exposures to light must be made automatically and must be of short duration.

2. Talbot's law has been found to hold (within the limits of accuracy of measurement) throughout the visible spectrum. The range of frequencies covered was from 10 to 60 interruptions per second.

3. For moderate intensities of illumination and for a range of intensities 1 : 18, the relation between the incident energy (E) and the galvanometer deflection (P) has been found to be represented by the expression

$$D = K \cdot E^B.$$

As a consequence of determining the various values assumed by B , it is found that from $\lambda 450$ – $650\mu\mu$ the deflections are proportional, approximately, to the square root of the energy, while from $\lambda 700$ – $830\mu\mu$ the deflections are directly proportional to the energy. An attempt is made to account for the behavior of "hard" and "soft" cells.

4. As a consequence of the variations of B with λ it is shown that the form of the sensibility curve varies with the absolute intensity of illumination. It is furthermore shown that, under very faint illumination, the selenium cell is most sensitive toward yellowish-green light, while, under intense illumination, the cell is by far the most sensitive toward red light. An explanation of the cause of disagreement between various investigators in their determination of the position of maximum sensibility is attempted.

5. The conditions are defined under which the selenium cell may be safely used to measure the intensity of white light.

JOHNS HOPKINS UNIVERSITY,
February, 1912.

PROCEEDINGS
OF THE
AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE SIXTY-SECOND MEETING.

THE spring meeting of the Physical Society was held in the Jefferson Physical Laboratory, Harvard University, Cambridge, on Saturday, April 27, 1912, with President W. F. Magie in the chair.

The following papers were presented:

The Optical Properties of Crystalline Spheres formed by Precipitation in Gelatine. H. W. MORSE.

Some Further Results of Precision Measurements of Refractive Indices as a Function of Temperature. (By title.) F. A. MOLBY.

Absolute Formulæ for the Attraction of two Coaxial Solenoids. (By title.) GEO. O. OLSHAUSEN.

The Calculation of the Maximum Value of the Force between Two Coaxial Circular Currents. (By title.) FREDERICK W. GROVER.

Absorption and Reflection Spectra of Metallic Sulphides. A. TROWBRIDGE.

Distortions in Spectral Luminosity Curves Produced by Variations in the Character of the Comparison Standard and in the Surroundings of the Photometric Field. HERBERT E. IVES.

The Electron Theory of Magnetism. D. F. COMSTOCK.

Direct Measurement of the Velocity of Kathode Rays and the Variation of Mass with Velocity. E. LEON CHAFFEE.

The Effect of a Magnetic Field on Photo-electric Emission. A. W. HULL.

The Kinetic Energy of the Electrons emitted by Photoelectric Action. O. W. RICHARDSON and KARL T. COMPTON.

The Law of Fall of a Drop through Air at Reduced Pressures and a Redetermination of e . R. A. MILLIKAN.

A Theoretical Correction to the Drop Method of Determining e . ARTHUR C. LUNN.

The Characteristics of Vowels. D. C. MILLER.

Changes in the Dielectric Constant Produced by Strain. E. P. ADAMS and C. W. HEAPS.

On Mechanical Force from the Magnetic Field of a Displacement Current. R. H. GODDARD.

Rotations in the Metallic Arc. W. G. CADY.

Note on the Variation from Lambert's Cosine Law of the Emission from Tungsten and Carbon at Glowing Temperatures. A. G. WORTHING.

The Chaffee Gap as Interrupter for an Induction Coil. Demonstration.
G. W. PIERCE.

Anomalous Magnetization in Iron. B. O. PIERCE.

Magnetic Induction in a Group of Oblate Spheroids of Iron. (By title.)
S. R. WILLIAMS.

The Relations between the Optical and Electrical Constants of Metals.
L. P. WHEELER.

The Diurnal Range of Temperature and Other Meteorological Elements at
Different Levels above Mount Weather. WM. R. BLAIR.

The Thermal Properties of Liquid Water up to 80° and 12,000 Kgm. per Sq.
Cm. P. W. BRIDGMAN.

Progress on a Redetermination of the Specific Heat of Steam. (By title.)
A. R. NOTTINGHAM.

A Lecture Experiment to Show Variation of Elasticity with Temperature
(Illustrated). A. TROWBRIDGE.

Variation of Electrical Resistance with Temperature. Oxides. A. A.
SOMERVILLE.

The Electric Spark. Control of the Condenser Discharge by Ultra-violet
Radiations. WILLIAM O. SAWTELLE.

The Relation between Current, Potential Difference and the Condition of the
Electrodes, in the Case of Surface Ionization in a Vacuum. O. W. RICH-
ARDSON and CHARLES SHEARD.

Heats of Dilution. W. F. MAGIE.

The Relation of Osmotic Pressure to Temperature. W. F. MAGIE.

The Maximum Intensity of Magnetization in Iron. (By title.) B. O.
PIERCE.

Oscillographic Study of Current Charges Produced by a Carbon Trans-
mitter in Wire and Wireless Telegraphy. G. W. PIERCE and E. L. CHAFFEE.

The Linear Expansion of Invar Steel between 100° C. and -190° C. (By
title.) F. A. MOLBY.

An Attempt to use Centrifugal Separation on Liquid Air. (By title.)
H. L. HOWES.

The Effect of Light on the Helix of Cathode Rays from a Wehnelt Cathode.
(By title.) C. P. KNIPP.

Multiple Reflection of Short Electric Waves from Screens of Tinfoil Reso-
nators. W. L. SEVERINGHAUS and W. S. NELMS.

An Accurate Potentiometer Method for Measuring Resistance. W. P.
WHITE.

The Addition of Luminosities of Different Color. H. E. IVES.

Outline of a New Method for the Determination of e/m in Metals. D. F.
COMSTOCK.

Adjourned at 4:30 P.M.

ERNEST MERRITT,
Secretary.

THE ELECTRON THEORY OF CONDUCTORS.¹

BY O. W. RICHARDSON.

THIS paper is a further development of the matter of previous papers communicated to the Society. (For Abstracts see *PHYS. REV.*, Jan. and Feb., 1912.) It is divided into two parts.

The first investigates some consequences of the hypothesis that the collisions of the electrons inside a conductor occur with centers of force which attract them with a force varying inversely as the cube of the mutual distance. Those electrons which contribute to the integrals which determine the thermoelectric properties can be divided into two groups, according as their orbits are closed or open. The additional hypothesis is introduced that the electrons whose orbits are closed do not contribute anything to the current of conduction. They may, of course, contribute to the polarization current.

The criterion which determines whether the orbit of an electron at any point is an open one, is that the kinetic energy should exceed the work of the attracting forces from infinity to the point in question. From this condition the proportion of the electrons, instantaneously present at a point where their potential energy is $\omega = -\phi$, which are engaged in carrying the current, can readily be calculated. It is found to be

$$\chi\left(\frac{\phi}{R\theta}\right) = \frac{2}{\sqrt{\pi}} \left[\sqrt{\frac{\phi}{R\theta}} e^{-\frac{\phi}{R\theta}} + \int_{\left(\frac{\phi}{R\theta}\right)^{\frac{1}{2}}}^{\infty} e^{-x^2} dx \right].^2$$

This calculation assumes that the distribution of velocity accords with Maxwell's law and is independent of the distribution of potential energy, in agreement with the results obtained previously.

It has been shown in a previous paper that ν , the number of electrons in unit volume at a place where the potential energy is ω , is

$$\nu = \frac{ne^{-\frac{\omega}{R\theta}}}{\int_{\tau} e^{-\frac{\omega}{R\theta}} d\tau} = \kappa e^{-\omega/R\theta},$$

where n is the total number of electrons which can become free in the total volume τ , and κ is a function of θ only. The number of current-carrying electrons is therefore

$$\int_{\tau} \nu \chi\left(\frac{\phi}{R\theta}\right) d\tau = \frac{2n}{\sqrt{\pi}} \frac{\int_{\tau} \left[\sqrt{\frac{\phi}{R\theta}} + e^{\frac{\phi}{R\theta}} \int_{\sqrt{\frac{\phi}{R\theta}}}^{\infty} e^{-x^2} dx \right] d\tau}{\int_{\tau} e^{\frac{\phi}{R\theta}} d\tau}.^2$$

¹ Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1912.

² These equations were affected by an error of calculation as originally given. They were changed to the above expressions in reading the proof April 20, 1912.

These formulæ lead to the right kind of variation with temperature in the case of poor conductors to agree with the results of experiment; but it is impossible to institute a quantitative comparison as there appears to be no independent method of determining the value of the integrals involved.

From this point of view the change of metallic resistance caused by a magnetic field might be caused very largely by the effect of the magnetic field in altering the proportion of open to closed orbits and would therefore not necessarily exhibit any close correlation with the magnitude or sign of the Hall effect.

The second part of the paper deals with the theory of the photoelectric effect. It was shown previously that if $N(\nu)$ is the number of electrons emitted when unit energy of frequency between ν and $\nu + d\nu$, and otherwise having the characteristics of the complete ethereal (black-body) radiation is incident on a body, at temperature θ , which only emits electrons under the influence of photoelectric action, then

$$\frac{c}{4} \int_0^\infty N(\nu) E(\nu) d\nu = A \theta^{\frac{3}{2}} e^{-\frac{\omega}{R\theta}}, \quad (1)$$

where c is the velocity of light. $E(\nu)$ is the complete radiation formula at temperature θ , A is a constant which is different for different substances but does not involve θ , ω is the heat absorbed when one electron is allowed to escape from the substance without performing external work and R is the gas constant for a single molecule. Now it appears that a thermodynamic investigation of the specific heat of electricity σ leads to the conclusion that

$$\frac{\partial \omega}{\partial \theta} = \frac{3}{2} R - e\sigma$$

and it may be shown that for all known substances $e\sigma$ is much smaller than $3R/2$. If we may disregard $e\sigma$ on account of its smallness we can put

$$\omega = \omega_0 + \theta \frac{\partial \omega}{\partial \theta} = \omega_0 + \frac{3}{2} R\theta,$$

where ω_0 does not involve θ and the right hand side of (1) then reduces to

$$A \theta^{\frac{3}{2}} e^{-\frac{\omega_0}{R\theta}}.$$

Assuming as a sufficient approximation that $N(\nu)$ is independent of θ we may expand it in powers of ν^{-1} . Substituting Wien's formula

$$E(\nu) d\nu = \nu^3 \varphi\left(\frac{\nu}{\theta}\right) d\nu$$

and expanding the exponential on the right, one finds, after equating coeffi-

cients of powers of θ , that

$$N_\nu = \frac{4}{c} \frac{A}{\nu^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{n! \int_0^{\infty} \varphi(x) x^{1-n} dx} \left(\frac{\omega_0}{R\nu} \right)^n. \quad (2)$$

The integrals in the denominators of (2) are not, however, mere numbers. According to the radiation formulæ of Planck and Wien they contain a parameter h/R , where h is Planck's constant. By changing the variable to $z = (h/R)x$ one finds

$$N_\nu = \frac{c^2 A}{2\pi} \frac{h}{R^2 \nu^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{n! \int_0^{\infty} z^{-n+1} x(z) dz} \left(\frac{\omega_0}{\nu h} \right)^n, \quad (3)$$

where the integrals are now pure numbers. For example if Planck's formula is used $x(z) = (e^z - 1)^{-1}$. Hence

$$N_\nu = \frac{A}{\nu^2} \varphi \left(\frac{\omega_0}{\nu h} \right),$$

where $\varphi(\omega_0/\nu h)$ is a function of $\omega_0/\nu h$ only. Thus the curves giving the relative emission of electrons under the influence of isotropic radiation of varying frequencies may be constructed for all substances when the curve for any one substance and the values of ω_0 have been determined. Also the maximum and minimum values of N_ν will be given by particular values of $\omega_0/\nu h$, the same for all substances.

In a rather similar manner it may be shown that the mean kinetic energy T_ν of the electrons emitted by light of frequency ν is of the form $\nu \Phi(\omega_0/h\nu)$ where Φ is a function of $\omega_0/h\nu$ only. Φ is different from φ but is the same for all substances.

These conclusions cannot be said to be established with any degree of mathematical rigor. It might be objected that the physical terms which have been neglected would, if included, possibly entirely change the nature of the solutions. I do not, however feel that that is the case. On the other hand the properties of the integrals which enter into (2) and (3) might cause the series to become divergent or non-existent. Using Wien's radiation formula as a sufficient approximation, the integrals in (3) become Γ functions, being in fact $\Gamma(2)$, $\Gamma(1)$, $\Gamma(0)$, $\Gamma(-1)$, etc., in succession. Thus the series disappears except for the first two terms. However, this result is entirely dependent on the fact that the index of the power of θ on the right-hand side of the integral equation is an integer. This cannot be considered to be necessary on theoretical grounds and empirical reasons might be brought forward in favor of substituting $\theta^{\frac{1}{2}}$ for θ^2 . In that case the Γ functions are no longer infinite and it can be shown that the series are convergent when $\omega_0/h\nu$ is less than unity. If ω_0 has the values calculated from thermionic measurements this fraction is less than unity for such frequencies as give rise to photoelectric

emission. The fraction is also of the order unity. The condition

$$\frac{h\nu}{\omega_0} \geq 1$$

is also required for emission to occur on Einstein's theory.

So far as I am able to judge the experimental data hitherto accumulated do not conflict with the results of these calculations. More direct tests are being carried out at present.

PALMER LABORATORY,
PRINCETON, N. J.

THE IONS FROM HOT SALTS.¹

BY O. W. RICHARDSON.

THE first part of this paper deals with further measurements of the specific charge e/m of the ions from salts. The principal object of the investigation has been the determination of the nature of the negative ions which are emitted at low temperatures. Experiments have been made on CdI_2 , BaI_2 , SrI_2 , CaI_2 , Fe_2Cl_6 , and MnCl_2 . The negative thermionic currents from the iodides of the alkaline earth metals are surprisingly large, and apparently larger at low temperatures than those given by the corresponding oxides. Thus a layer of barium iodide only a few square millimeters in area was found to give a negative emission of about two microamperes before it was visibly hot. Heavy negative ions from all these substances have been detected, but as a rule they are mixed with an excess of electrons.

On account of various difficulties satisfactory measurements of e/m for the heavy ions have so far only been obtained in the cases of CaI_2 , SrI_2 and BaI_2 . In the case of all these substances the electric atomic weight of the heavy ions is in good agreement with the view that the ions are atoms of iodine carrying a single electronic charge (or, of course, molecules carrying two charges). The proportion of heavy ions to electrons is greatest when the salts are first heated and falls off with lapse of time. The proportion of heavy ions to electrons also diminishes as the temperature is raised, independently of the time effect.

It seems probable that in the case of these compounds the heavy ions are primary ions and are not formed by the combination of the atoms or molecules of the gas with electrons, since in the case of calcium iodide, freshly heated at a low temperature, all the ions had an electric atomic weight corresponding to I_- and no electrons could be detected. If this is so it would seem to be the only case known in which a negatively charged atom occurs as a primary gaseous ion.

Generally speaking the large negative emission which has been observed is not accompanied by any appreciable positive emission. In the case of SrI_2 ,

¹ Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1912.

in order to measure e/m for the positive ions, it was found necessary to heat the salt to a very much higher temperature. The value of the electric atomic weight observed agreed very well with Sr_+ and is therefore in accord with Davisson's measurements with other salts of this metal.

Various time changes have been observed which are being investigated further.

It is interesting to observe that the salts which most readily form heavy negative ions are those containing the extremely electronegative element iodine, whilst those which most readily form positive ions are those of the extremely electropositive heavy alkali metals.

The second part of the paper is devoted to a brief discussion of the time changes which the ionization from heated salts exhibits. It was first shown in the case of the haloid salts of zinc, by Garrett and Willows, that the ionization at constant temperature increased to a maximum and subsequently diminished. The effects could be explained on the hypothesis that two successive decompositions took place, the first being unaccompanied by ionization and the second resulting in the emission of ions. Recently G. C. Schmidt,¹ who has examined the case of cadmium iodide in detail, has advanced the view that these successive actions take place in the vapor of the salt. It does not seem to the writer that the facts can be explained on such a simple hypothesis. On such a view it is difficult to see why the ionization should diminish with time when an excess of salt is present and the pressure and temperature are kept constant. For, under these conditions the supply of the CdI_2 vapor is maintained at a steady rate by the salt present. It would seem that in order to explain these effects it is necessary to take into account the existence in the solid salts of unstable forms which have not been detected by chemical methods. This conclusion is supported by the fact observed by Schmidt that the distilled salt is not so efficient as that which has not been distilled.

PALMER LABORATORY,
PRINCETON, N. J.

DISTORTIONS IN SPECTRAL LUMINOSITY CURVES PRODUCED BY VARIATIONS IN THE CHARACTER OF THE COMPARISON STANDARD AND IN THE SURROUNDINGS OF THE PHOTOMETRIC FIELD.²

BY HERBERT E. IVES.

THE ordinary procedure in obtaining spectral luminosity curves by either the equality of brightness or the flicker photometer is to compare the spectral colors with a constant standard of arbitrarily chosen color. Thus Koenig compared all the spectrum against wave-length $.535\mu$, Abney used the white light of an arc, the writer has used the light of a standard incandescent lamp. Some previous experiments having led to the suspicion that one's

¹ Ann. der. Physik, Vol. 35, p. 401, 1911.

² Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

estimation of relative brightness of a set of colors such as the spectrum is altered by a change in the color or character of the comparison standard, a series of experiments have been carried out to clear up this point.

For the incandescent lamp light used in the earlier experiments of this series¹ were substituted a red light, and a green light, chosen so as to give the same intensity of illumination as the "white light." A large number of luminosity curves were then obtained by both the equality of brightness and flicker methods for a high and a low illumination and for large and small photometric fields, using the three colors of comparison field. These showed:

1. By the equality of brightness method the luminosity curves are subject to large shifts and distortions depending upon the color of the comparison field. These shifts are not consistent either in direction or amount in repetitions of the experiment, appearing therefore to consist in disturbance of the observer's judgment, or criterion of equal brightness. By adhering to one comparison light for a long period it has been found that a fairly constant criterion is formed, and consistent results are obtained which might lead one to believe the criterion a correct one. This common belief is however untenable upon considering the family of different curves shown by these experiments to be obtained by the use of several different comparison sources. Particularly disturbing to this belief is the fact that no one of those curves may be depended upon to be reproducible after the observer's criterion is upset by work with another comparison source and he has formed a criterion of brightness with it. The equality of brightness method luminosity curves of Koenig and Abney may easily be considerably shifted from what we may designate as "normal" (to be determined by some other method) due to this cause.

2. Using the flicker photometer and making the same changes in the color of the comparison standard no variations were found in the luminosity curves larger than the errors of measurement.

Following these experiments similar ones were performed with a bright field surrounding the photometer field, in place of the black space usual in optical instruments. Illuminations, field size, and color of comparison source were varied as before. These showed:

1. By the equality of brightness method additional distortions similar to those produced by changing the comparison field color result. Again there appeared to be no regularity or constancy to these shifts. Varying the brightness of the surrounding field hence acts also merely as a judgment-disturbing factor.

2. With the flicker photometer no variations were found in the luminosity curves, greater than the errors of measurement.

3. With a small field, bright surroundings considerably increase the sensibility of equality of brightness settings.

4. The Purkinje effect (equality of brightness method) and the reverse of that effect (flicker photometer) are dependent in magnitude only on the size

¹PHYS. REV., XXIX., 4, p. 441; PHYS. REV., XXXII., 6, p. 561.

of the photometric field, and not on the total area of illuminated field, in the case where a bright surrounding field is used.

The method of obtaining spectral luminosity curves by steps of small hue difference was investigated. Starting at either end of the spectrum the brightness of adjacent portions was measured, passing by small wave-length differences to the other end. This method was found extremely susceptible to cumulative errors and to errors caused by favoring almost imperceptible differences in the two portions of the photometric field. In order to avoid all possibility of these errors the procedure was adopted of measuring the ratio $\lambda/(\lambda - \Delta\lambda)$ haphazard at numerous points in the spectrum, each measurement being performed so that a complete reversal of all parts took place. For $\Delta\lambda$ was chosen $.004\mu$. From the plot of these ratios a luminosity curve was built up, which, by its method of determination, minimized the possibility of disturbances such as those above considered. At the same time a flicker luminosity curve was obtained. These showed:

At high illuminations (30 m.c.) for a small field (size of fovea) the spectral luminosity curve derived by measurements of small steps of negligible hue difference agrees within the errors of measurement, with the curve obtained by the flicker photometer.

The conclusion drawn from this work is that the flicker photometer, under the conditions named, eliminates all disturbing subjective factors and indicates the true brightness of different colors. This confirms the conclusion drawn as extremely probable from the previous work with five different observers, whose mean equality of brightness curve agreed with their mean flicker curve, at high illuminations.

THE ADDITION OF LUMINOSITIES OF DIFFERENT COLOR.¹

BY HERBERT E. IVES.

AN indispensable characteristic of a method of measurement is that the sum of the measured values of the parts shall equal the measured value of the whole. In order for this to be so it is necessary that things measured equal to the same thing shall measure equal to each other. But this latter property is not alone sufficient, for the process of physical summation may introduce qualities not present in a manner to be measurable in the parts. If these properties are possessed then the ordinary arithmetical processes of addition, subtraction, multiplication and division may be confidently performed with the results of the measurements. In the measurement of most physical quantities and the ordinary photometry of lights of the same color no question of the validity of the measurements arises upon this score. The result of adding two illuminations may be obtained by adding their separately measured values. But in the photometry of lights of different colors, in view of the uncertainties of estimation, and in view of the different results given by

¹ Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

different methods it is not permissible to assume that luminosities may be dealt with as ordinary physical quantities.

Spectral luminosity curves have been obtained during the course of the writer's work on heterochromatic photometry, by four different methods, viz., equality of brightness, flicker, critical frequency, and visual acuity. A set of such curves simultaneously obtained, in which the reference standard was the light of an incandescent lamp, showed considerable differences in area. The visual acuity curve was five times the mean area of the others. They in turn differed twenty to thirty per cent. from each other. The large area of the visual acuity curves is due to the chromatic aberration of the eye, owing to which a monochromatic light is far more efficient for defining detail than is a complex light such as the "white" reference standard. Because of this large *quality factor* visual acuity is useless as a method of light measurement. The differences in the areas of the other curves are due to more obscure psychological causes.

Previous work having shown that the two most sensitive methods—equality of brightness and flicker—gave identical results when psychological elements are eliminated, the important test, from the present standpoint, was whether in these methods the arithmetical properties hold. Under conditions where the psychological elements are absent it has been found that brightnesses measured equal to the same measure equal to each other.

There remains to be tested the summational quality. Is there in these photometric methods any quality factor such as chromatic aberration in the case of visual acuity? The most satisfactory test consists in the measurement of the dispersed light of a source color by color against the undispersed light. The sum of the brightnesses of the colors should equal the brightness of the undispersed light. Such an experiment was made by Tufts some years ago, but the conditions of illumination, field size, etc., were far from those indicated as best by the writer's work.

A special slit on the spectrometer made it possible to obtain successively adjacent portions of the spectrum to be measured against the "white" comparison standard, by the flicker method. At the end of the measurements, the slit was opened until the whole spectrum fell upon the eye-slit, when both sides of the photometric field appeared of the same color, and a photometric match could be obtained by all methods of photometry with necessarily identical results. The experiment was carried out at an illumination of 300 illumination units, photometric field 2° diameter, with bright surroundings.

The result proved the physical and arithmetical summations identical.

It follows from this and the previous work reported before the Physical Society that in the flicker photometer we possess an instrument which under proper conditions offers a practical solution of the problem of heterochromatic photometry. There remains only to be determined the luminosity curve of the normal eye, by measurements on numerous individuals, in order that standard conditions for the photometry of different colored lights may be specified and made the subject of legislation.

With the completion of this work will be rendered possible the establishment of a logical primary standard of light, namely, a definite intensity of radiation of maximum luminous efficiency, in accordance with the proposals of Ives, Strache and Houstoun.

ACCURATE POTENTIOMETER METHOD FOR MEASURING RESISTANCE.¹

BY WALTER P. WHITE.

IN this method, the resistance is made part of a Wheatstone's bridge which is not changed during the observations. The lack of balance, which may be relatively large, is not allowed to send a current through a galvanometer, but is measured by putting a potentiometer in the place usually occupied by the galvanometer alone. The method is best suited for measuring resistance changes rather than absolute values. It is obviously more accurate than the ordinary potentiometer methods, since the electrical quantity to be measured is much smaller, and changes in the battery current produce far less effect. Its advantage over the simple Wheatstone bridge lies in the fact that no troublesome contact resistance changes occur during the measurements, and that the general advantages of the potentiometer can be secured along with some of the benefits of the Wheatstone bridge.

It does not avoid the difficulty from temperature change in the coils which occurs in Wheatstone bridge work of high precision, but it facilitates the use of some methods which diminish this difficulty also. For instance, in temperature measurement it facilitates the use of resistance thermometers with two (or four) coils in the bulb, half of them of wire of low temperature coefficient. The suggestion may be worth while that some of the disadvantages which appear to have prevented the use of thermometers of this type might be obviated if the metal of low temperature coefficient should be the alloy of silver and platinum, which would endure much the same heat treatment as the pure platinum and very much more than would be admissible with manganin.

GEOPHYSICAL LABORATORY,
CARNEGIE INSTITUTION OF WASHINGTON,
WASHINGTON, D. C.,
April 23, 1912.

ON SECONDARY IONIZATION PRODUCED BY THE IMPACT OF POSITIVE IONS ON SOLIDS.¹

BY O. W. RICHARDSON AND CHARLES SHEARD.

THE incentive to the present investigation was an observation recorded by one of the authors² to the effect that when a fresh metal wire is positively charged and heated in a vacuum the relation between current and electro-

¹ Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

² O. W. Richardson, *Phil. Mag.*, Vol. 6, p. 80, 1903.

motive force is approximately linear, although there are no negative ions present. Since one would expect saturation to occur with a quite small potential under these circumstances this phenomenon seemed to call for further examination. The experiments have been made with three forms of testing vessels, (1) a glass tube in which the electrodes were a loop of heated platinum wire and a platinum plate, (2) a brass cylinder with an axial platinum wire, (3) a brass cylinder with two parallel platinum wires which could be heated and tested separately. All the currents were very small and were measured by a delicate electrometer. The pressure recorded on the McLeod gauge was between the limits 0.00005 and 0.0002 mm. in all the experiments.

In every case when the apparatus was newly set up the relation between current and potential was approximately linear from +40 to +400 volts, although as a rule there was a slight convexity towards the voltage axis. This relation did not hold below 40 volts; saturation usually occurred below 5 volts and in some instances the current with 3 volts was distinctly greater than with 30 volts. The increase in the currents above 40 volts is evidently due to some new source of ionization being brought into play by the field and it is much too large to be attributable to ionization by collisions with the molecules of the residual gas.

The large additional current at high voltages was found to gradually disappear with continued usage of the tube if the platinum wire was kept heated and charged positively. It was also cut down very considerably if the tube was placed in a magnetic field. These effects were not observed if the negative emission from the heated wire was tested in a similar manner.

When the large additional current at high voltages had disappeared with usage of the tube it could be restored by the following processes:

1. Heating the positively charged wire to a higher temperature than that at which it had been tested previously.
2. Allowing a current of negative electrons to pass from the wire to the other electrode.
3. Allowing the apparatus to stand with air at atmospheric pressure in it. The effect of letting air in was found to give similar results whether the tube was scraped inside or not and it could not be removed by continued pumping unless a discharge of positive ions was allowed to pass to the electrode at the same time.

The results can all be explained on the hypothesis that the various agencies 1-3 deposit a layer of material on the opposite electrode which readily emits electrons when bombarded by the positive ions. The effect of the magnetic field shows that the secondary ions thus emitted are electrons and this conclusion is confirmed by the fact that similar effects are not observed when the primary discharge is negative instead of positive.

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J.

THE PHOTOELECTRIC EFFECT.¹

BY O. W. RICHARDSON AND KARL T. COMPTON.

THE writers have recently been engaged in an investigation of the magnitude and distribution of the total kinetic energy of the electrons emitted when light falls on metals, considered as a function of the frequency of the light and of the nature of the metal.

The light from a Heræus quartz mercury arc lamp after passing through a Hilger monochromatic ultra-violet illuminator fell on a small narrow strip of the metal to be investigated. This was placed at the center of an exhausted glass bulb provided with a quartz window. The inside of the bulb was silvered and was connected to the insulated quadrants of a delicate electrometer. The current which flowed into the electrometer against various opposing potentials was measured. As the field is everywhere radial these currents give the magnitude and distribution of the kinetic energy of the emitted electrons directly. The correction for the reflected electrons has not yet been determined, but is probably small with this arrangement, as the chance of a reflected electron being returned to the central electrode is small.

The experimental results may be analyzed and exhibited graphically by plotting the number of electrons having a given energy against the energy. These curves differ somewhat in form in different cases but they are nearly symmetrical about the axis of mean energy. The number also is a maximum near this value, so that the mean energy is the most probable value of the energy. Equal deviations from the mean have also nearly equal probabilities. The probability of an electron having an amount of energy between given limits changes very rapidly in the neighborhood of the maximum energy and also in the neighborhood of zero energy. The range of the energy of the electrons emitted is approximately a linear function of the frequency; *i. e.*, the maximum energy is a linear function of the frequency.

In a paper read before the meeting of the Society on March 2, 1912, one of the writers gave reasons for believing that the average kinetic energy \overline{T}_ν of the electrons emitted by the action of light of frequency ν might be expected to be capable of representation for all substances by a relation of the form

$$\overline{T}_\nu = \nu F\left(\frac{w_p}{\nu}\right), \quad (1)$$

where F denotes a universal function whose form depends on the energy distribution in the black-body spectrum and w_p is the energy required to set free a negative electron from the material denoted by the suffix p . The equation (1) is derived from the solution of two integral equations, one between equivalent expressions for the number of electrons emitted in a given time and the other between equivalent expressions for the energy lost in a given time. Ac-

¹ Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

cording to equation (1) the mean energy of the electrons emitted by light of frequency ν when it falls on different bodies is determined entirely by ν and a single parameter w_p characteristic of the body. Thus if the graph of \bar{T}_ν against ν were known for any one substance it could at once be predicted for any other, provided the appropriate value of w could be determined. Provided the electrons which participate in the photo-electric effect are in the same class as those which give rise to the thermionic emission the w 's are closely connected with the intrinsic potentials of the substances, the relation between them being, in fact, if the suffixes m and p refer to two different materials,

$$\frac{w_m}{e} - \frac{w_p}{e} = V_m - \theta \frac{\partial V_m}{\partial \theta} - \left(V_p - \theta \frac{\partial V_p}{\partial \theta} \right). \quad (2)$$

If, for the present, we disregard the terms depending on $\partial V/\partial \theta$ as small (this is legitimate since $(\partial/\partial \theta)(V_m - V_p)$ is equal to the thermo-electric power of a circuit consisting of the two metals) then the differences of w/e are equal to the contact differences of potential. Moreover the absolute value of w may be obtained, although perhaps only approximately at present, from the results of experiments on the variation with temperature of the thermionic emission and on the heat developed when electrons are made to condense into a metal. Such experiments show that the value of w/e for platinum is not far from 5 volts. We have therefore independent methods of obtaining both the absolute and relative values of w/e for different substances.

It can be shown that (1) may be written in the equivalent forms

$$\bar{T}_\nu = v\varphi_1 \left(\frac{v_p}{v} \right) \quad (3)$$

and

$$\lambda \bar{T}_\lambda = \varphi_2 \left(\frac{\lambda}{\lambda_p} \right), \quad (4)$$

where φ_1 and φ_2 are universal functions, λ is wave-length and $v_p = c/\lambda_p$ is a frequency which is proportional to w_p . According to Planck's radiation formula $w_p = hv_p$ where h is Planck's constant.

The experimental results have been compared with (4) by plotting $\lambda \bar{T}_\lambda$ as ordinates against λ/λ_p as abscissæ. The first metal tested showed that when $\lambda \bar{T}_\lambda$ was plotted against λ the curve was a straight line which intersected the axis of λ at a certain point. Calling this point λ_p the equation to the curve when plotted against λ/λ_p thus becomes $\lambda \bar{T}_\lambda = k(1 - \lambda/\lambda_p)$. The values of λ_p for the other substances were determined by making one of the experimental points fall on this line. When this was done all the other points were found to lie on the line in the case of the following metals: sodium, aluminium, magnesium, zinc, tin and platinum. In the case of sodium, however, we were not able to determine the average energy directly on account of photoelectric fatigue. The average was estimated from determinations of the maximum energy of the particles in this case. This procedure, however, gave pretty good

values when applied to the other substances. The behavior of bismuth and copper, which gave very small currents, appeared to be exceptional. The points for these two metals lay on a line through the origin having a different slope. The constant h for the metals Na, Al, Mg, Zn, Sn and Pt was $C \times 2.9 \times 10^{-27}$ whereas for Bi and Cu it appeared to be about $C \times 1.9 \times 10^{-27}$, where $C = 3 \times 10^{10}$ cm. per sec. The value of Planck's constant h is 6.55×10^{-27} in the same units ($1 = 1 \text{ erg} \times 1 \text{ sec.}$).

It will be seen that the values of λ_p which were used were obtained entirely from the photo-electric measurements. Assuming the truth of Planck's radiation formula values of λ_p may be calculated from the radiation constant h , the absolute value of w for any one metal and the contact differences of potential. Taking $w/e = 5$ volts for platinum the calculated values of λ_p are given in the first row of the following table and those from the photo-electric measurements in the second row.

Metal.	Na	Al	Mg	Zn	Sn	Bi	Cu	Pt
λ_p (calculated from w)	52.6	36.0	34.6	33.3	31.0	29.4	28.0	27.3
λ_p (photo-electric)	57.6	39.5	36.5	36.1	33.8	33.1	29.7	29.0

These numbers have been obtained quite independently, and no attempt has been made to adjust the data, so that the agreement shown in the table is really quite good. Values of λ_p were also obtained by trying to find the longest waves which would give rise to any photoelectric emission. These agreed with those given in the table. The agreement in the case of bismuth and copper is not in itself an argument either for or against the theory as the points for these substances did not fall on the general curve.

The values which we have found for the maximum energy of the emitted electrons T_{\max} have also been analyzed by plotting λT_{\max} against λ/λ_p . Although the points are a little irregular they all fall fairly near a line cutting the axis of abscissæ at $\lambda/\lambda_p = 1$, whose equation is $\lambda T_{\lambda} = ch(1 - \lambda/\lambda_p)$, where c is the velocity of light and h is Planck's constant. They could be better represented by a linear equation with a smaller value of h than that given by the radiation formula (about 10 per cent. lower). We do not, however, wish to emphasize this difference, pending further investigation, as we realize that the accurate measurement of the maximum energy is a rather difficult problem. So far as they go, the measurements of the maximum energy may be considered favorable to Einstein's¹ theory of the photo-electric effect combined with the hypotheiss that the difference in the work P for different substances is determined by the contact difference of potential.

If the laws which we have found to connect the frequency of the light with the maximum and mean energy of the liberated electrons hold up to the highest frequencies, it follows that the frequency ν of homogeneous Roentgen rays can be obtained from either of the equations

¹ Ann. der Physik., Vol. 17, p. 146, 1905.

$$v - v_p = \frac{\bar{T}}{2.9} \times 10^{27} = \frac{T_{\max}}{6} \times 10^{27},$$

where \bar{T} is the mean energy, and T_{\max} the maximum energy, of the electrons emitted when the Roentgen rays fall on a metal. For these high frequencies v_p can be neglected compared with v . Sadler¹ and Beattie² have shown that the energy of the electrons emitted when Roentgen rays fall on different substances is determined solely by the character of the Roentgen rays and is independent of the nature of the substance. This is in accordance with the requirements of the above formulæ.

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J.

HEATS OF DILUTION.³

BY WILLIAM FRANCIS MAGIE.

WHEN an aqueous solution containing a gram-molecule of an electrolyte is diluted by the addition of unit volume of the solvent the heat evolved (reckoned positive) or absorbed (reckoned negative) is the heat of dilution. It diminishes as the volume of the solution increases and changes remarkably with the temperature. A well-known thermodynamic argument shows that the rate of change of the heat of dilution with the temperature equals the negative rate of change of the heat capacity of the solution and the solvent with the volume of the solution, as it is increased by the transfer of solvent to the solution.

This theorem has been verified as closely as the simple method of observation used would permit, for solutions of the chlorides of sodium, potassium, ammonium, barium and strontium, within the range of concentrations from 4 to 0.5. The concentrations are expressed in an arbitrary unit by $100/N$, where N is the number of gram-molecules of water containing one gram-molecule of the solute. For the last three substances the heats of dilution are generally negative at the lower temperatures used (about 5° C.) and positive at the higher temperatures (about 24° C.). Observations have been made at temperatures indicated from these observations as the transition temperatures (at which there is no heat of dilution) for certain concentrations and the indications of the theory confirmed.

From the fact that the formula connecting the temperature change of the heat of dilution with the volume change of the heat capacity is verified for finite ranges of temperature, as well as from the direct observations of Teudt, it appears that the change of heat capacity with the volume is independent of temperature. This being assumed we have

¹ Nature, Vol. 81, p. 516, 1909; Phil. Mag., Vol. 19, p. 337, 1910.

² Camb. Phil. Proc., Vol. 15, p. 416, 1910.

³ Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

$$l = -a\theta + e,$$

in which l is the heat of dilution, a , the rate of change of heat capacity with the volume, θ , the absolute temperature, and e , a function of the volume but not of the temperature. The factor a is negative and proportional to the rate of change of the dissociation, measured by the electrolytic conductivity, with the volume. The first term is positive and corresponds to an evolution of heat. It may be explained, on the theory previously supported by the author, by supposing that when dissociation takes place, the water released from the molecules which dissociate has its heat capacity diminished, and the water associated with the newly formed ions has its heat capacity also diminished, so that the number of degrees of freedom of the water is diminished for both reasons, and the energy associated with the degrees of freedom which are thus removed from the solution is released in the solution in the form of heat. This energy, at these temperatures, is proportional to the absolute temperature. The term e is negative and corresponds to an absorption of heat. It is to a first approximation proportional to the rate of change of dissociation with the volume. When reduced to complete dissociation the heat absorbed is of the same order of magnitude as the heat of combination of the elements composing the solute. The term e may therefore contain, as a principal part of it, the heat of dissociation.

THE RELATION OF OSMOTIC PRESSURE TO TEMPERATURE.¹

BY WILLIAM FRANCIS MAGIE.

IF we represent by p the osmotic pressure, by θ the absolute temperature, and by a the rate of change of heat capacity of an aqueous solution of an electrolyte with the volume (see the previous paper) we may write the following relation

$$\frac{d^2p}{d\theta^2} = \frac{a}{\theta}.$$

Assuming that a is independent of the temperature we obtain by integration

$$p = a\theta(\log \theta - 1) + b\theta + e,$$

in which a , b , e are functions of the volume but not of the temperature.

By a simple thermodynamic argument we can deduce from this the following expression for the heat of dilution,

$$l = -a\theta + e.$$

Now a is known from observations on the specific heats of solutions, e can therefore be determined from the observed heats of dilution, and b from values of the osmotic pressure calculated from observations of the depression of the freezing point. The formula can then be tested by calculating with it the osmotic pressure at the boiling point and comparing the result with the osmotic

¹ Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

pressure calculated from observations of the elevation of the boiling point. The only data at present available are those for sodium chloride. The observations of Thomsen furnish the heat capacities, those of the author, the heats of dilution, those of Kahlenberg, the freezing and boiling points. This example confirms the formula very precisely for solutions containing one gram-molecule of salt to 50, 100, and 200 gram-molecules of water.

Furthermore we can calculate from the formula for osmotic pressure, when the constants are known, the ratio of the vapor pressure over the pure solvent to that over a solution of a given strength at different temperatures. According to von Babo's law this ratio should be constant. From the theoretical formula for this ratio, its constancy at different temperatures is immediately evident if we assume, as has commonly been done, that the osmotic pressure is directly proportional to the absolute temperature. With the more complicated relation of osmotic pressure to temperature expressed in the proposed formula, von Babo's law can only be tested by direct calculation for each solute. In the case of sodium chloride, for which the constants of the formula have been obtained, it turns out that the law is verified, the departures of the ratios for different temperatures between 0° and 100° C. from the mean being never so great as those which would arise from errors of observation.

The form of the formula for osmotic pressure is the same as that for the negative rate of change of the free energy with the volume (equal to the pressure) of a gas which conforms to van der Waals' equation.

The constants of the formula evidently depend upon the interactions between the parts of the solute and the solvent. The formula for the osmotic pressure is consistent with the view that the osmotic pressure is not a purely kinetic pressure, but is due to forces acting between the parts of the solute and the solvent.

THE LAW OF FALL OF A DROP THROUGH AIR AT REDUCED PRESSURES AND A REDETERMINATION OF e .¹

BY R. A. MILLIKAN.

IN work previously reported² it was shown for the first time that Stokes' Law of fall was insufficient to account for the motions of small drops through air and a corrected law of the form

$$X = 6\pi\mu av \left(1 + A \frac{l}{a} \right)^{-1} \quad (1)$$

was proposed and shown to hold in air at atmospheric pressure for large variations in a . By observing the motions of oil drops in gravitational and electrical fields, this law has now been tested for pressures between 5 mm. and 760 mm. and found to hold accurately so long as $l/a < .4$, *i. e.*, so long as

¹ Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

the radius of the drop is more than twice the mean free path of the gas molecule. When $l/a > .4$ the above equation fails and must be replaced by one containing higher powers of l/a . The value of A in (1) has now been found with a much greater precision than had before been possible. It is found to be .848 instead of .817, the value previously reported. This coupled with a redetermination of all of the factors involved in e has made it possible to obtain a value of this constant the probable error of which is about .1 per cent. The new value of e is 4.810×10^{-10} .

VARIATION OF ELECTRICAL RESISTANCE WITH TEMPERATURE.

V. — OXIDES.¹

By A. A. SOMERVILLE.

A CONTINUATION of work the beginning of which was reported at the New York meeting of the American Physical Society, held March 2, 1912. The resistance of oxides is determined by placing the powder in a porcelain tube, inserting nickel terminals into the tube, putting the whole thing into a tubular resistance furnace and measuring the resistance between the nickel terminals with a wheatstone bridge, when the furnace is at any temperature from room temperature to 1100° C.

The materials that have been studied are—

Zinc oxide.....	ZnO
Ferric oxide.....	Fe ₂ O ₃
Cupric oxide.....	CuO
Cuprous oxide.....	Cu ₂ O
Magnesium oxide.....	MgO
Manganese dioxide.....	MnO ₂
Aluminum oxide.....	Al ₂ O ₃

All are insulators under ordinary conditions, but the resistance decreases as temperature increases until most of them at 1100° C. are fairly good electrical conductors. The general form of the temperature-resistance curves is the same, being something like that of a parabola, but the actual value of the resistance of various materials at the same temperature differs widely.

CORNELL UNIVERSITY,
ITHACA, N. Y.

¹ Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

NEW BOOKS.

- Transactions of the International Union for Coöperation in Solar Research.* Manchester: The University Press, 1911. Vol. I., pp. viii + 231. Price, \$2.50; Vol. II., pp. viii + 244. Price, \$2.50; Vol. III., pp. xii + 257. Price, \$2.50
- Applied Physics for Secondary Schools.* By V. D. HAWKINS. New York: Longmans, Green and Co., 1912. Pp. ix + 199.
- An Introduction to General Thermodynamics.* By HENRY A. PERKINS. New York: John Wiley and Sons, 1912. Pp. xix + 247. Price, \$1.50.
- Introduction to Analytical Mechanics.* By A. ZIWET AND P. FIELD. New York: The Macmillan Company, 1912. Pp. ix + 378. Price, \$1.60.
- Lectures delivered at the Celebration of the Twentieth Anniversary of the Foundation of Clark University.* By VITO VOLTERRA, ERNEST RUTHERFORD, ROBERT WILLIAMS WOOD and CARL BARUS. New York: G. E. Stechert and Co., 1912. Pp. v + 161. Price, \$2.00.
- Über die Gesetze der Wärmestrahlung.* By W. WIEN. Leipzig: Johann Ambrosius Barth, 1912. Pp. i + 21. Price, Mk. 1.
- Die Resonanztheorie des Hörens.* By ERICH WAETZMANN. Braunschweig: Vieweg and Sohn, 1912. Pp. xii + 162. Price, Mk. 5.
- Elementare Mechanik.* By GEORGE HAMEL. Leipzig: Teubner, 1912. Pp. xvii + 634. Price, Mk. 16.

THE PHYSICAL REVIEW.

CONVECTION AND CONDUCTION OF HEAT IN GASES.

BY IRVING LANGMUIR.

PART I. HISTORICAL.

THE loss of heat by convection from a heated body has apparently always been looked upon as a phenomenon essentially so complicated that a true knowledge of its laws seemed nearly impossible. A. Oberbeck¹ gives the general differential equations for this problem but finds it impossible to solve them for actual cases. L. Lorenz² for the case of vertically placed plane surfaces is able to obtain some approximate solutions which agreed fairly well with some of the older experimental work. But as an illustration of the view taken even recently by one familiar with practically all the literature I might quote from a paper by A. Russell:³ "The phenomenon of the convection of heat at the surface of a body immersed in a cooling fluid is one which does not lend itself readily to mathematical calculation. If the fluid be a gas the variations of the pressure, density, and velocity at different points of the gas so complicate the problem that little progress towards a complete solution has yet been made."

In his own paper Russell then feels compelled to make the following simplifying assumptions in dealing with this problem. "The liquid is supposed to be opaque to heat rays. It is also supposed to have *no viscosity* (*italics mine*). The liquid therefore slips past the surface of the solid. In addition it is supposed to be incompressible. Hence we should only expect the solutions to give roughly approximate values when applied to the problems of spheres and cylinders being cooled by currents of air."

¹ Ann. Phys., 7, 271 (1879).

² Ann. Phys., 13, 582 (1881).

³ Phil. Mag., 20, 591 (1910).

Kennelly¹ who made very elaborate measurements of the "Convection of Heat from Small Copper Wires" also finds the theory involved very complicated and is satisfied to derive empirical laws to express his results. He says "The lateral conduction through the air is negligible because the air does not remain at rest but expands and flows convectively. Consequently we may safely ignore conductive thermal loss." "Convection loss from the wire is a hydrodynamic phenomenon, involving the flow of air past the surface of the wire, and the amount of heat which this moving stream can carry off. Very little seems to be known quantitatively about convection."

Such views as these are not conducive to the finding of simple laws if such exist.

The writer has long felt that the assumption that the effect of the viscosity of the gas is negligible is unwarranted. In the case of convection from small wires it has seemed rather that it is one of the most essential of the factors involved. The writer's views on this were given in his thesis on some reactions around glowing Nernst filaments,² from which the following extracts are taken.

"In the case of electrically heated glowing filaments the rate of loss of energy is equal to the watts input. If, as is often the case, the radiation loss may be calculated, this may be subtracted and one thus obtains the energy lost by convection and heat conduction through the gas. Now according to the kinetic theory the viscosity of a gas increases with the square root of the absolute temperature; the driving force of the convection being proportional to the difference of density between the hot and cold gas, increases only very slowly with increasing temperature. Therefore in the immediate neighborhood of the filament the flow of gas is small and the heat must be carried away practically only by conduction."

"It would seem, however, as though heat conduction alone would come into account up to a distance of about 0.2 mm. from the center of the wire. It is highly probable that at very high temperatures, for example 2200°, the motion of the gas in the immediate neighborhood of the wire would not perceptibly increase but probably decrease, while at the same time the heat conductivity of the gas would increase very greatly. (For example the heat conductivity, k , at 2300° K. is 27×10^{-5} while at 273° K. it is only 4.7×10^{-5} .) Thus even at a distance from the wire where the motion of the gas is considerable, the conduction will be more important than the convection."

¹ Trans. Amer. Inst. E. E., 28, 363 (1909).

² Über partielle Wiedervereinigung dissocierten Gase im Verlauf einer Abkühlung. Inaugural Dissertation, Göttingen, 1906.

"Therefore up to a distance of a few tenths of a millimeter from the glowing body one may consider the heat to be carried only by radiation and *conduction*."

"If W is the rate of energy loss per cm. of length of the wire and k the coefficient of heat conductivity, a the radius of the wire and T_0 its temp., then for the temp. T at a distance r from the axis of the wire the relation holds:"

$$(1) \quad T_0 - T = \frac{W}{2\pi k} \ln \frac{r}{a}.$$

Some later experiments by the author in this research laboratory¹ led to some very interesting results as to the heat losses in hydrogen from tungsten wires at very high temperatures. In connection with this work it was highly desirable to know the laws of heat "convection" more definitely, so the work described in the present paper was undertaken to test out the theory advanced in the above mentioned dissertation and, if the results should warrant it, to develop the theory further and give it more definite form. Several considerations had made it seem probable that the above theory would be fairly close to the truth. For example it had been noticed that the watts loss from a wire was very nearly independent of the position of the wire, that is, whether it were placed vertically or horizontally. Now the lines of flow of the heated air around the wire would be totally dissimilar in these two cases. Yet it was found that the energy necessary to maintain a piece of pure platinum wire at any given temperature (resistance kept constant) never differed by more than 6-8 per cent. for the vertical and the horizontal wire and at a bright red heat or above the difference became negligibly small. This was strong indication that the heat loss was dependent practically only on heat conduction very close to the filament and that the convection currents had practically no effect except to carry the heat away after it passed out through the film of adhering gas.

The thickness of the film of gas through which the conduction takes place can be calculated from equation (1) if the temp. of the wire, its diameter and the heat conductivity are known. This last quantity however varies considerably with the temp. and there is little data available on the heat conductivities of gases at very high temperatures.

PART II. THEORETICAL. HEAT CONDUCTIVITY OF GASES AT HIGH TEMPERATURES.

The literature on the heat conductivity of gases is relatively meager compared with the wealth of material on the viscosity of gas. For-

¹ Trans. Amer. Electrochem. Soc., 20, p. 335 (1911).

tunately the kinetic theory furnishes us a means of calculating the heat conductivities from the viscosities.

Meyer¹ gives the relation:

$$k = 1.603hc_v,$$

k = heat conductivity, h = viscosity, c_v = specific heat (per gram) at constant volume.

Eucken in a recent paper² shows that the constant (which will be denoted by K) for the monatomic gases, helium and argon, is 2.50 instead of 1.603 and that for diatomic gases, H_2 , O_2 , N_2 , and air it is 1.90. He also shows that K is independent of the temperature over a wide range.

In view of the serious difficulties involved in measurements of heat conductivities of gases and the ease and accuracy with which the viscosities can be measured it seems highly probable that the heat conductivities calculated in this way are much more reliable than those measured directly.

Viscosity of Gases.

The variation of the viscosity of gases with the temperature has been the subject of many careful researches in the last few years. In every case Sutherland's formula seems to agree within the experimental error with the results, at least in all cases of gases above their critical temperatures.

Sutherland's formula may be written

$$(2) \quad h = \frac{KT^{\frac{1}{2}}}{1 + \frac{C}{T}}.$$

The values of K and C for H_2 , air and Hg are:

Gas.	$10^6 K$	C	Observer.
H_2	6.6	77	Fisher, PHYS. REV., 24, 385 (1907).
Air.....	15.0	124	Fisher, PHYS. REV., 29, 106 (1909).
Hg.....	65.0	960	See below.

The viscosity of mercury vapor has been determined by Koch³ and his results were confirmed by Noyes and Goodwin.⁴ These results do not agree with Sutherland's formula, probably partly because of errors made at low temperatures and partly because the temperature of observation was much below the critical temp. of mercury. Koch gives for the

¹ Kinetic Theory of Gases.

² Physik. Zeitschr., 12, 1101 (1911).

³ Ann. Phys., 10, 857 (1883).

⁴ PHYS. REV., 4, 207 (1896).

highest temperature at which he made measurements:

$$\text{at } 380^{\circ} \text{ C. } 10^3 h = 0.654,$$

or extrapolating from his results:

$$\text{at } 420^{\circ} \text{ C. } 10^3 h = 0.718.$$

This last result is probably far enough above the boiling point so that the viscosity is nearly normal and for temperatures above this, Sutherland's formula would probably give the correct values. A. O. Rankine¹ shows that

$$C = \frac{T_c}{1.15},$$

where T_c is the critical temp. As no data are available for the critical temp. of mercury it may be roughly calculated. The ratio of the boiling points (in $^{\circ} \text{K.}$) and critical temps. for most liquids is nearly constant, about 1:1.7. Thus the critical temp. of mercury would be about 1100°K. Hence $C = 1/1.15 \times 1100^{\circ} = 960$. From this and the viscosity, the value of K may be easily calculated. $K = 65.0 \times 10^{-6}$. This should be considered only a rough approximation.

Specific Heats.—The most reliable data seem to be those of M. Pier² who gives for the actual molecular specific heat (constant volume) at the temp. T (absolute):

$$\begin{aligned} \text{For H}_2 \quad c_v &= 4.454 + .0009T, \\ \text{air} \quad c_v &= 4.654 + .0009T, \\ \text{Hg} \quad c_v &= 2.98. \end{aligned}$$

For the calculation of the heat conductivity we need the specific heats per gram. Assuming the "molecular weight" of air to be 28.8 we get

$$\begin{aligned} \text{For H}_2 \quad c_v &= 2.21(1 + .0002T), \\ \text{air} \quad c_v &= 0.1614(1 + .0002T), \\ \text{Hg} \quad c_v &= 0.0149. \end{aligned}$$

Heat Conductivities.—For hydrogen and air $k = 1.90hc_v$.

For mercury vapor at high temperatures we may safely assume $k = 2.5hc_v$ although Schleiermacher³ found experimentally a value of 3.15 for the constant at a temp. of 203°C.

¹ Proc. Roy. Soc. London, A 84, 181-92 (1910).

² Z. f. Electrochem., 15, 536 (1909), and 16, 899 (1910).

³ Ann. phys., 36, 346 (1889).

We thus obtain from the data for h and c_v

$$(3) \quad \text{For H}_2 \quad k = 28 \times 10^{-6} \sqrt{T} \cdot \frac{1 + .0002T}{1 + \frac{77}{T}},$$

$$(4) \quad \text{air} \quad k = 4.6 \times 10^{-6} \sqrt{T} \cdot \frac{1 + .0002T}{1 + \frac{124}{T}},$$

$$(5) \quad \text{Hg} \quad k = 2.4 \times 10^{-6} \sqrt{T} \cdot \frac{1}{1 + \frac{960}{T}}.$$

These equations should hold especially well at very high temperatures, when the gases are far above their critical temperatures.

But in the problem of the convection of heat from a hot wire, the difference in temperature between the wire and the atmosphere around it is often so great that we cannot consider the heat conductivity as being constant. We shall need to take into account the variation of the heat conductivity in the different layers of hot gas around the wire.

In any problem in heat conduction where steady conditions prevail we may write:

$$(6) \quad \frac{dq}{ds} = k \frac{dT}{dx},$$

where dq = heat flowing per second through the area ds , k = heat conductivity, T = temperature, x = distance measured perpendicular to the surface ds .

If the heat flux is uniformly distributed over the whole surface s then we may separate the variables and integrate the equation as follows:

$$(7) \quad \int k dT = q \int \frac{dx}{s},$$

where k is a function of T only, and s is a function of x only.

If we measure the rate of loss of heat in watts (W) we have:

$$(8) \quad W = \frac{4.19 \int k dT}{\int \frac{dx}{s}}.$$

Cylindrical Wires.—Let us consider a wire of diameter a surrounded by a cylindrical film of gas of a diameter b . Let T_2 be the temperature of the wire and T_1 the temperature of the gas at the outer surface of the

film, *i. e.*, at a distance $\frac{1}{2}b$ from the center of the wire. Then, if l is the length of the wire,

$$(9) \quad \int \frac{dx}{s} = \frac{1}{2\pi l} \ln \frac{b}{a}.$$

If W be the watts of heat energy conducted away from the wire *per unit* of length then

$$(10) \quad W = \frac{4.19 \times 2\pi}{\ln \frac{b}{a}} \int_{T_1}^{T_2} k dT.$$

For convenience place

$$(11) \quad \varphi = 4.19 \int_0^T k dT,$$

and place

$$(11a) \quad s = \frac{2\pi}{\ln \frac{b}{a}}.$$

Whence

$$(12) \quad W = s(\varphi_2 - \varphi_1).$$

Plane Surface.—Consider a plane surface of area s with an adhering film of gas of the thickness B . Equation (8) then becomes

$$(13) \quad W = 4.19 \frac{s}{B} \int_{T_1}^{T_2} k dT = \frac{s}{B} (\varphi_2 - \varphi_1).$$

The function φ can be readily calculated and plotted as a function of T . For any gas the heat conductivity k can be put in the form

$$(14) \quad k = A(1 + \alpha T) \frac{T^{\frac{1}{2}}}{1 + \frac{T}{C}};$$

as a very close approximation we have

$$(15) \quad \varphi = 4.19A(1 + 0.6\alpha T) \int_0^T \frac{T^{\frac{1}{2}} dT}{1 + \frac{T}{C}},$$

and (accurately)

$$(16) \quad \int_0^T \frac{T^{\frac{1}{2}} dT}{1 + \frac{T}{C}} = \frac{2}{3} T^{\frac{3}{2}} - 2CT^{\frac{1}{2}} + 2C^{\frac{3}{2}} \tan^{-1} \sqrt{\frac{T}{C}}.$$

The values of φ for air, hydrogen and mercury vapor from 0° K. up

to high temperatures have been calculated (by slide rule) and are given in the following table:

TABLE I.

Table of φ , in Watts per Cm., as Function of Absolute Temp. ($^{\circ}$ K.).

$^{\circ}$ K.	Hydrogen.	Air.	Mercury Vapor.
0 $^{\circ}$	0.0000	0.0000	
100 $^{\circ}$.0329	.0041	
200 $^{\circ}$.1294	.0168	
300 $^{\circ}$.278	.0387	
400 $^{\circ}$.470	.0669	
500 $^{\circ}$.700	.1017	0.0165
700 $^{\circ}$	1.261	.189	.0356
900 $^{\circ}$	1.961	.297	.0621
1100 $^{\circ}$	2.787	.426	.0941
1300 $^{\circ}$	3.726	.576	.1333
1500 $^{\circ}$	4.787	.744	.1783
1700 $^{\circ}$	5.945	.931	.228
1900 $^{\circ}$	7.255	1.138	.284
2100 $^{\circ}$	8.655	1.363	.345
2300 $^{\circ}$	10.18	1.608	.411
2500 $^{\circ}$	11.82	1.871	.481
2700 $^{\circ}$	13.56		.556
2900 $^{\circ}$	15.54		.636
3100 $^{\circ}$	17.42		.719
3300 $^{\circ}$	19.50		.807
3500 $^{\circ}$	21.79		.898

Theory of Conducting Film.

Let us assume that the viscosity of the gas causes the heat to flow from a hot wire as though there were around the wire a stationary cylindrical film of gas (of diameter b) through which heat is carried only by conduction.

If we know the watts lost by a wire per cm. of length we are now in a position to calculate the diameter b of this film.

It is to be expected that b will vary with the diameter of the wire. Mr. E. Q. Adams of this laboratory has derived a relation between b and the diameter of the wire, a , which has been well verified by the experimental results. Mr. Adams' derivation is:

$$\text{Derivation of } b \ln \frac{b}{a} = 2B.$$

"The effective thickness of the film of air near a plate or wire is the distance the heat must travel before the heat flux due to temperature difference becomes negligible compared with that due to convection.

"At constant pressure the temperature and the temperature gradient at the outside surface of the film are assumed to be independent of the diameter of the wire.

"Consider now the analogy between the cases of heat conduction from a wire and from a plane under similar conditions of temperature, etc.

"Let r = the distance of any isotherm from the axis of the wire.

"And x = the distance of the corresponding isotherm from the plane.

"While r varies from $a/2$ to $b/2$, x varies from 0 to B .

"Since the temperature gradient at the surface of the film is assumed independent of the radius, at this point:

$$dr = dx.$$

"Elsewhere, since *within* the film convection is considered to be negligible, the total heat flux is constant, and since the conducting area is proportional to r and the heat conductivity *at the same temperature* independent of it, the temperature gradient is inversely proportional to r .

"Whence:

$$(18) \quad dr = \frac{r}{b} dx.$$

"Since the comparison is between points at the same temperature only, the temperature coefficient of heat conductivity does not enter at all.

"Integrating between limits:

$$\ln \frac{b}{a} = \frac{2B}{b}.$$

"Multiplying by b :

$$(19) \quad b \ln \frac{b}{a} = 2B.$$

Calculation of the Energy Loss from Cylindrical Wires.

From (11a)

$$\ln \frac{b}{a} = \frac{2\pi}{s}.$$

Substituting in (19)

$$(20) \quad \frac{2\pi b}{s} = 2B$$

or

$$\frac{b}{a} = \frac{s}{\pi \frac{a}{B}}$$

But from (11a)

$$\frac{b}{a} = \epsilon^{\frac{2\pi}{s}}.$$

Whence

$$(21) \quad \frac{s}{\pi \frac{a}{B}} = \epsilon^{\frac{2\pi}{s}}$$

or

$$(22) \quad \frac{a}{B} = \frac{s}{\pi} \epsilon^{-\frac{2\pi}{s}}.$$

From this a curve may be drawn giving a/B as a function of s .

The following table gives values of s and a/B from which such a curve may be plotted (slide rule calculation).

TABLE II.

s	a/B	s	a/B	s	a/B	s	a/B
0.0	0.0	5.0	.453	10	1.696	30	7.738
0.5	0.735×10^{-6}	5.5	.558	12	2.263	32	8.370
1.0	0.594×10^{-3}	6.0	.671	14	2.844	34	8.995
1.5	0.725×10^{-2}	6.5	.788	16	3.438	36	9.622
2.0	2.752×10^{-2}	7.0	.908	18	4.040	38	10.25
2.5	.0644	7.5	1.032	20	4.645	40	10.87
3.0	.1176	8.0	1.160	22	5.263	42	11.50
3.5	.185	8.5	1.291	24	5.877	44	12.14
4.0	.265	9.0	1.424	26	6.505	46	12.77
4.5	.354	9.5	1.561	28	7.122	48	13.40
5.0	.453	10.0	1.696	30	7.738	50	14.03

If we know the value of B for any gas, that is, if we know the thickness of the film of gas in the case of a plane surface then we can very easily calculate the watts loss per unit of length from a wire of any diameter. The calculation is as follows:

Given.— B , the thickness of film for plane surface; a , the diameter of the wire.

Method.—1. Calculate a/B . 2. Look up the corresponding value of s from the above table (or on curve). 3. Look up the values of ϕ corresponding to the temperature of the wire and to the temperature of the gas some distance from the wire.

Result.—Then W , the watts lost per cm. of length of the wire, will be

$$W = s(\varphi_2 - \varphi_1).$$

Variation of B with the Pressure, Temperature and Nature of the Gas.

Although the effect of these factors on b , the thickness of the film around a cylindrical wire, would be complicated and difficult to foresee, yet it would seem probable that B , the thickness of the film for a plane surface, would vary in some simple way. The most natural assumption seems to be that B would be proportional to the viscosity of the gas and inversely proportional to its density. For it is the viscosity that causes the existence of the film and it is the difference of density between hot and cold gas (proportional to the density itself) that keeps the film from becoming indefinitely large.

PART III. EXPERIMENTAL.

Calibration of Platinum Wire.

Twenty feet of platinum wire, .020" in diameter, was especially prepared for us by J. Bishop from the purest platinum.

We specified that it should have a temperature coefficient of electrical resistance of .0038 (from 0°–100° C.) but actually we found it to have only .00350. The purest platinum obtainable from Hereaus has a temperature coefficient of .0039. Nevertheless, we decided that this platinum would fill our needs.

Part of this wire was drawn down, through diamond dies, to the following sizes:

.010, .005, .0027 and .0016 inches.

The wires were annealed and the resistance of three of them was determined at the temperatures 0°, 100° and 445°, and the constants of Callendar's formula were calculated and found to be

$$\begin{aligned}\alpha &= .00350, \\ \delta &= 1.720.\end{aligned}$$

Slight differences were observed between the different wires, but as no very great accuracy was sought it was assumed that the above constants would give the resistance of all of the wires.

In a previous paper¹ the author has shown that above a temperature of about 1100° C. the resistance of platinum no longer follows the parabola of Callendar, but is practically linear. The ratio between the hot resistance and the resistance at 0° C. was calculated from the parabolic

¹ J. Am. Chem. Soc., 28, 1357 (1906).

formula up to 1300° K., and then continued as a straight line. A few points from the curve obtained are tabulated below.

Ratio of Resistance at T° to Resistance at 273° K.

<i>T</i>	273	473	673	873	1073	1273	1500	1700	1900
<i>R/R</i> ₀	1.000	1.688	2.328	2.919	3.463	3.958	4.742	5.273	5.804

The accuracy of this calibration was such that the errors in the temperatures undoubtedly do not exceed 20° at 1300° K. and perhaps 50° at the melting point of platinum.

TABLE III.

Wires Used for Experiments.

The data for these wires are tabulated below.

Wire.	Diameter Inches.	Diameter Cm.	Resistance per Cm. at ° C. Ω/Cm.	Specific Resistance Cm. Cube. Microhms.	Temp. Coef. α, from 0° C.	δ, Callendar Equation.
I.	.00159	.00404	.882	11.26	.0035	1.72
II.	.00272	.00691	.2878	10.80	"	"
III.	.00497	.01262	.0878	11.00	"	"
IV.	.00987	.02508	.0218	10.77	"	"
V.	.02004	.0510	.00572	11.62	"	"

The diameters were found by weighing measured lengths of wire on a sensitive balance, and assuming the density of the platinum to be 21.48. These results agreed well with measurements with a micrometer.

Free Convection from Horizontal Platinum Wires in Air.

A piece of the wire about 40–50 cm. long was held horizontally between clamps in a wooden box open at the side. It was found that very steady readings could be obtained if the wire was merely protected from draughts by placing a few large screens around it. For convenience a box was used. It was about 1 meter long, 30 cm. high and 15 cm. deep. The wire was placed about 10 cm. from the top.

Direct current from a 125-volt line was passed through the wire. The current was measured by a calibrated ammeter. The voltage was measured with a voltmeter, connected to fine platinum wire leads which were welded to the hot wire at points far enough from its ends to avoid the cooling action of the latter.

From the volts and amperes the watts per cm. of length and the resistance were calculated. The cold resistance was measured by a Wheatstone bridge. The resistance at 0° C. was calculated, and the

TABLE IV.

Sample Series of Observations on Free Convection from Pt Wire in Air.

Wire No. II. Diam. .00691 cm. Total length 43.9 cm. Length between voltmeter lead 37.32 cm. Wire horizontal. Room temperature 300° K.

Volts.	Amperes.	Watts Cm.	Resist. $\frac{R}{}$ at ° C.	Temp. ° K.
1.59	0.131	.00356	1.13	308
2.72	0.217	.0158	1.16	320
3.92	0.288	.0302	1.27	350
5.00	0.34	.0429	1.37	380
6.80	0.415	.0758	1.53	425
9.40	0.499	.1257	1.75	490
14.35	0.615	.2305	2.17	620
18.9	0.679	.344	2.59	760
25.8	0.771	.534	3.12	945
28.1	0.800	.603	3.28	1010
33.9	0.861	.788	3.66	1155
38.8	0.915	.952	3.95	1275
42.9	0.958	1.12	4.18	1370
45.1	0.98	1.183	4.30	1420
47.2	1.00	1.265	4.40	1460
69.6	1.22	2.275	5.31	1850
72.5	1.235	2.40	5.48	1920 ¹

TABLE V.

Wire No.	Diam. Cm.	Length Cm.	Volts.	Amps.	Watts Cm.	Resist. $\frac{R}{}$ at ° C.	Temp. ° K.	Remarks.
I.	.00404	33.94	12.08	0.25	.089	1.62	450	Burnt out.
			47.9	0.45	.635	3.56	1112	
			70.0	0.53	1.082	4.41	1470	
			102.0	0.63	1.892	5.40	1890	
II.	.00691	—	—	—	given	above	—	
III.	.01262	36.5	5.52	0.95	.144	1.820	512	Burnt out.
			18.1	1.60	.794	3.54	1105	
			28.5	1.95	1.524	4.56	1530	
			47.5	2.62	3.41	5.65	1995	
IV.	.02508	37.45	3.60	2.30	.221	1.92	545	Burnt out at thin spot.
			11.12	3.74	1.11	3.64	1145	
			14.5	4.24	1.64	4.19	1375	
			19.1	4.85	2.48	4.84	1650	
			23.7	5.42	3.44	5.35	1868	
V.	.0510	44.5	2.19	4.75	.234	1.815	510	Not burnt out.
			6.83	8.00	1.227	3.31	1020	
			11.99	10.60	2.85	4.45	1485	
			17.70	13.00	5.175	5.35	1868	

¹ Burnt out.

ratio of the hot resistance to that at 0° C. was calculated and from this the temperature was determined from the calibration curve obtained as described above.

About the same number of observations were made with each of the other wires. A few only of these, taken at random, are given in the following table:

All the observed values of watts/cm. (87 observations) were plotted against temperature and *smooth* curves drawn as nearly as possible through the points. With two exceptions the maximum deviation of the observed watts/cm. from that of the curve was 3 per cent. and in most cases the deviation was less than 1 per cent. So it is evident that the convection currents were steady and that draughts of air were not influencing the results.

The following data were taken from the smoothed curves as obtained above.

TABLE VI.

Total Energy Losses from Horizontal Platinum Wires in Air (300° K.) in Watts per Cm.

Wire No.	Diam. Cm.	Temp. ° K.							
		500	700	900	1100	1300	1500	1700	1900
I.	.00404	0.11	0.24	0.41	0.61	0.84	1.14	1.54	2.13
II.	.00691	0.12	0.29	0.48	0.72	0.99	1.33	1.79	2.48
III.	.01262	0.13	0.31	0.53	0.79	1.11	1.46	1.95	2.71
IV.	.02508	0.17	0.39	0.68	1.02	1.45	2.00	2.68	3.55
V.	.0510	0.22	0.52	0.90	1.42	2.03	2.89	4.10	5.65

Radiation from Platinum.

The total radiation per sq. cm. of surface from a black body at temperature T is

$$5.32 \left(\frac{T}{1000} \right)^4 \text{ watts.}$$

Or, the radiation from a wire of a cm. in diameter is

$$16.7a \left(\frac{T}{1000} \right)^4 \text{ watts per cm.}$$

Now platinum is far from being a black body. Lummer and Kurlbaum¹ have determined the ratio between the radiation from platinum and

¹ Verh. Phys. Ges., Berlin, 17, 106 (1898).

that of a black body and found

Temp. ° K.	Ratio Pt : Black Body. Per Cent.
492	3.9
654	6.0
795	7.5
1108	11.2
1481	15.4
1761	18.0

From these data a curve was plotted and the radiation from the platinum wire used in these experiments was calculated as follows:

TABLE VII.

Energy Radiated from Platinum Wires in Watts/Cm.

Wire No.	Diam. Cm.	Temp. ° K.							
		500	700	900	1100	1300	1500	1700	1900
I.	.00404	.000	.001	.004	0.011	0.026	0.05	0.10	0.17
II.	.00691	.000	.002	.007	0.019	0.044	0.09	0.17	0.29
III.	.01262	.001	.003	.012	0.034	0.080	0.17	0.31	0.53
IV.	.02508	.001	.007	.024	0.067	0.159	0.33	0.62	1.06
V.	.0510	.002	.013	.049	0.137	0.323	0.67	1.25	2.15

Subtracting these corrections from the total watts lost in air we get

TABLE VIII.

Energy Conducted from Platinum Wires by Air, or "Convection" Losses in Watts/Cm.

Wire No.	Diam. Cm.	Temp. ° K.							
		500	700	900	1100	1300	1500	1700	1900
I.	.00404	0.11	0.24	0.41	0.60	0.81	1.09	1.44	1.96
II.	.00691	0.12	0.29	0.47	0.70	0.95	1.24	1.62	2.19
III.	.01262	0.13	0.31	0.52	0.75	1.03	1.29	1.64	2.18
IV.	.02508	0.17	0.38	0.66	0.95	1.29	1.67	2.06	2.49
V.	.0510	0.22	0.51	0.85	1.28	1.71	2.22	2.85	3.50

From these data the thickness of the film of air, B , for a plane surface was calculated as follows:

$$s = \frac{W}{\varphi_2 - \varphi_1}$$

(φ_1 being taken at 300°).

Then from a curve giving the relation between s and a/B (equation 22) the corresponding value of a/B was found. From this B was calculated.

TABLE IX.

Thickness of Air Film for Plane Surface Calculated from Table VIII.

Wire No.	Diam. Cm.	Temp. ° K.								Mean.
		500	700	900	1100	1300	1500	1700	1900	
I.	.00404	.27	.43	.43	.47	.56	.47	.40	.25	.41
II.	.00691	.31	.32	.38	.40	.43	.47	.38	.26	.37
III.	.01262	.42	.42	.44	.49	.56	.69	.69	.47	.54
IV.	.02508	.30	.37	.37	.41	.45	.45	.51	.56	.43
V.	.0510	.28	.30	.33	.33	.36	.37	.35	.36	.34
Mean		.31	.37	.39	.42	.49	.49	.47	.38	.41

In drawing conclusions from the above table it should be borne in mind that a small error in the $W/cm.$ will make a very large variation in B , as will be more clearly shown later.

Two facts stand out clearly from the above table:

1. The thickness of the film, B , calculated for a plane surface does not vary with the diameter of the wire. That is, within the experimental error, the expression

$$b \ln \frac{b}{a} = 2B$$

gives the relation between the thickness of the film and the diameter of the wire.

2. The film thickness B is surprisingly independent of the temperature. Considering the possible errors in the temperature measurements owing to the wires not being separately calibrated by resistance, it appears probable that B is independent of the temperature within the experimental error. To see if this is so and to judge the accuracy of the results the watts per cm. were calculated from equation (21) assuming the value of $B = 0.43$ cm. (a weighted mean of the above values of B).

TABLE X.

Calculated Energy Loss by Convection. $B = 0.43$ cm.

Wire.	Diam.	500	700	900	1100	1300	1500	1700	1900
I.	.00404	0.10	0.24	0.41	0.62	0.85	1.12	1.42	1.74
II.	.00691	0.11	0.27	0.46	0.69	0.95	1.25	1.58	1.96
III.	.01262	0.13	0.31	0.53	0.79	1.09	1.44	1.81	2.24
IV.	.02508	0.15	0.36	0.64	0.94	1.30	1.72	2.17	2.67
V.	.0510	0.19	0.45	0.78	1.16	1.61	2.11	2.68	3.30

By a comparison of Table X. with Table VIII. it will be seen that the differences are relatively small, in fact probably within the experimental error. For example the greatest deviation is with wire III. at 1700°, the calculated power loss being 1.81 watts/cm. whereas 1.64 was found by experiment. This is an error of 10 per cent. but corresponds to an error in the resistance of the wire of 4.5 per cent. or an error in temperature of about 90° at 1700°. In nearly every other case the errors are much smaller than this. The general tendency for the calculated $W/cm.$ for wire V. to be less than the observed may be connected in some way with the fact that the specific resistance of this wire is considerably higher (5 per cent.) than that of the others (see Table III.). An error in the temperature measurements of about 4 per cent. (*i. e.*, 40° at 1300°) from this cause would account for the discrepancy in the calculated $W/cm.$

Free Convections from Tungsten Wires in Hydrogen.

In a previous paper¹ the relation between power consumption and temperature for tungsten wires in hydrogen was studied. In those experiments the temperatures were determined for the most part from the change in the resistance of the wire and from its known temp. coefficient. In a few cases these results were checked by photometric measurements.

These experiments have now been repeated with much more care and in each case the temperature was determined both by resistance and by candle power except at such low temperatures that the candle power could not be measured. The two ways of measuring the temperature gave nearly identical results. The energy lost by radiation was calculated from the formula

$$\frac{W}{la} = 39.4 \left(\frac{T}{1703} \right)^{4.74},$$

W , watts; l , length in cm.; a , diameter in cm.

This formula has been derived in the course of a careful study of the radiation from drawn tungsten wires in exhausted lamps.

The following table gives the $W/cm.$ loss from tungsten wires in hydrogen corrected for radiation by the above formula. Most of the experiments were made with the wire vertically suspended in a tube about 5 cm. diam. Moderate variation in the size of the tube had little effect.

The results in column marked I. are those previously published.

Column II. gives results obtained by the same observer that obtained the results of column I., but in a different series of experiments.

¹ Trans. Amer. Electrochem. Soc., 20, 225 (1911).

TABLE XI.

Wire: Drawn Tungsten .0045 Cm. Diam.

Temp. °K.	Watts per Cm. Observed.			Calculated $W/Cm.$	Ratio.
	I.	II.	III.		III./IV., V.
500	—	0.42	—	0.48	—
700	—	1.10	—	1.11	—
900	—	2.00	—	1.90	—
1100	2.5	2.90	2.2	2.84	0.77
1300	3.5	3.70	2.9	3.9	0.75
1500	4.5	—	3.9	5.1	0.76
1700	5.9	—	5.0	6.4	0.77
1900	7.9	—	6.9	7.9	0.88
2100	10.1	—	8.9	9.5	0.94
2300	13.0	—	11.2	11.2	1.00
2500	17.4	—	16.0	13.0	1.23
2700	24.8	—	24.5	15.0	1.63
2900	36.4	—	39.0	17.3	2.26
3100	56.4	—	60.2	19.4	3.11
3300	96.2	—	88.2	21.8	4.06

In column III. are the results obtained in the recent experiments referred to. At low temperatures they are probably not any more reliable than those of columns I. and II., but at high temperatures (above 2300°) where the candle powers were used for the temperature estimation, the results of column III. are much more trustworthy than those of column I.

The calculated $W/cm.$ given in column IV. were obtained as follows:

For air at atmosphere pressure and room temp. B is equal to 0.43 cm., the weighted mean obtained from Table IX. We assume, for different gases or the same gas under different conditions, that B would vary directly as the viscosity and inversely as the density at the outside surface of the film. At 300° K. the viscosity of hydrogen from equation (2) is 0.496 that of air. The density is .070 that of air. Hence B for hydrogen should be $0.496/0.070$ or 7.1 that of air, that is, for hydrogen at 1 atmo.

$$B = 3.05 \text{ cm.}$$

For a wire .0045 cm.

$$\frac{a}{B} = .00145,$$

whence

$$s = 1.131.$$

The $W/cm.$ (calculated) given in Table XI. are obtained by multiplying this value of s by the values of $\phi_2 - \phi_1$ for hydrogen (see Table I.)

calculating φ_1 for the temp. 300° K. From the value of s by (IIa) it is found that b , the effective diameter of the film of stationary gas around the wire, is 1.14 cm.

Up to the temperature of 2000 the observed value of $W/\text{cm.}$ in columns I. and III. are less than the calculated. This is to be expected from two causes: (1) The gas in the tube is at a temp. above 300° K. which would increase φ_1 and decrease $\varphi_2 - \varphi_1$, and with it, W . (2) The walls of the tube would prevent to some extent "free convection" and would tend to increase b , thus decrease s and therefore W .

On the whole at temperatures below 2300° the agreement is strikingly good in view of the fact that the results are calculated from experiments with platinum wires in air and that no arbitrary constants have been employed.

The increasingly large deviations above 2300° are due to dissociation of hydrogen into hydrogen atoms. This was suggested in the previous paper and has now been amply verified. The results on the measurement of the extent of the dissociation and the heat of the reaction were given in a paper presented before the Washington meeting of the American Chemical Society, December, 1911. These results will soon be published in the Journal of the Society.

Free Convection from Tungsten Wires in Mercury Vapor.

The apparatus used in this experiment is illustrated in Fig. 1. A is a glass tube 2.5 cm. in diameter, containing mercury at its lower end which serves to supply the mercury vapor and also to make electric contact with a platinum wire fastened to the lower end of the tungsten wire W . The wire W about 7 cm. long is welded at its upper end to a heavy platinum wire which is fastened to a steel rod F through which the current is supplied. The tube A is wound with resistance wire with leads $E-E$. Another coil with leads GG is placed around that part of the tube containing the mercury. Asbestos insulation K prevents too great heat loss from this lower end whereas a second glass tube C placed around A serves to prevent heat loss from the central part of the tube. The procedure of the experiments was as follows: The entire apparatus was exhausted to less than 1 mm. pressure. Pure hydrogen was admitted to about $2/3$ of an atmosphere pressure and the mercury was raised to boiling by means of the coil GG . Then sufficient current was applied to the winding EE to prevent condensation of the mercury except above the level of the asbestos placed at J . Mica disks B served to prevent hydrogen gas from diffusing down into the mercury vapor. Liquid air was placed around the tube, L , to dry the hydrogen and to dry out the whole system while exhausted before admitting the hydrogen.

In the first few experiments no mica disks were used, for it was thought that the strong blast of mercury vapor would prevent any hydrogen

from diffusing down into the heavy mercury vapor. But it was found that the energy consumption by the wire increased with about the 15th power of the absolute temperature and that the tube *A* which was of lead glass blackened opposite the wire *W* because of reduction of the lead. By using the mica disks the energy consumption became nearly linear and showed no tendency to increase abnormally at high temperatures. But experiments since that time showed that the correction for radiation which we then applied was too large. So the results obtained with our new values for radiation do show a distinct tendency to increase at very high

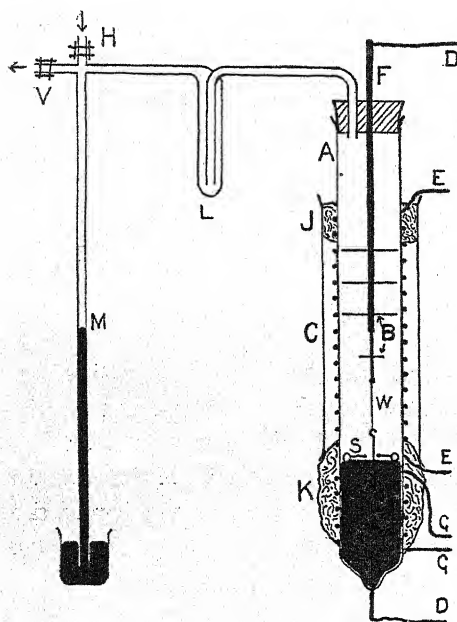


Fig. 1.

temperatures. However, this is believed to be due entirely to minute traces of hydrogen which did make their way down through the mercury vapor. In fact in several trials, even with the mica disks, the tubes showed signs of blackening due to lead reduction.

Convection from Tungsten Wire in Mercury Vapor.

Temp. of Wire ° K.	Watts per Cm.		Con-vection.	$\frac{B}{\text{Cm.}}$	Watts/Cm. Calculated.	
	Total.	Radiated.			$B = .0784.$	$B = 0.47.$
1500	0.61	0.13	0.48	.083	0.487	0.15
1700	0.90	0.26	0.64	.072	0.624	0.19
1900	1.24	0.45	0.79	.074	0.777	0.24
2100	1.71	0.76	0.95	.077	0.943	0.29
2300	2.29	1.19	1.10	.084	1.125	0.34
2500	3.04	1.74	1.30	.082	1.315	0.40
2700	4.24	2.54	1.70	.055	1.52	0.46
2900	5.82	3.47	2.35	.034	1.74	0.53
3100	7.74	4.64	3.10	.022	1.97	0.60
3300	9.94	6.00	3.94	.016	2.21	0.67
3500	12.37	7.62	4.75	.013	2.46	0.75

Experiments will probably be undertaken with mercury vapor in the absence of foreign gases to verify this conclusion and obtain more accurate data on the energy loss in mercury vapor.

The following tables gives the results of the final experiment with mercury vapor at atmospheric pressure and a wire of .0069 cm. diameter.

The values of B are calculated as in the case of convection in air. The average value of B up to about 2700° is 0.0784 so this value is used as a basis for calculating the watts/cm. as in next to the last column. It is seen that the agreement between the observed and calculated values is excellent up to about 2700° , but that above this the energy loss is much greater. This is probably due to traces of hydrogen as mentioned above.

If we assume that for different gases the thickness of film, B , for a plane surface should be proportional to h and inversely proportional to the density then we calculate that B should be 0.41 cm. For at 600° K. the density of mercury vapor is 3.46 times that of air and its viscosity is 3.27 times that of air. The watts/cm. calculated on this basis are about 69 per cent. too low. It is possible however that the blast of mercury vapor may have caused a greater loss of energy from the wire than would have occurred with "free convection."

At any rate the order of magnitude of the results is right and up to 2700° K. there is no perceptible temperature coefficient to the value of B .

SUMMARY AND CONCLUSIONS.

It has been shown that:

1. The loss of heat from wires by free convection takes place exactly as if there were a film of stationary gas around the wire, through which the heat is carried entirely by conduction.
2. The thickness of the film is independent of the temperature of the wire, but probably increases with increasing temperature of the surrounding gas.
3. The loss of heat from very small platinum (also copper) wires by radiation is negligibly small up to temperatures of several hundred degrees.
4. The thickness of the film of gas varies in a simple way with the diameter of the wire, namely,

$$b \ln \frac{b}{a} = 2B,$$

B being a constant for any gas, b diameter of film of gas, a diameter of wire.

5. The rate of convection of heat from any wire is equal to the product of two factors, one the shape factor s involving only the diameter of the wire and the constant B (for any gas); and the other, a function φ of the heat conductivity of the gas.

Thus if W is the energy loss from wire in watts per cm., then

$$W = s(\varphi_2 - \varphi_1),$$

where s may be found from the equation

$$\frac{s}{\pi} e^{\frac{-2\pi}{s}} = \frac{a}{B}$$

and

$$\varphi = 4.19 \int_0^T k dT.$$

k is the heat conductivity of the gas at the temperature T in cal./cm. ° C. φ_2 is taken at the temperature (T_2) of the wire, and φ_1 is taken at the temperature (T_1) of the atmosphere.

6. Tables are given by which curves may be plotted showing the relation between φ and T and between a/B and s .

7. The fact that B is found to be independent of the temperature is a strong indication that Sutherland's formulæ may be applied to the heat conduction of gases up to extremely high temperatures.

In a subsequent paper the author will show that the formulæ here developed agree extremely well with Kennelly's results on the "Convection of Heat from Small Copper Wire," that for air the thickness of film B varies inversely as the 0.75 power of the air pressure and that for forced convection B varies inversely as the 0.75 power of the wind velocity. It will also be shown that a similar relation holds for the value of B when the temperature of the atmosphere around the wire varies from 90° up to 800° K.

Several experiments will also be described in which the presence of such a "stationary" air film is demonstrated. The thickness of the film has been determined by direct measurement and the temperature distribution around the filament has been studied with care.

The author wishes to express his indebtedness to Mr. S. P. Sweetser, Mr. H. Huthsteiner and Mr. E. Q. Adams for most of the experimental work in connection with this investigation.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.,
SCHENECTADY, N. Y.,
January 27, 1912.

ON THE CONDUCTION OF ELECTRICITY AT CONTACTS OF DISSIMILAR SOLIDS.

BY ROBERT H. GODDARD.

INTRODUCTION.

THE fact that the resistance to the flow of current across the contact of dissimilar solids depends upon the direction of the current was first observed by Ferdinand Braun¹ in 1874, with metallic sulphides against metals. Braun showed later, 1877-8, that the effect was not due to counter E.M.F. of polarization, as the sulphide suffered no change in weight. Also, current from the secondary of an induction coil passed simultaneously decreased the resistance in both directions.

The more recent contributions to the subject have been inspired, in most cases, by the application of such unilaterally-conducting contacts, usually called "crystal rectifiers," to the detection of electromagnetic waves. The name comes from the fact that contacts having this property are usually those between a metal and a crystalline substance. Many such crystals are known. Among those which have been most carefully studied are the silicon-steel, carbon-steel, and aluminium-tellurium rectifiers of L. W. Austin.² The first gave phenomena, very reproducible below 0.2 volt, A.C.; the direction of the rectified current being from steel to silicon, except with one specimen which gave opposite rectification for all points on the surface. In every case the thermo-E.M.F. was opposite to the rectified E.M.F.

The carbon-steel rectifiers were made by placing a steel needle in contact with a cored arc-light carbon. Graphite was found unsatisfactory. A lamp filament against a light carbon also furnished a satisfactory rectifier. The phenomenon was not, however, regular with direct current. The rectified current was from steel to carbon.

The aluminium-tellurium rectifier, discovered previously by Austin,³ exhibited a marked peculiarity. For low voltages, up to 0.6 volt, 8.0 amperes, the rectified current was in the direction of the thermo-E.M.F.; *i. e.*, Al to Te; whereas above this voltage, the rectified current was in the opposite direction, and increased with increasing voltage. Large contacts, such as a No. 20 aluminium wire melted into a block of tel-

¹ Ferdinand Braun, *Pogg. Ann.*, 153, p. 556, 1874.

² L. W. Austin, *Bull. Bur. Standards*, 5, 1, pp. 133-147, 1908.

³ L. W. Austin, *PHYS. REV.*, 24, pp. 508-520, 1907.

lurium, gave marked unilateral conductivity with direct, but unsatisfactory rectification with alternating, currents.

An extensive series of experiments has been undertaken by G. W. Pierce. The first piece of work¹ was on the carborundum rectifier. The resistance of this substance is much higher than that of those just described, so that a rectified current of the order of 10^{-3} amperes requires an E.M.F. of 20 volts. With a pressure on the crystal of 500 grams, one specimen gave a rectification of 4,000 : 1. On increase of pressure the resistance fell in both directions but the rectification, *i. e.*, the ratio, decreased. There was still some rectification, however, even when the crystal was driven well into the electrode. Best results were obtained when one of the surfaces of the crystal was platinized, to give low resistance, the rectified current then being in the direction of crystal to metal. With a current of $\frac{1}{2}$ ampere, the rectification was but 1.6 : 1.1, the crystal grew hot, and the contact surfaces luminous. The temperature coefficient of resistance of carborundum resembles that of a salt solution more than that of a metal.

An hysteresis effect was observed, apparently due to a slow building-up of the current, which disappeared after a few reversals—the current on rise of E.M.F. being less than on return to zero. An attempt was made to observe a back E.M.F. The rectifier was connected alternately, 120 times a second, with a source of potential of 35 volts, and a capillary electrometer. A persistent but very small reading, 0.002 volt, was indicated.

The second piece of work² was performed with certain minerals—brookite, anatase, and especially molybdenite. For each of these the rectified current was from metal (usually copper) to crystal, and of the order of 0.01 ampere for 10 volts. The extent of the rectification depended upon the circumstances of contact, as regards position on the surface of the crystal, and pressure at the point.

Of particular interest were the experiments with molybdenite, undertaken to show, by oscillographic records, the existence of a counter E.M.F. existing for a short period of time. The apparatus consisted of a Braun tube, the luminous spot of which was focused on a sensitive film on a rotating drum. This drum was driven in synchronism with the current which passed through the rectifier and the deflecting magnets of the Braun tube. The drum was revolved until successive exposures had combined to give a sufficient intensity to the film. These oscillograms showed that the rectified cycles led their respective voltage-phase

¹ G. W. Pierce, *Phys. Rev.*, 25, pp. 31-60, 1907.

² G. W. Pierce, *Phys. Rev.*, 28, pp. 153-189, 1909.

cycles at three positions. But these could be obtained by calculation, from the resistance and inductance of the circuit. In fact, no departure in amplitude or phase existed between the rectified cycle and the voltage-phase (applied) cycle that was not accounted for by the inductance of the oscillographic apparatus, or by the current-voltage curves of the rectifier with steady currents. In other words, if there were any terms contingent upon heating or other effects which involve an integral of a function of the current with respect to the time, this integral attained its final value in about $1/6,000$ second, corresponding to about 1 mm. on the original photograph.

In a further piece of work,¹ Pierce examined the rectifying properties of iron pyrites. This substance had been studied previously by Braun, who found that the difference in the conductances in the two directions increased with increase of current; and with continued passage of the current, the larger conductance decreased. Pierce found this only with currents large enough to heat the contact.

The oscillograms, like those taken with the molybdenite rectifier, showed that the current through the rectifier had the same phase and amplitude it would have if the rectifier were replaced by a resistance producing the same amplitude, *i. e.*, there was no apparent integrative action. This rectifier showed the irregular action manifested by all the solid rectifiers previously investigated, namely, that for some adjustments of the contact, the greater current was from copper to crystal, whereas for other adjustments, *e. g.*, with a different pressure, or at a different point on the crystal, the greater current was from crystal to copper. With fixed adjustments, the current and E.M.F. were constant.

A study of the galena rectifier has been made by A. E. Flowers.² When the E.M.F. is gradually increased from a low value, "break-downs" occur; *i. e.*, the current jumps suddenly to a higher value. If the E.M.F. is further increased, a large jump of current eventually occurs, the metal point sinks into the galena, and rectification is destroyed. These break-downs occur more frequently if the E.M.F. is reversed at each increase, and the contact will stand a rather high E.M.F. if it is removed temporarily, as soon as the breakdown occurs.

In confirmation to Pierce's observations, certain parts of the galena crystal were found to be non-rectifying. Further "a non-rectifying surface was often found to have others beneath and parallel, when layers were split off, but scratching or scarring a surface usually spoiled more or less completely its rectifying properties." Heating destroyed the

¹ G. W. Pierce, *PHYS. REV.*, 29, pp. 478-484, 1909.

² A. E. Flowers, *PHYS. REV.*, 29, pp. 445-560, 1909.

effect, although it was sometimes regained by further heating, disappearing again as the temperature was still further increased. The effect did not depend upon the metal at the contact.

The author emphasizes the time taken to build up the current on closing the circuit, observed with some crystals, especially on heating. It was found that an artificial rectifier could be made by allowing sulphur to burn on the end of a copper wire for a moment, and afterward placing this sulphur against lead (brass was unsatisfactory) and reversing a current through the contact a few times.

Few theories of the foregoing phenomena have been suggested. The effects were first thought to be of thermoelectric origin but Austin and Pierce showed that heating could not account for the magnitude of the effects observed. Pierce¹ considers that all rectifiers come under more or less the same class of phenomena, and that there may be some connection between thermo-E.M.F. and rectification, as all rectifying crystals have large thermo-E.M.F.'s against the common metals. Flowers concludes that the effect, with galena, is due to electrochemical deposition of a resisting film.

ALLIED PHENOMENA.

Closely related to the crystal rectifier is the electrolytic rectifier—a fine platinum wire touching the surface of an electrolyte. Although the contact area is of the same order, there are two marked differences; oscillograms made by G. W. Pierce² show evidence of a positive E.M.F., for about $1/1,500$ second, greater than the E.M.F. immediately following; further, for best results, a steady current should be superposed on the alternating current to be rectified. Austin³ considers that heat is one of the factors, with, probably, chemical action and electrostatic attraction across the gas film.

The aluminium valve, or rectifier, which consists of a cell containing one of certain electrolytes, one electrode being an aluminium plate, is more amenable to experiment than the rectifiers previously described, owing to its size. The hindrance to the passage of the current has been shown⁴ to be due to back E.M.F. This is produced slowly on a fresh plate, 10 minutes being required to obtain the full valve effect at 15 volts. Increase of temperature decreases this E.M.F. Careful experiment⁵ has shown that the back E.M.F. together with the drop due to ohmic resistance is equal to the total impressed voltage even when the latter is 20

¹ G. W. Pierce, *PHYS. REV.*, 29, pp. 478-484, 1909.

² G. W. Pierce, *PHYS. REV.*, 29, pp. 56-70, 1909.

³ L. W. Austin, *Bull. Bur. Standards*, 2, pp. 201-224, 1906.

⁴ S. R. Cook, *PHYS. REV.*, 15, p. 23, 1904.

⁵ S. R. Cook, *PHYS. REV.*, 20, pp. 312-321, 1905.

or more volts. On removing this applied voltage, the counter E.M.F. falls off to about half its value in 10 seconds.

There are two suggested explanations for the low resistance of the valve when aluminium is cathode. Guthe¹ suggests that hydrogen ions are liberated between the film and the metal, and these can pass through the more or less solid film of Al_2O_3 on the metal, whereas, when the current is in the opposite direction, the large negative ions from the solution cannot penetrate this film. Schulze² considers that the solid film merely serves to hold a gas between it and the metal. Electrons from the metal can pass readily through this gas whereas the large negative ions from the solution meet with resistance. This view is more or less supported by experiments made under pressure³ which show that the rectification becomes less, temporarily, under pressure. When the gas is condensed, there should be less distance for the large ions to travel.

Many coherer phenomena suggest strongly that films play an important part in conduction at small contacts. This is very evident in some cases. For example, very sensitive coherers are formed⁴ by lead in contact with *oxidized* copper, or copper alloy. Eccles⁵ has advanced the theory that the action of iron-mercury, and oxidized iron-iron coherers depends entirely on the heating of the film of oxide by the electrical oscillations and the current due to the applied p.d. By developing the theory mathematically the necessary results are shown to follow. Further, it has been found⁶ that a platinum wire placed upon two others coheres, if an oscillatory spark is set up in the neighborhood. The same effect may be produced with a broken carbon filament in an incandescent lamp globe, provided a considerable resistance is placed in series. The coherence of the platinum wires may also be produced by heating each junction to a red heat.

In certain other cases, the rôle of the film is not so evident. For example⁷ a tantalum wire, 0.05 mm. in diameter touching mercury forms a very sensitive coherer, which cannot be made permanently to cohere. Further, Shaw and Garret⁸ have found that when a coherer formed of two copper wires touching each other is made to cohere by a spark some distance away, the wires if separated (requiring a small but measurable pull) and held apart for a few moments will not recohere when

¹ Guthe, *PHYS. REV.*, 15, p. 327, 1903.

² Schulze, *Ann. der Physik*, 28, p. 787, 1909.

³ Carman and Balzer, *PHYS. REV.*, 30, pp. 776-781, 1910.

⁴ M. Hornemann, *Ann. der Physik*, 14, 1, pp. 129-138, 1904.

⁵ W. H. Eccles, *Phil. Mag.*, 19, pp. 867-888, 1910.

⁶ Majoli, *N. Cimento*, 10, pp. 552-585, 1905.

⁷ L. H. Walter, *Proc. Roy. Soc., Ser. A*, 81, pp. 1-8, 1908.

⁸ Shaw and Garret, *Phil. Mag.*, 8, p. 164.

placed together again, but will do so if the current be reversed. The wires may again be separated and the action repeated several times.

STUDY OF VARIOUS CONTACTS.

In the present investigation it seemed advisable to try a number of rectifying substances against various metals, in order to see if any general conclusions could be drawn. The apparatus consisted simply of a source of E.M.F. of one or two volts, connected through a small resistance to a milliammeter and a switch which served to reverse the current passing

Substance.	Greater Current Flows from —		Conductances Diverge.		Conductances Approach.	
	Metal to Crystal.	Crystal to Metal.	The More Rapid Change is the—			
			Rise.	Fall.	Rise.	Fall.
C	Al Fe	Pb Si			Si	
Te		Mg Al Zn Cu Fe Ag Sn Pb Pt Si Galena	Mg Al Zn Pb Ag Galena	Cu Pt Si		Na
Si	Al Zn Cu Fe Sn Pb Pt		Pb Pt	Al Zn Cu Fe Sn		
Galena	Cu Fe Pb Pt	Al Sn Ag Si Zn		Al Zn Cu Fe Pb Pt		Cu Si
Magnetite	Zn Pb Pt Te	Al Cu Ag Si		Pb Te Si		Zn Cu Pt

through the rectifier. The drop across this switch was measured by a voltmeter with a range of three volts. Direct current was used in order to bring out any gradual change that might appear while the current was being applied. Where there was uncertainty regarding the direction of the rectified current, a number of contacts were made to determine the general direction.

The result of these experiments is given in the following table, in which certain general tendencies are apparent. The conductances, in most cases, grew farther apart the longer the current passed, the rise being more rapid with some metals and the fall with others, as the table indicates.

While the current was passing the conductances usually changed together; *i. e.*, while one was rising or falling, that in the opposite direction was rising or falling, as could be seen by sudden reversals. This was not always true, as one conductance sometimes remained stationary. Usually the conductance in the direction which had undergone greater change recovered the faster. Scraping a metal to remove possible oxide sometimes increased the mean conductance but decreased the difference.

PERMANENT CHANGES OF CONDUCTANCE.

The gradual changes that have just been described are indicative of something like a change of structure at the contacts, due to the passage of the current, and an attempt was made to learn if the conductance of the contact was the same after current had passed in either direction. To test this point, rectified currents of the order of 1×10^{-2} amperes were passed through the contact, which could be immediately connected in series with a galvanometer and a high resistance, so that a current would pass of the order 1×10^{-5} amperes, which did not produce rectification.¹

The current due to thermo-E.M.F. was eliminated by reversing the current through the contact and taking the mean value. The difference in the deflections, after the larger current had passed in one direction or the other, varied up to about 30 per cent. of the lesser, depending somewhat on the circumstances of contact. The deflections were for the most part steady, and were, for most substances, greater after the current had passed from metal to crystal, as the following table will show.

That these changes of resistance, indicated by galvanometer deflections, did not take place instantly was shown by passing the larger current

¹ Throughout this paper, excepting the part that deals with the oscillographic study, the "rectification" was measured by applying a direct current, first in one direction and then in the other direction, the time of application being long enough to take a reading of the current.

in one direction, then noting the deflection with the smaller current—for example Te to Fe_3O_4 , deflection 13.8—then “making” the larger current in the opposite direction for very short intervals, by letting the ends of two wires touch while moving rapidly past each other—giving, successively, deflections, 13.2, 12.6, 12.0, the last being the same as that obtained for this direction previously. In some cases, for example, PbS, Zn, the deflections after long exposure to current were less than after a shorter exposure.

Substance.	Deflection Greater After Current has Passed from	
	Metal to Crystal.	Crystal to Metal.
Te	Fe Pb Pt Si Galena	Al
Si	Al Cu Fe Si	Zn Pb Pt
Galena	Al Zn	Pt Si
Magnetite	Al Fe Pb Pt Si	Zn Te Galena

Before the larger current was first applied, the deflection was usually smaller than afterward, and in certain cases was not steady, there being a slow rise or fall of deflection irrespective of whether the small current were on or not. The effect was not of a very definite nature and seemed to depend upon the circumstances of contact. Most of the substances and metals gave steady deflections; those showing a fairly certain rise and fall respectively were: Fe_3O_4 , Pb; Fe_3O_4 , Si; PbS, Te; Si, Si; and Te, Pt; Fe_3O_4 , Pt; PbS, Zn. The effect is very similar to a spontaneous rise observed by the writer in studying the conductivity of powders,¹ namely, for powdered BaS.

EFFECTS WITH LARGE CURRENTS.

In considering the general effects by currents of from 0.5 to 9 amperes, it will be convenient to call the direction of current from metal to crystal,

¹ R. H. Goddard, *PHYS. REV.*, 28, p. 411, 1909.

A, and the reverse direction, *B*. In most cases the current in the direction *A* showed the greater irregularity, the current in the direction *B* being perfectly steady in some experiments. An exception was C, Fe_3O_4 , for which the current in direction *A* took steadily the largest value *B* attained in its frequent jerks. These irregularities for *A* were of a definite kind, with some contacts. For example, with galena against Cu, Ag, Pb, Pt, and once with Te, Pt, the current kept falling gradually and then rising with a jerk. With Si, Zn; Si, Pb; and PbS, Ag, these irregularities were accompanied by acoustic effects. In the first two contacts a hissing noise accompanied the passage of current in the direction *A*, while there was silence for *B*, except for occasional clicks. The PbS, Ag contact emitted a musical trill for the direction *A*. Also, with other contacts, there was occasionally a click on opening the circuit, and on reversing.

During the passage of a large current, there was visible sparking for the direction *A*, most noticeable with Te, Al; Te, Mg; Te, Zn, but lacking with Te, Ag; and Si, Al. Also, if the contact was glowing, the color was usually brighter for the direction *A*, especially with Si, Al; Si, Pt; Si, Zn; Te, Zn; Te, Al; but duller with Fe_3O_4 , Al; Fe_3O_4 , Zn. The portions of some substances close to the contacts showed colored deposits after the passage of considerable current. With Si, Zn the color was yellow, with C, Al, white, and with Fe_3O_4 , reddish.

The direction of greater current, *i. e.*, of rectification, changed in many cases when the current was increased to 0.5 ampere or more, the general tendency being to change from the direction *A* to *B*. Tellurium, for which the rectification is in this direction, gave no change on increase of current, with any metal. Silicon exhibited this change with Al, Ag, Fe, Zn, and Pt, with currents equal to and greater than 0.45, 1.3, 0.50, 0.55, and 0.50 amperes, respectively. Different settings gave other, but not widely different, values. In the case of Zn, at 1.7 amperes, a high resistance state occurred—equivalent to the establishment of a high resistance at the contact. The current fell to 3×10^{-3} ampere, and the direction of rectification changed to *A*. Magnetite against Al, Zn, Pb, and Pt, showed similar change of direction for currents from 0.5 to 0.8 ampere. The effect was not observed with Cu. Galena gave results of less certainty, although the direction was *B* with all the metals, Mg, Al, Zn, Cu, Fe, Sn, and Pb for two amperes.

On some occasions, immediately after reversing from the direction of larger current, the current remained large for a second or so, then falling to a low value. This was noticed at times with PbS, Ag; Te, Al; Te, Cu; Si, Pt; and Si, Zn. This lag lasted longer, the longer the current

had been maintained before reversing. The lag occurred in both *A* and *B* directions, for different substances; and the conductance fell from the value just preceding, in some cases, and rose to a higher, in others.

Often, when several amperes were passing, the current would be reduced suddenly to but a milliampere or so, at 20 volts, with the substances still in contact, and the rectification sometimes still persisting. This state, independent of the direction in which the current had been flowing, was permanent, as the substances could be pressed gently together without destroying the high resistance. The phenomenon was marked with Si against Mg, Al, Zn, Cu, and Pt; also with magnetite against Al, Zn, and Cu.

The rectification was less for small currents, after the passage of large currents, than before.

MICROSCOPIC EXAMINATION DURING PASSAGE OF LARGE CURRENTS.

Certain light effects were observed to take place coincidently with jerks and irregularities observed with the current-measuring instruments. The most striking phenomena were obtained with silicon. A microscope of 50 diameters magnification was used. The appearance, with Pt, may be seen from Fig. 1. For the direction *A*, at 1.8 amperes and 3.8 volts the tip of the platinum wire, *a*, was dim, while the adjacent Si, *b*, was bright. On reversing, *b* became less bright, while *a* grew nearly as red as *b* after the lapse of about 0.5 second. The voltage simultaneously rose from 3.5 to 3.8 volts, the current remaining closely the same. This was repeated at least a dozen times. With increase of current, the

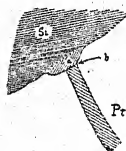


Fig. 1.

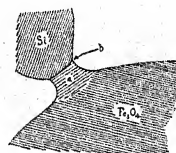


Fig. 2.

contact "broke down," and gave about the same current for each direction. With another setting, Si, Pt gave no rectification although the contact appeared brighter for the direction *A*. On the other hand, certain contacts gave rectification but no light; *e. g.*, sometimes Si, Zn and Te, Pt.

The contact Fe_3O_4 , Si had the appearance shown in Fig. 2. For the direction *A*, with current of 1.2 amperes at 11 volts, the region *a* became red, the light spreading quickly from the junction, *b*, between the Si and Fe_3O_4 . On reversing, blue sparks appeared for an instant along

the line *b*, when the current fell to 6×10^{-3} amperes at 24 volts. These changes could be repeated. Curiously, there were sparks for an instant on closing the circuit in the direction *B*, and also in the direction *A*, even if the circuit was opened for five minutes before reversing.

If, after breaking the circuit in the direction *A*, connection was made with a milliammeter, a sudden throw of 50 divisions was obtained, falling at once to 15, and thence slowly to about 2. A very short application of the larger current gave a throw indicating 15 milliamperes—the fall being more gradual. Increase of the larger current destroyed the rectification.

A contact of Fe_3O_4 , Al showed blue sparks for the direction *A*, followed by redness of the metal. For current in the opposite direction, the metal was less bright, and the mineral at the contact glowed white hot. In one experiment, with a current of one ampere, a bright spot moved slowly around, a short distance from the center of the contact. With Pt, the current for *A* was fairly steady, while that for *B* was unsteady, and lowest during sparking.

Tellurium, with Mg and Al, gave continuous sparking for the direction *A*, and but a single spark on closing the circuit in the direction *B*. With Pt, however, asymmetry was obtained without sparking even at 1.5 amperes. Arc-carbon gave less definite results. Al generally gave sparking and less current for the direction *B*. Sometimes with Fe there was sparking only for current in the direction *B*. At other times there were occasional flashes for both directions. Carbon against Si, with currents of two amperes or more, gave intermittent sparking for both directions, the direction of greater current being uncertain. The direction was also uncertain for C, Fe_3O_4 , but although there was not continuous sparking for the direction *B*, the sparks were usually larger for this direction of the current.

With PbS the light did not appear continuously, and experiments made to determine the light effect were therefore uncertain. The sparking usually took place, however, in the direction *B*.

In a number of special experiments, the effect of large currents could be seen without the aid of a microscope. A small cube of PbS, placed between the ends of two No. 50 aluminium wires grew red hot and showed blue sparks where the current entered, the metal being redder on this side, also, when a current of two amperes was passed through. Increase of current made the sparking more energetic, and when 4.5 amperes were reached, the high resistance state previously mentioned was produced. A small piece of Fe_3O_4 placed between the wires behaved in a similar manner,

except that it was more difficult to keep the current flowing. In a similar experiment with Si, the element grew red hot, with a much brighter point where the current entered.

A peculiar effect was also obtained when two pieces of silicon, one or two cm. long, were placed in contact and a current of one or two amperes passed through the junction. The piece on the negative side, even if thinner, was brighter from the contact to a cm. or more away. A small blue spark appeared on separating the two pieces, and it was found impossible to produce an arc. After the experiment the pieces were fused together. The effect disappeared when sufficiently large currents, *e. g.*, 25 amperes, were passed through the junction. This was very different from the behavior of two pieces of Fe_3O_4 in contact, in which there was usually no sparking, or difference in color on reversing. Once, when rectification appeared at 3 amperes, with the greater current in the direction *A*, the contact appeared redder as a whole when the current passed in this direction.

EFFECTS IN HIGH VACUA.

The foregoing experiments, while suggesting electrolytic, or allied, action, at least made certain the necessity of performing the experiments with chemically clean surfaces. For this purpose a number of methods were tried. First, an attempt was made to clean the substance by using it as a cathode in a discharge tube, and causing it to give off a cathode deposit. The result was unsatisfactory, as the effect was uncertain—a result which is in accord with the literature of the subject. Melting the substance by passing a heavy current through a piece in which was a narrow portion, was found to present too many difficulties. Cutting the specimen by means of cut-nippers operated through a ground joint was found too complicated to be practical.

The method finally employed was a simple one, and may be understood by an examination of Fig. 3 (*a*). The first step in the procedure was to exhaust the system by connecting the ground joint, *J*, with a Gaede mercury pump. When the highest vacuum the pump would give (without freezing mixtures) was attained, tested qualitatively by the discharge tube, the cock was closed and the system removed from the tube leading to the pump. In the first experiments, the cock was not used—the tube being sealed off—but the vapor of the vacuum wax used was later found not to give different results from those obtained with sealing. The substances studied had previously been set in solder, if poorly conducting, and placed in a holder, as (*b*), Fig. 3. In the figure is shown a metal, filed into the required shape, and held in place by a screw. This holder,

containing a piece of iron wire (shown dotted), could be moved from the outside by a 6-inch permanent horse-shoe magnet; and as this wire was placed off center, the holder could be rotated through any angle. A continuous electrical connection between the holder and the electrodes in the end of the tube was secured by soldering to them a closely-wound coil of No. 48 bare copper wire, about $\frac{1}{8}$ inch in diameter. The holder could be moved with ease where desired, and as many as four specimens could be kept in the bulb, B_1 , without serious entanglement.

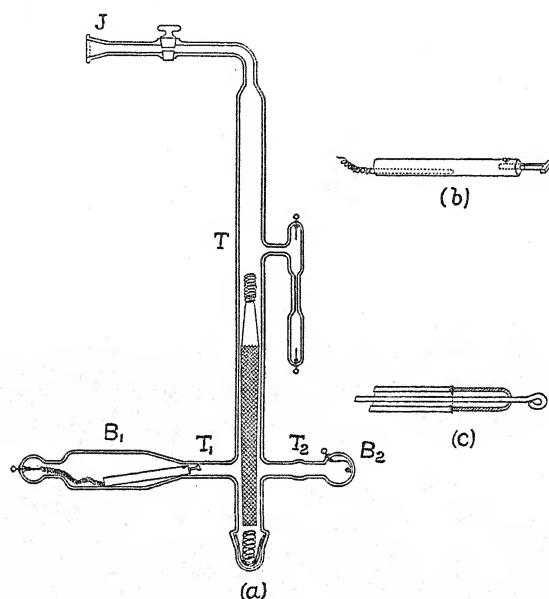


Fig. 3.

The specimen could be cleaned by filing, in which case it was allowed to rest against the file, the holder being in the tube, T_1 , and the whole system was shaken back and forth to permit motion of the file relatively to the specimen. The file, of circular section, was prevented from breaking through the tube by a spring fastened to the top, and another spring, one end of which was pulled out, bent into the form of a U, and placed in the tube with the ends of this U pressing into two dents in the wall of the tube. This served to keep the spring in place.

Another method of securing a fresh surface, more satisfactory than by filing, was to break off the end of the specimen by dropping the file upon it as it protruded from the side tube, T_1 . This was facilitated by filing a small indentation in the specimen near the holder. Metals were bent back and forth until they broke, by turning the holder through 180°

before each impact. In this way, with a soft metal like aluminium, a clean surface 1 mm. square could be obtained. Of course, before placing specimens in contact, the tube T was inverted so that the file did not hinder passage from T_1 to T_2 . In the figure is shown a small bulb, B_2 , containing tellurium in contact with a platinum wire which served as electrode. Owing to the comparatively low melting point of tellurium, it could be melted, in vacuo, through the glass. Substances which had to be broken or filed were contained in another bulb, similar to B_1 , which was fastened to the tube, T_2 , after B_2 had been removed. To get at any holder or specimen it was, then, merely necessary to cut off the tube at the point where the letter T_2 is situated.

In order to use, in some experiments, a much larger current than ordinary vacuum-tube electrodes will stand, the special electrodes shown in Fig. 3 (*c*) were made. A ring of platinum foil was fastened to the glass tube by means of soft blue glass. This platinum was then copper-plated, and a short lead tube, into one end of which a stout copper wire had been soldered, was soldered to the coppered platinum. Such an electrode was capable of holding a Crookes vacuum, and the amount of current was limited only by the size of the copper wire. This method of soldering lead to platinum was taken from Strasser.¹

RESULTS IN VACUO.

The substances used in the apparatus just described were Mg, Al, Fe, Pb, against Te, Si, galena, and natural graphite. In the open air, Te and the common fused Si gave good rectification for each metal, in directions B and A , respectively. The effect was strongest with Mg, Al, and Fe. With considerable pressure on a clean Mg surface, Te gave once a small preponderance of current in the direction A . The other substances, galena and natural graphite, gave rather uncertain action at best—the currents in the two directions often being equal—but such rectification as was obtained was usually in the direction B except with PbS, Al and PbS, Pb. The thermo-E.M.F. of these metals was from carbon to metal through the junction. Two kinds of silicon were used: the common fused silicon and some pure crystals, obtained from Eimer and Amend and Bausch and Lomb. The former always gave rectification in the direction A , while some crystals gave rectification nearly always in the direction B . The pure crystals were about 3 mm. long and not more than 0.3 or 0.5 mm. in thickness. The fused silicon was broken into pieces of about this size, and both set in solder. The thermo-E.M.F. for both kinds of silicon was the same; *i. e.*, Al to Si through the junction.

¹ B. Strasser, Ann. der Physik, 31, pp. 890-918, 1910.

With currents and voltages of from 10 to 50 milliamperes and 1 to 3 volts, the following effects were observed in vacuo. Tellurium showed the same action with all the metals tried. Before the metal had been filed, the rectification was the same as in air. Afterward, the currents were exactly equal for a given contact and for those in the immediate neighborhood. There was often a slightly greater current for *A*, the direction opposite to the rectification in air, giving rise to an E.M.F. difference of 0.01 to 0.04 volt, *in the direction of the thermo-E.M.F.* This held good at 40 or 50 milliamperes or more. For some contacts there was rectification in the direction *B*, of the order of 5 or 10 per cent. of the total current. Whenever a part of the metal which had not been filed or broken touched the tellurium, marked rectification appeared. The possibility of there being a short circuit was eliminated by touching the platinum wire leading to the tellurium, when the current was very much increased. Experiments in which the tellurium was broken or melted, and the metals broken or filed, all gave the same results. This held good even when the contact was one of considerable resistance. The only metal that gave uncertain action at times was aluminium. Magnesium, on the other hand, gave at all times consistent results. The probable reason is that the former contained traces of silicon, for on dissolving it in aqua regia, a small brownish residue remained, insoluble in hot aqua regia or in hot dilute nitric acid. The magnesium, on being dissolved in hydrochloric acid, gave no visible residue.

The pure silicon gave rectification in direction *B* to a small extent, and at times gave none at all, whereas the fused silicon usually gave considerable rectification in the direction *A*, very rarely giving as low a difference as 0.1 volt, and never giving the same current for the two directions. The resistance, also, was usually above that for the contacts with pure silicon. The test with pure silicon was difficult, as it was hard to break off the ends of the small, needle-like, crystals without making the fracture close to the surface of the solder.

Rectification with galena was uncertain, as in air, but more current was obtained in the directions *A*, *B*, and *A*, with Al, Fe, and Pb respectively. With graphite there was no rectification with any metal, save for occasional slight differences in either direction. In all the foregoing experiments, the tests were made with slow reversals.

Some of these experiments were so suggestive of an action due to a resisting film of oxide, especially those with tellurium, that experiments with melted tellurium were carried out in air, as a check upon the work. When a current of 2 amperes at 10 volts was passed through tellurium, melted in a quartz tube, with Pt and Al electrodes the current quickly

fell—when the direction was that of higher resistance—to about $\frac{1}{2}$ ampere. On opening the circuit and closing it again, the value was still $\frac{1}{2}$ ampere, but if the circuit was opened, and 1.5 volts applied in the same direction, the current was too small to give a readable deflection; *i. e.*, < 0.0002 ampere. On reversing, however, the deflection grew slowly until it reached 8×10^{-3} ampere. The large current, passed in the opposite direction, resulted in a deflection at once, on applying the smaller E.M.F. in the same direction.

When the quartz tube was placed in a thermostat, and the aluminium wire broken off under the surface of the melted tellurium, the results were not very satisfactory owing to the heat conducted away by the aluminium wire. On one occasion, however, the current was sensibly the same in both directions, but rose in the direction *B*, and fell in the other direction. A more satisfactory arrangement was had by melting a piece of tellurium in the open air, on a piece of sheet aluminium hammered into the form of a shallow dish. As the tellurium melted, a small quantity of greenish fumes was given off, after which it remained quiet, "wetting" the aluminium surface. When a platinum wire touched the melted tellurium, and a current was passed through, it was found to be steady and closely the same for both directions. The resistance rose somewhat when the substances were cooled. An aluminium wire touching the solid tellurium now formed a rectifier—the contact with the sheet aluminium serving as a low resistance contact. When the whole was heated the aluminium wire gave good contact, and when the drop of tellurium was lifted by means of it, and replaced, rectification was obtained in the direction opposite to that just preceding.

EXPERIMENTS IN VARIOUS GASES.

Contacts, not chemically clean, carrying small currents showed no loss of rectification when placed in various gases. With large currents, certain effects were observed. In nitrogen there was less bright illumination than in air; *e. g.*, with Te and Al or Mg. Oxygen gave brighter illumination than air, and Mg in contact with Te was ignited by a sufficiently heavy current.

A three-way cock in the system of tubes connected with the Gaede pump made filling the apparatus, Fig. 3 (*a*), with gas a simple matter. The system was exhausted to a Crookes vacuum, while the gas in question passed from the generator and purifying tubes, through the three-way cock. The cock was then turned to permit the flow of gas into the system at any rate desired. After removing the apparatus from the pump, the specimens were broken off or filed as previously described.

HYDROGEN.

Hydrogen was made by electrolysis of KOH solution in a water voltmeter and was dried by being passed through a tube containing glass beads and phosphorus pentoxide. Tellurium showed rectification of one or two hundredths of a volt in the direction *B* with all the metals, except once when Pb showed a small preponderance of current in the direction *A*. Fused silicon showed rectification for *A*, with all the metals. Carbon gave currents nearly the same for the various metals, while galena usually gave the greater current for *B* with Al, and *A* with Fe, although in the latter case the readings were often the same, as was the case with Pb.

CARBON DIOXIDE.

Carbon dioxide was generated by the action of hydrochloric acid on marble, purified by passing through potassium hydroxide solution, and dried with phosphorus pentoxide. Tellurium gave usually the same current in the two directions with all the metals, there sometimes being a difference in favor of the direction *B* of one or two hundredths of a volt. The difference, before filing, amounted to tenths of a volt. Fused silicon gave rectification in the direction *A* with every metal. Galena gave much the same values of current, with all the metals, there being rectification in the direction *B* for some contacts with Fe. Graphite, also, showed no rectification except a few times for *A* with Al and Fe.

NITROGEN.

Nitrogen was produced by heating a solution of ammonium chloride and sodium nitrite. It was cleaned by passage through chromic acid and potassium hydroxide solutions, and dried with phosphorus pentoxide. Tellurium showed currents in the two directions with Fe at various currents. The same held with Pb. The currents were the same for Al, with an increase in the direction *B* for some positions. Magnesium gave equal currents, and currents greater for *B* or *A*, depending on the position. The pure Si (Bausch and Lomb) gave the same current in both directions, while the potential varied as much as one hundredth of a volt. This held good for Al as well as the other metals. The fused Si, on the other hand, gave definite rectification in the direction *A*, being especially large with Al. Galena against Mg and Fe gave greater current in the direction *A*, Al in the direction *B*, while with Pb, the currents were either equal, or that for *A* was greater. With graphite the currents in the two directions were equal, except for occasional increases for *A* with all four metals.

OXYGEN.

The oxygen was prepared by heating equal parts of pure potassium chlorate and manganese dioxide, and was freed from any moisture by passage through phosphorus pentoxide. An escape tube, extending several centimeters into mercury was provided to guard against excessive pressure. Tellurium (previously melted in vacuo) with oxygen gave rectification with Mg and Al as well as in air, after a few reversals. The same held true for Fe and Pb, but to a somewhat less extent. Pure Si gave more or less rectification in the direction *B*, for various values of current, although sometimes the currents were the same. Fused Si rectified in the direction *A* with all metals, and all contacts except once with Fe when both currents were equal. Galena gave rectification in the direction *B*, strongly with Mg and Al, usually with Fe, and more or less with Pb. With graphite, the currents were sometimes in the direction *A* and sometimes equal; Pb gave, occasionally, a greater current in the direction *B*.

CONDUCTIVITY OF METAL FILINGS IN AIR.

As a check upon the previous work, a study was made of the conductivity of metal filings in air and in vacuo. Eccles¹ observed that the coherer does not obey Ohm's law, and the writer² has found asymmetry with the coherer, although the phenomenon is not so definite nor so easy to obtain as the deviation from Ohm's law. A case of asymmetry with two copper wires³ has already been mentioned.

In the present experiments, filings of copper, aluminium, and iron, produced and studied in air, were first examined. The deviation from Ohm's law is considerable, Fig. 5 (*a*) and (*b*). Asymmetry was observed with each of these metals, the current being as much as 45 per cent. greater in one direction than in the other. If the current were kept in one direction for some time, the conductivity rose, but fell in the other direction, as could be seen by making occasional reversals. Aluminium did not show this last effect well, and, in fact, needed considerable pressure to have current flow at all.

APPARATUS FOR PRODUCING METAL FILINGS IN VACUO.

In order to produce and examine, in vacuo, a sufficient quantity of filings, the apparatus shown in Fig. 4 (*a*) was designed. A file fastened by a joint to an iron rod, to increase the inertia, was enclosed in a long glass tube, *T*, with springs to prevent the file breaking through the ends

¹ Eccles, *Electrician*, 47, pp. 582, 715, 1901.

² R. H. Goddard, *Phys. Rev.*, 28, p. 411, 1909.

³ Shaw and Garret, *Phil. Mag.*, 8, p. 164.

of the tube. To this tube were attached various other tubes: a Crookes tube to test the vacuum, and a tube, T_1 , one end of which could be attached by the ground joint, to the Gaede pump, while the other end—joining T —supported the specimen to be filed, shown enlarged, Fig. 4 (c).

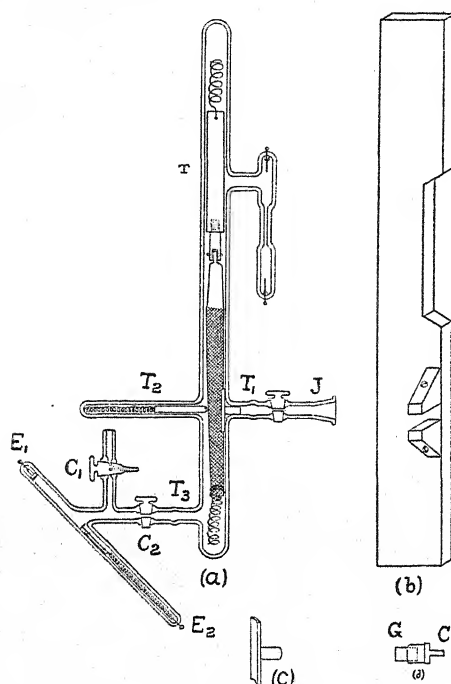


Fig. 4.

In order to press the file against the specimen, an iron rod and a spring were placed in the tube, T_2 , the rod being provided at the end with a steel ball, which pressed against the file.

To produce filings, the apparatus was placed on the board, Fig. 4 (b) so that T_1 rested in the notch of (b), as near the junction with T as possible, and the whole was shaken back and forth. With this method of holding the apparatus there was no shearing stress on the glass, and filings could be obtained almost as rapidly as with a vise in air. In practice, however, the rate was not made excessive, for fear of heating the specimen, and thereby breaking the glass.

When a sufficient quantity of filings had been produced, the tube, T , was turned until the filings dropped through the tube, T_3 , into a kind of coherer, in which the conductivity could be tested by means of the electrodes E_1 and E_2 . These electrodes were of copper, of a size to fit the glass tube, with platinum foil soldered to the faces to give good

contact. The electrode, E_2 , was connected to the platinum leading-in wire with a fine, flexible spiral of copper wire, and could be moved from outside the tube by a magnet. Small quantities of hydrogen and oxygen, produced electrolytically, and dried with phosphorus pentoxide, could be admitted to this coherer by way of the three-way cock, C_1 , after C_2 had been closed.

In taking the apparatus apart it was necessary simply to open the upper end of T , and cut off T_1 and T_3 at the points where the letters, T_1 , and T_3 , are placed. When the file and specimen were introduced or removed, the rod holding the steel ball was pushed out of the way by a stiff wire, inserted between the specimen and the wall of T_1 , the end of the wire engaging the crimp that held the ball in place. After each experiment, all filings were removed from the tubes and the surface of the file.

CONDUCTIVITY OF METAL FILINGS PRODUCED IN VACUO.

With copper, in vacuo, there was no asymmetry, nor change of conductivity with time, nor deviation from Ohm's law as observed in air—there being, in fact, a slight falling-off of current¹ with E.M.F., Fig. 5 (c). Occasionally, however, as the voltage was increased in steps of about 1/500 volt, the current jumped to a higher value, thereafter obeying the same law. This is in agreement with the theory of Eccles that deviation from Ohm's law is due to turning of the irregular particles under the influence of the electrostatic field, thus furnishing more contacts at higher voltage. It shows, further, that a film is necessary, to prevent fusions, *i. e.*, good contacts, before this turning can occur. Sparks from a Hertz oscillator several feet away reduced the resistance to a low value.

When oxygen, prepared as above described, was admitted, the resistance increased about 25 times, the deviation from Ohm's law appeared, Fig. 5 (d), and asymmetry was observed, there being a tendency for the conductivity in the direction in which the current was flowing to be higher than in the other direction, as in air. When, however, hydrogen was admitted to the copper filings produced in vacuo, there was no noticeable asymmetry, and the current-voltage characteristic, Fig. 5 (e), was the same as in vacuo. On removing the hydrogen by opening the cock, C_2 , and then admitting oxygen (repeating the operation several times, to remove all the hydrogen) deviation from Ohm's law appeared, Fig. 5 (f). In all experiments, the filings were, of necessity, moved about when gases were admitted.

¹ The resistance of the coherer has been found to increase with increasing temperature, owing, probably, to expansion and breaking of some of the contacts. This may account for the deviation in vacuo, if increased heating, due to increasing the current, can produce the same effect.

Aluminium and magnesium did not give results so satisfactory, as a certain amount of pressure was required to give conduction—though much less than that required in air or oxygen. Aluminium filings produced in vacuo, gave deviation from Ohm's law as with copper—Fig. 5

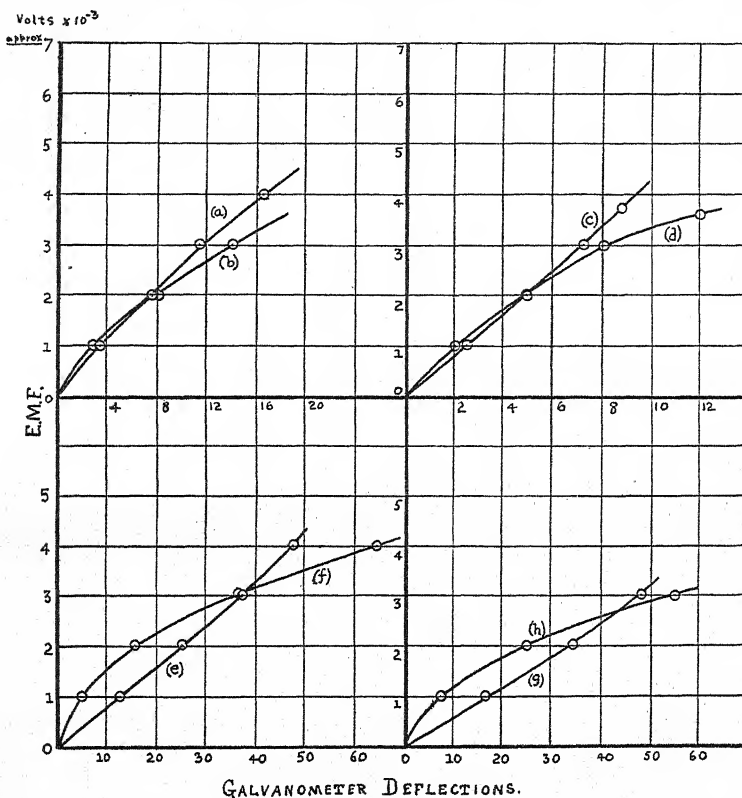


Fig. 5.

(a) Copper, in air; (b) iron, in air; (c) copper, in vacuo; (d) copper, in oxygen after vacuo; (e) copper, in hydrogen after vacuo; (f) copper, in oxygen after hydrogen; (g) aluminium, in vacuo; (h) aluminium, in vacuo.

(g); although occasionally as (h), which, since pressure is necessary for conduction, is easily explained by turning of the particles. No asymmetry was observed. When oxygen was admitted the resistance increased greatly, and the deviation from Ohm's law was large. Hydrogen, acting on filings produced in vacuo, necessitated a little greater pressure for conduction, gave deviation from Ohm's law as in vacuo, and no asymmetry.

It was thought that impurities in the aluminium might necessitate the application of pressure, but it was found that magnesium filings required pressure at all E.M.F.'s, even when no ground joints, *i. e.*, vacuum wax, was used. Magnesium behaved in other respects like aluminium.

ODOR OF ALUMINIUM AND MAGNESIUM FILINGS.

It may be of interest to note, in passing, that the filings of aluminium and magnesium, when produced in vacuo, gave a strong odor somewhat resembling garlic even when the tube was not opened until over a week after performing an experiment; and the odor in the tube still persisted a day or so after air was admitted. A piece of either metal, filed in the air, lost its odor in less than half a minute.

CONDUCTIVITY OF GALENA, POWDERED IN VACUO.

In order to test whether or not the peculiarities of conductivity across contacts of chemical compounds depend upon the surrounding gas, galena was filed in vacuo, and the resulting powder examined. The apparatus was similar to that shown in Fig. 4 (a), except that there were no cocks, C_1 and C_2 .

The galena, Fig. 4 (g), was first set in a block of solder, after which this block was soldered to a piece of copper, turned in a lathe to the form shown in section, (c). This was fastened to the holder previously described, Fig. 3 (b), and in place of a spring to furnish pressure of the file on the galena, a lead rod was placed in the tube, back of the holder, so that the pressure of the galena on the file could be varied by varying the angle at which the apparatus was held.

In air, galena filed to a powder, showed deviation from Ohm's law, Fig. 6 (a), and asymmetry—there being a difference in conductivity

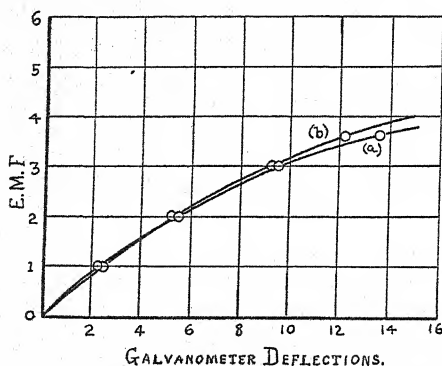


Fig. 6.

Powdered galena. (a) In air; (b) in vacuo.

of 10 to 15 per cent. for the two directions. The conductivity in the direction of an applied steady current increased faster than that in the opposite direction, observed by reversals, independent of which conductivity was the greater. In vacuo the deviation was still present,

Fig. 6 (*b*), together with asymmetry of the same nature and magnitude as in air.

OSCILLOGRAMS OF RECTIFIERS AT HIGH FREQUENCIES.

The foregoing experiments show that a film of some sort is necessary to produce peculiarities of conduction at contacts, and it remains to determine how such a film acts. The experiments which follow have thrown some light on this question, although not as much as was anticipated. It has already been shown by Pierce¹ that no effect is present for a time longer than $1/6,000$ of a second that is not accounted for by the voltage-current characteristics of the rectifier for steady currents. In view of the fact that crystal detectors are being used as receivers of waves of 200,000 or more frequency, and further as Merritt² has demonstrated that the silicon detector will act for a wave-length as short as 15 cm., it seemed advisable to obtain oscillograms for frequencies of several hundred thousand, if possible.

APPARATUS.

The method will be understood by an examination of Fig. 7. A Poulsen arc was used to produce the high-frequency oscillations, as being the best means of producing a continuous train of undamped oscillations, except by using a high frequency alternator. The arc was used by Dr. Story³ in a research on the Poulsen arc. A resistance, P_1 , and an inductance, not shown, were in series with the arc. In the high frequency circuit was a variable oil condenser, C , and an inductance consisting of three coils, S , S_1 , and S_2 , of the same shape and size. These were arranged so that all the current passed through S and a variable carbon resistance, P_2 , and then branched, part of the current going through a group of rectifiers in parallel, R_1 and the coil S_1 , and the other part, through an equal number of rectifiers, R_2 , arranged so as to give rectification in the opposite direction, and thence through the coil S_2 . The coils

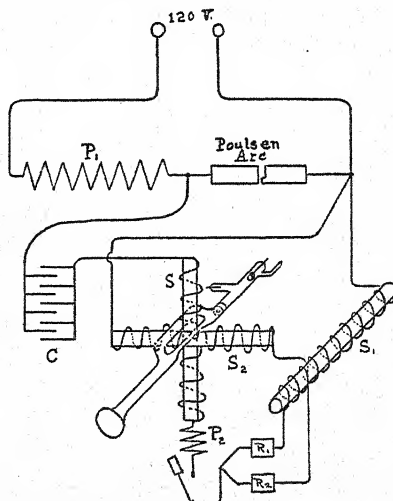


Fig. 7.

¹ G. W. Pierce, *PHYS. REV.*, 28, pp. 153-189, 1909.

² Ernest Merritt, *PHYS. REV.*, 32, p. 630.

³ W. E. Story, Jr., *PHYS. REV.*, 30, pp. 236-261, 1910.

S and S_2 were made to act at right angles to each other, on the beam of cathode rays in a Braun tube, whereas S_1 was a dummy coil. It is easily seen that the total current in the circuit remained the same, for if the rectifiers R_1 hindered the passage of current through S_1 , the rectifiers R_2 , acting in the opposite direction, would let current through S_2 , and vice versa. The rectifiers were made as follows: a large number of pieces of the crystal, to which there was good contact (*e. g.*, by setting the pieces in solder, or clamping them under brass plates) were fastened to a board. A groove was made around the rectifiers, near the edge of the board, and holes were drilled at intervals in the bottom of this groove. Contacts were made by inserting wires in the holes, and bending them until they touched the crystals. After all the wires had been separately adjusted the groove was filled with mercury, thus connecting the wires in parallel.

The Braun tube, made by the writer, was of the special form shown in

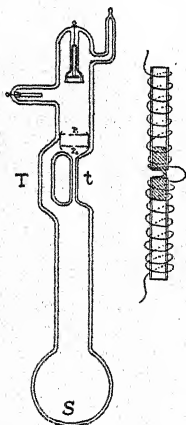


Fig. 8.

Fig. 8. The cathode beam was produced by a hole about $1/3$ mm. in diameter, in an aluminium disk shown in section, P_1 . In this disk was another, larger, hole to facilitate exhaustion of the tube. The rays passing through this second hole were stopped by the aluminium disk P_2 . Deflection took place in the tube, t , which was but 2 mm. inside, and 3.3 mm. outside, diameter. A larger tube, T , served to strengthen the apparatus and aid in evacuating the tube. The screen S was made by spreading powdered willemite, mixed with alcohol, on the inside of the bulb and letting it dry—after J. J. Thomson. The part of the main tube below T and t was covered with tin foil and connected, with the anode, to earth. The distance from P_1 to S was 40 cm., and a current of 0.3 ampere

in one of the coils gave a deflection of 6 mm. In practice, the current varied from 0.3 to 0.7 ampere.

Each coil was a single layer of cotton-covered copper wire, 0.5 mm. in diameter, wound on a glass tube, 1.22 cm. in diameter, and soaked in melted paraffine. They were 20 cm. long, with 17 turns to the cm. In order to bring the two halves of these coils as near the tube, t , as possible, they were held in place by the notched plugs of wood, shown in section in the figure.

Instead of using a high potential storage battery, it was found possible to use a Wimshurst electric machine, with or without a condenser in parallel, provided a high resistance formed by a pencil mark on ground glass, was placed in series. With the proper resistance (not easy to get)

the cathode beam was extremely steady, and the glow in the tube persisted for a minute or more after stopping the machine. Lines ruled on paper with india ink¹ were more satisfactory.

EXPERIMENTAL RESULTS.

The oscillogram, a Lissajous' figure, shown in Fig. 9 (a), was obtained with silicon against steel, under exceptionally good circumstances. There were 30 contacts in each group, R_1 and R_2 , and the calculated frequency was 410,000. The phenomenon could be repeated as many times

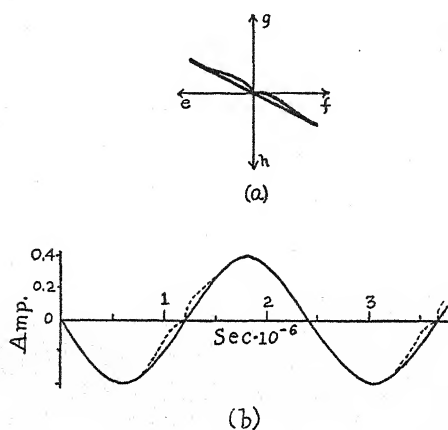


Fig. 9.

f , direction of total current; g , direction of greater rectified current; h , direction of lesser rectified current. Frequency 410,000.

as desired, but the arc did not remain steady longer than two or three seconds, too short a time, it was found, to affect a photographic plate. The failure of the arc to give oscillations for a longer period was no doubt due to the comparatively large resistance of the groups of rectifiers. At other times these curves were only feebly approached, which was true also, of the same number of contacts with copper, molybdenite and aluminium, tellurium.

The conclusions which are indicated are shown in Fig. 9 (b), where the deviation is from a sine wave form instead of the straight line in the Lissajous' figure. There is, apparently, an opposition to the current, or a back E.M.F., which appears for about 4×10^{-7} second, followed by an increase in current lasting the same length of time, when the current is reversed. It strongly suggests a crowding of ions against the film, and the springing-back of these ions at the instant the current is reversed.

¹ F. A. Aust. PHYS. REV., 32, pp. 732-733, 1911.

CRITICISM OF THE EXPERIMENT.

It seems likely that important information could be obtained by using a high-frequency alternator, thus making it possible to obtain photographic exposures of a minute or more. Perhaps, also, more persistent oscillations could be had with the arc by using a great many contacts in parallel, to reduce the resistance. In either case a large number of contacts would be necessary, each adjusted for maximum rectification, and for equality with the other contacts as to current in both directions. If the voltage were considerable, it might be necessary to place several of these groups in series. A smaller hole in the diaphragm, P_1 , could be used to advantage, with a long exposure.

CONCLUSIONS.

1. Pure elements give little or no rectification against pure metals, unless oxygen (or an active gas) is present. The evidence in support of this is as follows:

(a) Pure tellurium and pure silicon give practically no rectification against metals, in vacuo, hydrogen, nitrogen, and carbon dioxide, but behave in oxygen as in air. The conclusion is also supported by the action of an aluminium wire broken off under melted tellurium. Natural graphite is at best uncertain for slow alternations.

(b) A piece of tellurium, melted in air in contact with a piece of aluminium, shows unilateral effects or not, depending upon whether or not the tellurium adheres firmly enough to "wet" the aluminium surface.

(c) Copper filings in the form of a coherer, produced and examined in vacuo or hydrogen, show no asymmetry, and deviate from Ohm's law with an upward slope, but in oxygen give the well-known deviation observed with filings produced in air. Aluminium and magnesium do not, however, show such conclusive behavior.

(d) Currents of 0.5 ampere or more, in air, give evidence that whatever unilateral effects appear are due to the presence of oxygen, for the following reasons: the direction of greater current, for sufficiently large current, is from metal to substance regardless of the direction for small currents. This is the direction for tellurium for all currents; and this element, by (a), rectifies owing to the presence of oxygen. Further, for large currents there appear incrustations, with colors characteristic of the oxides of the substances used. The current in the direction of metal to crystal is the more unsteady, for large currents, as it is with tellurium.

2. Rectification, other than that due to the presence of oxygen, occurs at contacts of impure elements or chemical compounds.

(a) Impure, fused, silicon rectifies under all circumstances.

(b) The behavior of galena in air and vacuo also lends support to this conclusion, although it is, at best, an uncertain rectifier.

(c) Galena, filed to a powder in vacuo, shows deviation from Ohm's law exactly as in air. The deviation is of the same character as that observed previously by the writer for many other powders in air, the effect appearing under all circumstances except when the powders were subjected to great pressure.

(d) Chemical compounds, in air, usually rectify in the direction opposite to that with pure elements in air or oxygen.

3. The phenomenon is due, primarily, to the presence or formation of a film at the contact.

(a) Pure elements require the presence of oxygen (therefore of oxide) at the contact.

(b) The resistance of copper filings produced in vacuo increases 25 times on admission of oxygen. A similar change takes place with aluminium and magnesium filings.

(c) In many cases, with large currents, the current remains large, for a second or so, after reversing to the direction of lesser current. This indicates that the building-up of resistance requires a certain amount of time.

(d) In the experiment with melted tellurium, with large current, a small E.M.F., after application of a larger E.M.F., both in the direction of greater current, gave practically no current, but gave 0.008 ampere on reversing. This was evidence of a deposit which the small E.M.F. could gradually remove *if applied in the direction of greater current*. Also, since 10 volts gave $\frac{1}{2}$ ampere, while 1 volt gave no readable deflection, there is evidence that the larger current kept flowing either because the heat developed, or the E.M.F., or both, continually broke down the film.

(e) The experiment with Fe_3O_4 , Si, large current, in which a single spark was visible (microscopically) when the current was passed in the direction of lesser resistance, five minutes *after* the circuit had been broken with the current in the direction of higher resistance, indicates the breaking-down of a resistance at the contact. The continued sparking in the direction of higher resistance, at large currents, with this and other contacts, indicates that the irregularities observed for this direction, A, are due to breaking down and sparking. Further, the changes in brightness of the substances and metals, on reversal of current, are indicative of changes in the place of greatest heat production, or changes in heat conductivity, but are not so easy to interpret as the following special experiments.

(f) When galena, Fe_3O_4 , or silicon were placed between the ends of two aluminium wires, and a current of two amperes passed through, blue sparks appeared and the metal was redder, on the side the current entered the crystal. The entire substance was red hot. In this case a comparatively large region was heated, and if oxygen was deposited, as might be expected, on the anode wire, more heat should be generated, owing to greater resistance, at this surface. Hence the metal should be redder, as was observed. Continued sparking showed continued breaking down of the film—presumably of oxygen with Fe_3O_4 and silicon, and of sulphur with galena.

On the other hand, in the experiment in which a large current was passed through a small contact between two large pieces of silicon, the silicon on the side at which the current passed *from* the contact was redder. The probable explanation is that a *small* region was heated, owing to the very refractory nature of silicon, and that the heat generated in this small region passed more easily to the piece of silicon on which there was no film of oxide, since the thermal conductivity of the film was probably small compared with that of the hot silicon.

(g) The experiments mentioned at the beginning of the paper, concerning the change in resistance of rectifiers measured by a very small current, after a current of about 10^{-2} ampere had passed through the contact are indicative of the building up of some sort of permanent resistance, but the data do not show sufficient regularity to enable further conclusions to be drawn.

4. Solid rectifiers are, apparently, analogous to the aluminium valve, or electrolytic rectifier. In this rectifier a solid film is produced and large ions are packed against it, giving a back E.M.F., when the current passes in one direction, whereas smaller, negative, ions pass freely through the film when the current is in the opposite direction. The greatest point of dissimilarity is the continued breaking down of the resisting film, in the case of solid rectifiers, probably due to heating.

(a) The hysteresis effect observed by Pierce with carborundum, for the first few reversals, together with the fact, observed by the writer, that tellurium cleaned in vacuo required a few reversals before showing good unilateral effect, when oxygen was admitted, are both analogous to the slow building up of resistance with direct current, for a fresh plate in the aluminium valve, before an alternating current has been applied.

(b) The oscillogram at 410,000 frequency suggests the building up of a back E.M.F., which gives a slight additional current at the moment the current is reversed. This conclusion is, however, based upon a single experiment.

SUMMARY.

1. A large number of experiments with currents up to 9 amperes suggested the necessity of using contacts of as nearly chemically clean surfaces as possible.

2. By means of a glass apparatus which could be evacuated to a Crookes vacuum, it was possible to break or file the ends of substances and metals in vacuo, and measure the conductances. Proceeding in this way it was found that tellurium and pure silicon lost most or all of their power to produce rectification when cleaned as above, in vacuo, hydrogen, nitrogen, and carbon dioxide, but behaved in oxygen as in air. Fused silicon and galena gave rectification in vacuo and all gases; the former always, the latter often. Galena and natural graphite were found uncertain even under ordinary conditions.

3. Copper filings produced and examined in vacuo and hydrogen did not show the anomalies of conduction manifested in air and oxygen. Aluminium and magnesium gave uncertain results. Galena powdered in vacuo by filing showed the same anomalies as in air.

4. The experiment with copper filings supports the theory of Eccles, that the deviation from Ohm's law of the coherer is due to turning of the particles, due to electrostatic forces, so that the long axes point in the direction of the current. Although Ohm's law was practically obeyed, there were sudden increase in conductance on raising the voltage. There was little evidence of turning in general, however, unless a film of oxide was present—although Hertz waves greatly increased the conductance.

5. From the experiments mentioned in (2) and (3) it is concluded that rectification is of two kinds, "surface" and "body" rectification, and that the former takes place with pure elements in an active gas, and the latter with impure elements and chemical compounds, irrespective of the nature of the gas present.

6. Many experiments with contacts in air carrying large currents, and contacts in vacuo, showed phenomena which suggested that a film of some sort is necessary in order to have rectification. These experiments, together with an oscillogram of a number of silicon, steel rectifiers in parallel indicate that the action of the solid rectifier is like that of the aluminium valve, or electrolytic rectifier; *i. e.*, a film is formed which hinders the motion of certain ions—with this difference, that, in solid rectifiers the film is broken down by heat, or sparking, so that some current usually flows in the direction of higher resistance.

CLARK UNIVERSITY,

January 10, 1912.

AN ELECTRICAL METHOD OF MEASURING SMALL INTERVALS OF TIME.

By F. C. BROWN.

WHEN it is desired to measure the time between two mechanical operations, we have found that it may be carried out quite accurately by the use of resistances, battery, ballistic galvanometer, condenser, and contact keys. The principle is based on the relation between the electro-magnetic and the electro-static systems of units, and if we define the ohm in terms of the dimensions of a mercury column, the method is an absolute one. Aside from the advantage of having an absolute measure of time in terms of length, or of length, mass and resistance, the advantages of the method are simplicity and ease of manipulation. The interval of time to be measured may be diminished to the limits of dependable electrical contact, no doubt less than 0.001 second. How large the interval may be varies from 0.4 second to several seconds depending on the period of the galvanometer.

THEORY.

The theory of the method is essentially the same as that given in a previous paper on a method of measuring fluctuating resistances,¹ but as Dr. Erich Schneckenburg, of Berlin, has kindly pointed out that the method of derivation of equation (1) in that paper is not obvious, I shall briefly review the essential steps.

Suppose it is desired to know the interval of time, Δt , between the making of electrical contact at t_1 , and the time of breaking of contact at t_2 . It is to be shown that this time interval varies directly as the throw of a ballistic galvanometer which has been connected, as shown in Fig. 1, in a Wheatstone's bridge circuit during the interval.

Considering the battery resistance small compared to the other resistances, we obtain, by Kirkchhoff's and Ohm's Laws, the current through the galvanometer to be

$$i = \frac{E(aR - bx)}{(k_1x + k_2)^2}, \quad (1)$$

where

$$k_1 = Gb + aG + ab + aR + bR,$$

$$k_2 = aRG + bRG + abR.$$

¹ By Brown and Clark, *PHYS. REV.*, XXXIII., p. 53, 1911.

By differentiating equation (1) with regard to x , we have

$$\frac{di}{dx} = - \frac{E(bk_2 + k_1aR)}{(k_1x + k_2)^2}. \quad (2)$$

We may write this equation in the following form and keep within the accuracy of laboratory experimentation,

$$\Delta i = - \frac{E(bk_2 + k_1aR) \cdot \Delta x}{(k_1x + k_2)^2}, \quad (3)$$

where it is understood that if the value of Δx is relatively too large a small correction must be added.

Now if it is agreed that Δx is to be measured as the amount of variation of x from that value which gives no current through the galvanometer as required by equation (1), then obviously $\Delta i = i$, and since the negative sign is of no importance,

$$i = \frac{E(bk_2 + k_1aR) \cdot \Delta x}{(k_1x + k_2)^2}. \quad (4)$$

Since the quantity of electricity that passes through the galvanometer as the result of the closed circuit during the interval Δt is $Q = i \cdot \Delta t$, and since in a ballistic galvanometer $K \cdot d = i \cdot \Delta t$, where K is the usual constant as obtained by a standard condenser and a known electromotive force, it follows that,

$$\Delta t = \frac{K \cdot d \cdot (k_1x + k_2)^2}{\Delta x \cdot E \cdot (bk_2 + k_1aR)}. \quad (5)$$

The only qualification, that should be made to this result is, that either the galvanometer coil must not have moved over an appreciable distance during the time interval to be measured, or that the effective resistances in the closed circuit should be so large that the damping action can be neglected. I take it that it is quite generally understood that self induction in the coil of a ballistic galvanometer can be disregarded. At any rate the experimental results in this paper will obviously warrant the assumption.

If a standard condenser of capacity C is charged by the same electromotive-force E , and then discharged through the same galvanometer, thereby giving a deflection d' , we may write equation (5) in the form,

$$\Delta t = \frac{C \cdot d \cdot (k_1x + k_2)^2}{\Delta x \cdot d' \cdot (bk_2 + k_1aR)}. \quad (6)$$

We therefore have the interval of time between the two contacts.

measured in terms of the units of length and the units of resistance, and if we choose to define the unit of resistance in terms of the dimensions and the mass of a mercury column, our fundamental system of units may be regarded as solely based on the units of length and of mass. The relative merits of the unit of time as a derived unit will be discussed later.

EXPERIMENTAL TEST.

In order that the method may be workable it is sufficient to show that the time interval, between contacts at t_1 and at t_2 , varies directly as the deflection of the galvanometer, and that the computed value of the interval agrees with that observed. In our former paper the relation between the deflection and the time was recorded, but the linear relation did not hold for greater values of the time than 0.2 second. The lack of agreement beyond this point has since been found to have arisen from inaccurate theoretical calibration of the timing pendulum, and not to the fact that the galvanometer coil was not in its zero position while the entire quantity of electricity was impressed, as was previously supposed. Using correct values of the time as obtained by an experimental calibration of the pendulum, Messrs. T. Ingwaldson and J. S. Hilliard of our laboratory have obtained the observations shown on the curve in Fig. 2. Each point shown on the curve is the average of four observations. The accompanying table gives a fair representation of the reliability of the observations for a single setting of the keys. However it was found at the close of the observations that there was an error of about one mm. in the throw of the galvanometer due to the inaccurate setting of the keys.

TABLE I.

Time Interval Between Keys. Seconds.	Throw of Galvanometer, Mm.
0.009.....	6.0; 5.8; 5.5; 5.7.
0.045.....	26.5; 26.8; 26.8; 26.8.
0.38.....	222.0; 222.0; 221.8; 222.4.
0.09.....	53.0; 53.0; 54.0; 53.5.

The indicated values of the resistances used for balancing the bridge were, $a = b = x = 9,000$ ohms, and $R = 9,004.6$ ohms. The source of electromotive force was a storage battery of 38.0 volts. The value of R indicates the probable error in the resistances. In order to obtain the observations given above the value of x was changed to 9,100, thereby giving a value of $\Delta x = 100$ ohms. The resistance of the galvanometer was 950 ohms, and its period was 26.7 seconds.

The linear relation between the throw of the galvanometer and the time holds between 0.002 second and 0.32 second as accurately as the

particular apparatus was capable of testing. Of course for longer periods of time there would appear appreciable errors due to the swinging of the galvanometer coil in a closed circuit of high resistance. It is possible to correct for this error or to diminish it greatly by increasing the resistances and by increasing the period of the galvanometer. By increasing the mass of the galvanometer coil without increasing its inductance, *i. e.*, such as is done by adding weights to the coil, will clearly doubly lessen the relative error due to the cause above mentioned. But as adding mass in this way also decreases the sensibility, I am not prepared to say how far the last idea could be carried satisfactorily. To show the adaptability of the method for measuring intervals of time larger than those given in Table I., the period of the galvanometer coil was increased to 91 sec. by using a phosphor bronze strip suspension 1 meter in length.

TABLE II.

Deflections.

<i>1</i> Period.	Current Direct.	Reversed.	
0.9 second.	33.6	28.0	
	34.0	26.0	
	32.0 zero at 3.5	27.8	
	34.2	26.0	
	32.8	26.2	av. 30.1
2 periods.	56.5	65.5	
	57.0	64.5	
	55.8 zero at 5.0	64.1	
	55.0	64.1	
	55.4	66.1	av. 60.5
3 periods.	94.5	92.3	
	87.2	94.0	
	87.8 zero at 3.0	93.1	
	90.0	92.0	
	86.0	92.5	av. 90.7
4 periods.	109.2	116.0	
	108.0	118.2	
	110.5 zero at 4.0	117.0	
	110.2	117.5	
	110.5	118.4	av. 113.7
5 periods.	139.0	131.0	
	139.2	130.1	
	139.2	129.5	
	138.9	129.3	
	138.5	131.2	av. 134.6

The time interval was measured in terms of the number of swings of the pendulum. The keys were knocked down always at the middle of the arc. The period was about 0.9 second and the intervals were therefore some multiple of 0.9 sec. The observations are shown in Table II. The resistances used for balance in the bridge were 9,000 ohms each, and Δx was 500 ohms. A single dry cell was in circuit. When the galvanometer period is so large it is highly advisable to determine the zero position from the amplitude readings to the right and left such as is practiced in balance work. After the zero position is found it is very easy to

bring the coil to rest.

It is observed that up to 3 periods, *i. e.*, 2.7 seconds, the deflection is proportional to the time interval, but beyond this the deflection falls off quite rapidly.

However by replacing the 9,000 ohm resistances in *a* and *b* (see Fig. 1) with 100,000 ohm values each, the deflection was proportional to the time up to about 3.6 seconds as shown in the following table, where each recorded value is the average of ten observations.

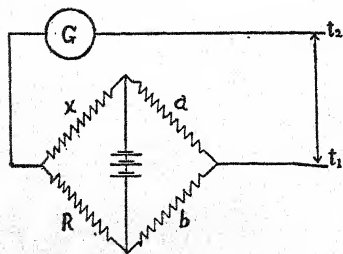


Fig. 1.

TABLE III.

2 periods	55.0
4 periods	110.7
5 periods	132.6

The foregoing results would seem to indicate that for large intervals of time it is more important to have a long period than it is to have extremely high resistances.

In order to test the adaptability of the method for absolute determinations of time, the constant of the galvanometer as obtained by equation (5) was compared with the value as obtained by the well-known battery and condenser method. By the current method the value was calculated to be $K = 1.77 \times 10^{-10}$ coulombs per mm. and by the second method, $K = 1.82 \times 10^{-10}$. This variation is quite within the limits of error of the capacity and the resistances used. As we did not have a standard condenser or resistances of the highest accuracy, it was not considered worth while to investigate the possible accuracy of the method any further.

As a practical laboratory method of measuring small intervals of time I believe that the scheme presented should be useful in certain kinds of

work, wherever it is feasible to mark time by electrical contacts. We believe that it is advisable to reserve in every laboratory a ballistic galvanometer of very slow period primarily for measuring time intervals. By posting at this galvanometer particular values of resistance and of

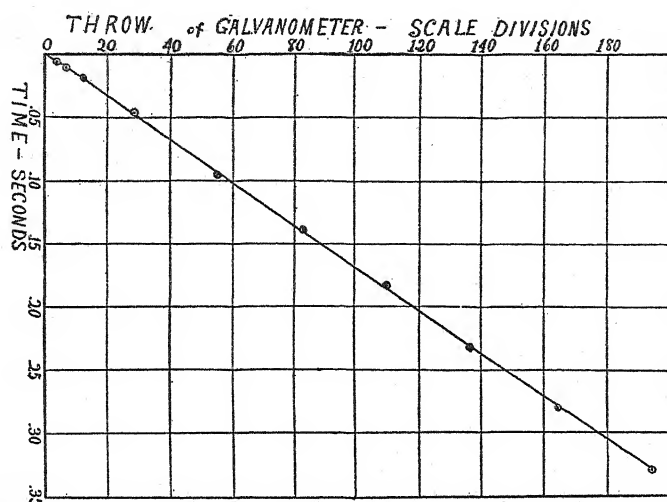


Fig. 2.

electromotive force to be used, and also the equivalent time interval, as calculated, for one division deflection, the troubles of calculations can be eliminated once for all. By taking only the most elementary precautions therefore the method should in many cases supplant a chronograph and standard clock. From this point of view the simplicity and quickness of manipulation are striking.

If it is desired to measure two or more time intervals simultaneously, several galvanometers and sets of keys may be connected in the same bridge circuit. The use of two galvanometers is illustrated, as used for measuring fluctuating resistances, in my previous paper on the recovery of the Giltay selenium cell and the nature of light-action in selenium.¹ If one observer wishes to read several time intervals simultaneously, he can do so by altering the periods of the galvanometers sufficiently so that he can have time to record each deflection.

THE ADVISABILITY OF THE SECOND AS A DERIVED UNIT.

Quite apart from the usefulness in the laboratory, are the considerations concerning the practicability of the method for determining the absolute unit of time. In this discussion any of the well-known methods used

¹ PHYS. REV., XXXIII., p. 412, 1911.

by standard laboratories might be substituted. The only question is whether it is best to have the second as a fundamental or a derived unit of time. A fundamental absolute unit should have two distinct merits. It should be one capable of being reproduced to a sufficiently high degree of accuracy, and it should be unvarying for all time. As is well known our present fundamental absolute unit of time fulfills the first requirement remarkably well, but there is considerable doubt as to how long it will remain fixed. Tidal action, the shrinkage of the earth, friction with meteorites and friction of the atmosphere with itself, in addition to friction of possible electromagnetic origin, all tend not only to vary the value of the second, but also to make it almost a hopeless task to satisfactorily correct the unit for future periods of time.

By way of comparison, there is no question but that we cannot reproduce the unit of resistance, and consequently the derived unit of time, with as great accuracy as we can reproduce the present absolute fundamental unit of time, but on the other hand we have no information that would lead us to believe that the unit of resistance, defined in terms of the dimensions and mass of a mercury column, would not remain fixed.

It seems wise to suggest therefore that it would be worth considering the advisability of adopting our present unit as a secondary standard, which might be rechecked and revised in the distant future, if the electrical method, with the ohm as the fundamental unit, should show the value of the second to be materially in error. At any rate the temporary establishment of the unit of time defined from as many sources as possible might lead to discoveries of far-reaching importance.

THE PHYSICAL LABORATORY,
THE STATE UNIVERSITY OF IOWA.

PHOTO-ELECTRIC POTENTIALS OF THIN CATHODE FILMS.

BY PAUL H. DIKE.

THE velocity with which electrons are emitted from a metal plate in a vacuum when illuminated with ultra-violet light has been investigated by several observers, beginning with Lenard, and has been studied with increasing activity in recent years. In the earlier investigations the velocities of the electrons were measured usually only as a subsidiary problem to the study of the current maintained by the stream of electrons given off. More recently, however, much work has been done on the study of the conditions affecting the initial velocities of the electrons.

Two general methods of procedure are open for the measurement of the electronic velocities. The minimum positive potential to which a plate must be raised to prevent the escape of the fastest electrons projected from its surface may be measured directly by allowing the plate to charge itself up to a maximum potential under the action of light, all other conductors in the neighborhood being earthed, or it may be determined indirectly by measuring the current produced, on exposure to the light, between the surface and a charged conductor placed near it, as a function of the difference in potential maintained artificially between the two conductors. By varying the potential on the second conductor a value is found for which the current becomes zero, the negative potential on the second conductor then being just capable of preventing the escape of any electrons from the surface.

For various reasons the results obtained by different observers for the value of the photo-electric potential are not very accordant, though they have one common feature, namely, the observed potentials are in general low. The stream of electrons projected from a metallic plate by the action of ultra-violet light may be considered as an aggregation of slow-moving cathode rays, and it has been shown only recently by von Baeyer and Gehrts that like other cathode rays they are subject to reflection and can produce secondary radiation on striking a surface, though they do this to a less degree than ordinary cathode rays and have a range of only a few centimeters. Hence, as might be expected, the size and shape of tube and the arrangement of the electrodes have been found to in-

fluence very strongly the results obtained. It is well known that the potentials may also be affected by the atmosphere and pressure in which the test was conducted, the wave-length of the light used, and the condition of the surface, but the potential has been found to be independent of the intensity of the incident light.

Some representative values are given below:

Observer.	Surface.	Max. Pot. Volts.	Remarks.
Ladenburg ¹	Platinum.	1.86	Electronic reflection neglected.
Ladenburg ¹	Copper.	1.69	Electronic reflection neglected.
Ladenburg ¹	Zinc.	1.12	Electronic reflection neglected.
Ladenburg and Markau ²	Platinum.	2.3	Electronic reflection reduced.
	Lampblack.	3.5	Electronic reflection reduced.
Hull ³	Lampblack.	3.5	Electronic reflection reduced.
Klages ⁴	Mercury.	2.3	Electronic reflection reduced.
Von Baeyer and Gehrts ⁵	Gold, copper and aluminum.	6.3	Electronic reflection reduced.
Millikan ⁶	Copper.	20	Tube not described.
	Aluminum.	12.5	

The observations reported by Millikan were carried out with plates which were kept for months in very high vacua with frequent prolonged exposure to intense ultra-violet light, giving unprecedentedly high potentials for iron, zinc, copper, silver and aluminum, in all cases many times higher than when the plate was first put in the tube. He apparently supposes that the light in some way cleans the surface, possibly by removing a gas layer, though such a result is quite the contrary of what would be expected from the usual behavior of a metal when so exposed to the light, a fatigue effect being in general observed.

It was with the hope of producing some similar effect by other means that the work to be described was undertaken. If the metal surface could be cleaned in vacuum, so as to free it from surface layers, or if it could be produced in vacuum and tested without exposure to any outside gases it seemed possible that the desired result might be achieved. The most feasible way of cleaning the plate appeared to be by means of the cathode discharge, using the plate to be cleaned as cathode, subjecting it

¹ Verh. der Deutsch. Gesell., 1907, p. 504.

² Verh. der Deutsch. Gesell., 1908, p. 562.

³ Am. Jour. of Science, XXVIII., Sept., 1909, p. 251.

⁴ Ann. der Phys., 31, 1910, p. 343.

⁵ Ber. der Deutsch. Phys. Gesell., 21, 1910.

⁶ PHYS. REV., XXX., Feb., 1910, p. 287.

to the bombardment of the canal rays and tearing off the particles which make up the cathode deposit. It has since been learned that this was a part of the process used by Millikan and by von Baeyer and Gehrts in preparing the plates which gave the high potentials, as well as by several other observers, notably Christler,¹ studying the photo-electric current rather than the potential.

Numerous tests were made of this method of preparing the surface, in two tubes of different types, in atmospheres of hydrogen and of nitrogen at very low vacua, using copper and iron as the metals to be tested. The results obtained showed that the potential to which either of these metals could be raised by the action of ultra-violet light from the quartz-mercury arc could be only slightly increased by prolonged use as cathode. The highest value observed for the photo-electric potential of copper was 4.0 volts, and for that of iron 3.1 volts. Copper polished with rouge gave a potential of 3.8 volts before being used as cathode.

As this line of attack did not yield the high potentials looked for, another process was tried, using the plate to be tested as a receptacle for a cathode deposit, and examining its photo-electric properties under various conditions and with varying thickness.

One end of a brass tube (Fig. 1) was closed with a piece of plate glass, *C*, into which the electrode leading to the electrometer was sealed by means of a hard cement, the inside of the plate being protected from the discharge by means of a brass shield, *D*, fitting loosely inside the tube. The tube was provided with two side tubes, one for admitting the light to the plate, *A*, and the other for an observation window, not shown in the figure, to permit watching the surface of the plate under test. A

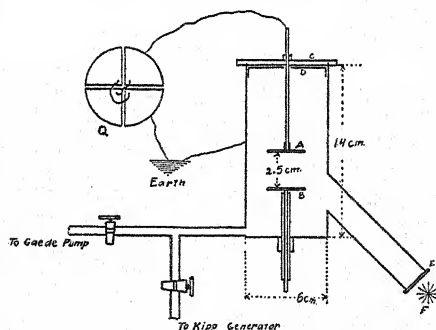


Fig. 1.

glass plate with a good optical surface formed the receptacle for the film, a connection being made from the front of the plate to the metal of the electrode by means of a bit of gold leaf, so placed as to be out of the path of the light. This plate was placed about three centimeters from an iron plate which was used as cathode and kept at a negative potential, while the glass plate and the walls of the tube were earthed in order to prevent "soak" of the charge into the insulation of the electrode. An initial test

¹ PHYS. REV., XXVII., 1908, p. 267.

of the glass plate before any discharge had been passed through the tube showed that it was photo-electrically inactive. After depositing for 90 minutes in an atmosphere of nitrogen using a current of from 2 to 5 milamperes from the 2,000 volt generator set, a test was made giving a potential of 3 volts. A second deposition, lasting one hour, gave 1.2 volts, and continued deposition, aggregating about four and one half hours in all, gave no increase in potential, the thinnest film giving the highest potential obtained. The film on removal from the tube was found to be very irregular in thickness and crossed by several interference bands, so that no good estimate of the thickness for any particular observation could be made.

As iron deposits with some difficulty and is liable to oxidation if the gas in the tube is not entirely free from oxygen, a cathode of sheet platinum was substituted for the iron in the next series of observations, so as to deposit a platinum film on the glass plate, the tube still containing an atmosphere of nitrogen. The glass plate gave no deflection of the electrometer when exposed to light from the mercury arc before deposition. During deposition of a film tests were made of the photo-electric potential at intervals of from one to ten minutes of actual time of discharge. During discharge the current was maintained as constant as possible by regulating the pressure in the tube. This was accomplished by admitting more nitrogen when the current decreased and connecting to the pump when the current increased. It was attempted to maintain an average current of 0.0025 ampere, but the fluctuations from the mean were large. At the end of a run, which was usually terminated by an accidental leakage of air into the tube, the plate was removed from the tube and the thickness of the film measured. This was done by making use of the interference fringes produced by placing a piece of optical glass in contact with the film under sodium light. A narrow strip of the film was scraped off across the center of the place and the fringe-shift from the glass to platinum was measured under a low power micrometer microscope, as a fraction of the distance between two successive fringes, which gives the thickness of the film in terms of the wave length of sodium light. To eliminate the possible difference of phase change on reflection from glass and from platinum the plate was replaced in the tube and a second layer of platinum deposited on the first and on the glass surface laid bare by scraping, giving two platinum surfaces from which to reflect. The fringe shift was again measured and found to be the same as before, within the limits of accuracy of the measurements, from which it was concluded that at least for these thin films the difference of phase change in reflection from glass and from metal is negligible.

In order to form an estimate of the thickness of the film at any time during its deposition it is necessary to make the assumption that the thickness of the film is proportional to the time of deposit, which can only be approximately true since it was impossible to keep the current absolutely constant, but no other way is known of determining the thickness of the extremely thin films in which we are here principally interested. The rate of loss of weight of the cathode due to discharge, and hence probably the rate of deposit of cathode film has been found by Granquist¹ to increase rapidly with decrease of pressure, to be proportional to the cathode fall but not to the total potential difference between cathode and walls and at constant pressure to be nearly proportional to the square of the current, while Holburn and Austin² find the rate of loss to be nearly proportional to the first power of the current. Since the pressure in the tube as well as the current was kept nearly constant the above assumption seems fairly well justified, though there is room for considerable error. Upon dividing the total thickness of film by the number of minutes of actual discharge we find the average rate of deposit per minute and from this we can determine the probable thickness at any time.

Three films were thus deposited and tested, the results being shown graphically in Figs. 3, 4 and 5, where ordinates represent photo-electric

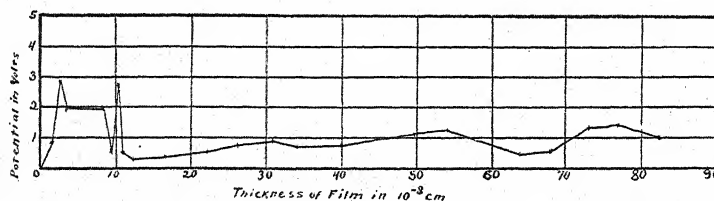


Fig. 3.

potentials in volts and abscissæ-thickness of film in 10^{-8} cm. It will be noticed that while the maximum potentials attained are very different for the three films, they occur at practically the same thickness of film in all cases, namely, 5×10^{-8} cm., and that the sudden drop to a minimum behaves in the same manner within the limits of error in estimation of thickness. This latter thickness, about 10^{-7} cm. is the same thickness of platinum film for which Patterson³ found a rapid increase in electrical conductivity, the specific conductivity being very low for films thinner than this. This enormously increased photo-electric potential of very

¹ G. Grunquist, Öfvers. k. Vet.-Akad. Förh. Stockholm, 54, pp. 575-594, 1897.

² Wiss. Abh. physik. Reichsanstalt., 4, 1904 (99-114).

³ Phil. Mag., Vol. 4, Dec., 1902, p. 652.

thin films was accompanied by a large photo-electric current as shown by the rapid charging of the electrometer system when the light was turned on, but both effects fatigued quite rapidly with continued illumination and did not recover on standing, that is, the electrometer was deflected

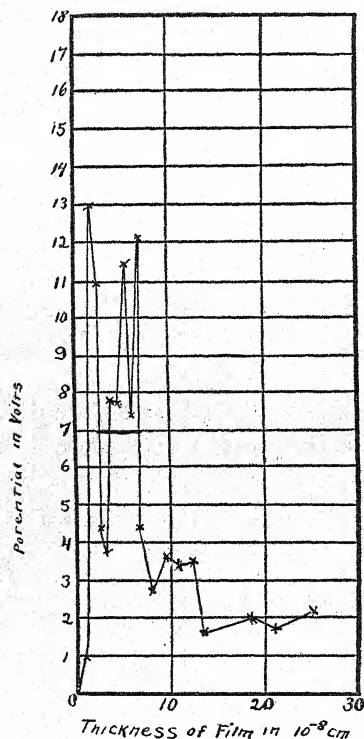


Fig. 4.

quite rapidly to a maximum and then gradually fell back. The thicker films giving lower potentials also gave smaller currents, but did not fatigue appreciably. There is apparently a critical thickness for platinum as deposited on glass beyond which it becomes less easy for the electrons to escape and more easy for them to migrate through the metal as shown by the increased conductivity.

The idea suggested itself that the high potentials might be due to the material on which the platinum was deposited, and to test this a quartz plate was substituted for the glass. This was also inactive before being deposited on, but a one-minute discharge sent the potential up to 18 volts and three minutes more sent it beyond to the limit of the electrometer scale, which corresponded to 30 volts. Depositing was continued, for a total of one hour, testing at intervals of one to two minutes of discharge, and high potentials were obtained throughout, averaging

about 25 volts but sometimes going beyond 30 volts and no minimum reached. On removing the plate from the tube a partial explanation of the peculiarities of this film was found in its irregular thickness as shown by the colors of the surface, which resembled interference bands. This was due, partially at least, to the fact that the plate was larger than the cathode, causing the deposit to be less dense toward the edges, and films of all thicknesses were present at the same time. There was reason also to doubt the purity of the gas during this test.

The type of tube so far used was not entirely satisfactory, since it did not allow sufficient flexibility of arrangement of the plates, and the cathode when close enough to the plate to give a good deposit was partly in the path both of the direct and of the reflected ultra-violet light, and moreover formed a good reflector for the electrons projected from the

plate, thus lowering the potentials obtained. A new tube (Fig. 2) was designed to avoid most of these difficulties, in which the cathode (*B*) could be rotated out of the way when not in use by means of a ground

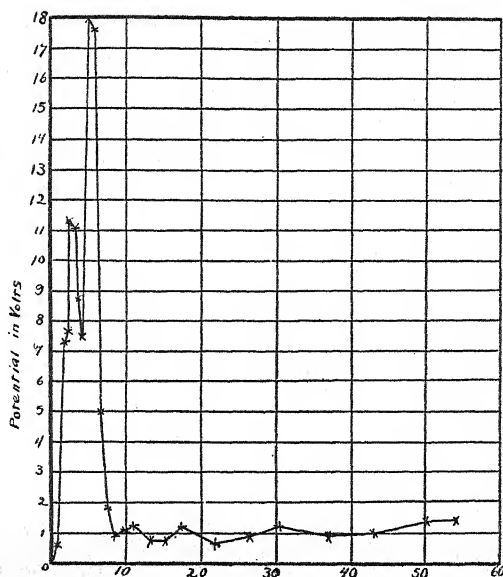


Fig. 5.

joint (*C*). This consists of a brass cone fitted to a conical hole in a heavy plate-glass end (*D*) of the tube. The tube itself was of brass as before, but much larger in diameter. The plate to be tested (*A*) was held in a light frame supported by a yoke (*F*) in such a way that it could be rotated about a vertical axis from outside the tube so as to present either face to the cathode discharge and to the ultra-violet illumination. The light entered the tube through a well-diaphragmed side tube and the beam of light was restricted to the central portion of the plate, which it struck at an angle of 45 degrees when the plate was set so as to be parallel to the cathode. Two other side tubes were provided, so placed that the light *transmitted* through the plate would pass out through one, and the light *reflected* from the plate through the other. All three

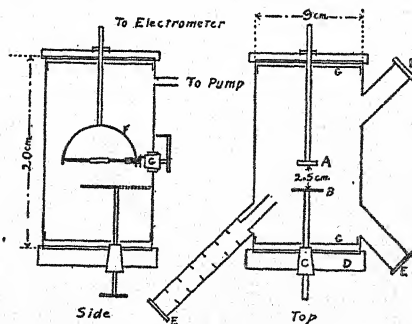


Fig. 2.

tubes were provided with quartz or fluorite windows (*EEE*). This was done in order to avoid as far as possible scattered light inside the tube, as well as to provide a convenient means of watching the plate during deposit. The electrode leading to the electrometer was insulated as before by passing it through a plate-glass end into which it was cemented. Both glass ends were shielded from the discharge by brass plates (*CC*) fitting loosely inside the tube. The platinum cathode, 3 cm. in diameter, was backed with mica and the supporting stem covered with a glass tube in order that all the discharge should leave from the front surface of the cathode.

Deposition took place in an atmosphere of hydrogen, and tests of photo-electric potential were made in a vacuum such that a spark from an induction coil would pass through 4 cm. of air rather than through a discharge tube connected with the main tube. The source of potential for deposition was the 2,000-volt generator.

The first test was made with platinum deposited on quartz, the results being shown graphically in Fig. 6. The points to be noted in regard to it are the continued high values for the potential, and the drop and

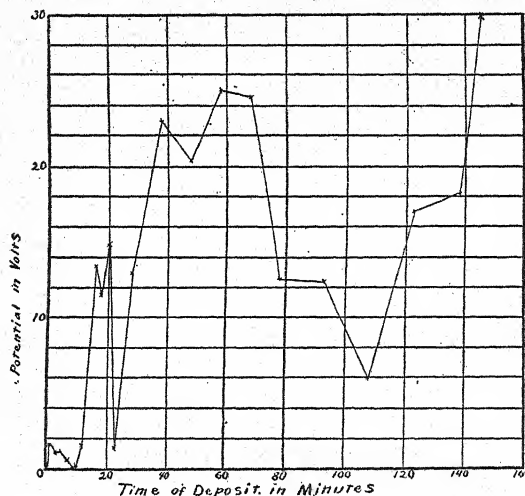


Fig. 6.

recovery after about two hours deposition. The drop coincided with the appearance of a nearly uniform blue coloration of the film. The colors changed with increasing thickness of film from blue to green, to red, and then to blue again, suggesting interference, but the total thickness of the film at the end of the series of tests did not much exceed a half wavelength of sodium light. With the appearance of the blue coloration a

new phenomenon was observed, an ageing effect. If the film was tested immediately after the discharge allowing only sufficient time for the vacuum to be reduced to its usual value during tests, practically no deflection of the electrometer would be obtained, but if it were allowed to stand without exposure to light for an hour or more a potential of two or three volts would be obtained on exposure to the light, and this would increase gradually with time of standing still, after 24 hours or more, the maximum of about 20 volts was attained. This increase took place even if hydrogen or air were allowed to enter the tube, in one case air being admitted up to atmospheric pressure without affecting the potential obtainable on re-exhaustion. The apparent drop in potential at the appearance of the blue coloration is probably due to a failure to allow time for the ageing to take place, since the effect of standing was not discovered till after the low readings had been recorded.

There was a very marked difference in the potentials obtainable from the same film according as the cathode was in position for discharge or not. With cathode up, that is, in position for discharge, the potential would be about 30 per cent. less than when the cathode was turned out of the way, as would be expected from the work of van Baeyer and Gehrts, since the reflection of the electrons was very greatly reduced in the second position.

The film in which the ageing effect was first noticed when removed from the tube appeared colored but fairly uniform with some dark spots. Upon examination under a high power microscope these spots were seen to be thin parts of the film, very small and surrounded with a peculiar radiate structure, so that even in this thick deposit there were thin parts, which may have been responsible for the continued high potentials.

Since a platinum film so thick as to be nearly opaque gave results so different from ordinary platinum, it was of interest to try a film deposited on sheet platinum instead of on glass or quartz. A quartz plate was used as a support for a piece of platinum foil which had been used in an electric furnace at high temperatures for a considerable length of time, the foil being attached to the quartz by means of sealing wax. The foil had a burnished surface and was not polished, but was thoroughly cleaned with French chalk and distilled water.

The results of the observations are shown in Fig. 7, where ordinates represent maximum potentials for a given film, and abscissæ represent times of discharge in minutes. The potentials above 30 volts were measured by means of a gold-leaf electrometer, reading to the nearest volt. These high potential readings could be repeated indefinitely and on successive days, until an addition was made to the thickness of the film.

The same ageing phenomenon occurred here as with the thick film of quartz. The film also showed coloration, and the disappearance of the color was taken as an indication that the film had been removed when the plate was used as cathode to clean it. The films on platinum are seen to combine properties of those on glass and on quartz. They

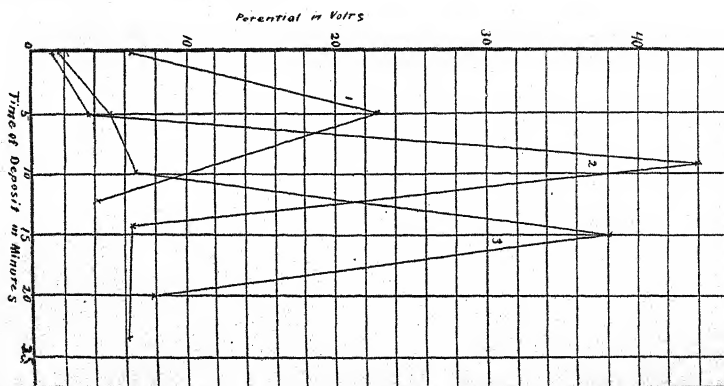


Fig. 7.

resemble those on glass in attaining a high maximum for a certain thickness of film, and at about the same thickness for both glass and platinum, while, like the films on quartz, they show the ageing effect to a very marked degree. The *initial potential* with the platinum foil, *i. e.*, before any film had been deposited, is of the same order as the potentials obtained by von Baeyer and Gehrts for gold, copper and aluminum with elaborate precautions as to vacuum, cleaning of the surface and avoidance of electronic reflection, while here the latter was the only point particularly considered. The high potentials recorded in Fig. 7 exceed any previously reported even by Millikan.

No explanation of these high potentials is obvious and that they are caused by anything else than the true photo-electric effect is hard to believe, since it would seem that the experimental conditions had been sufficiently varied to have disclosed any spurious effect. The potentials were obtained only when the light was on though the sensitiveness to light persisted in some cases for days. There were no insulated surfaces exposed to the discharge which could influence the electrometer, and in the last form of apparatus scattered light was reduced to a minimum. Any errors introduced, such as faulty vacuum, leakage of the electrometer system, electronic reflection, etc., would tend to reduce the potentials rather than increase them.

The films on glass which gave pronounced maxima for a thickness of

about 5×10^{-8} cm., falling abruptly with the increase of thickness to a low value, were colorless and had good mirror surfaces which showed no structure when examined with a high power microscope, while the films on quartz were colored, showed a lack of uniformity and had surfaces not at all like those of ordinary platinum. These colored films gave high potentials no matter how much the thickness was increased, at least up to $\frac{1}{2}\lambda$ for sodium light and in this respect also were different from ordinary platinum.

Hence it may be said that ordinary platinum gives unusually high values for the photo-electric potential only when in an excessively thin film of about 5×10^{-8} cm. thickness, which is of the order of magnitude of the accepted values for the diameters of gaseous molecules; while if it comes down in the form which gives colored films the potentials may be continuously high with increasing thickness of film. The cause of the coloration of the films deserves a special investigation, for interference does not seem to be an adequate explanation.

Ordinary platinum even after it has been rolled probably has some crystalline structure, which becomes more pronounced with long heating and annealing,¹ while our very thin films and the colored deposits probably lack such regular structure and may be considered amorphous. In the amorphous form the molecular bonds are free and may be points of readiest escape for the electrons which are shot off, or the electrons may resonate more freely to the vibrations of the ultra-violet light when the bonds are unsatisfied, acquiring a greater velocity. When the film becomes thick enough the molecular bonds unite to form elementary crystals, and the freedom of the electrons is diminished. When deposited on platinum the film goes down on a surface already crystallized and the molecules deposited do not readily join the old crystals but when in sufficient numbers unite to form new crystals group. The ageing effect, that is, the increase of electronic velocity with time in the case of the films deposited on platinum and on quartz, remains to be explained.

SUMMARY OF RESULTS.

1. It has been demonstrated that photo-electric potentials of a platinum surface can reach values more than twice as high as any hitherto reported, and many times higher than any previously obtained with the same source of illumination, namely, the quartz mercury arc.

2. These high potentials were observed only with very thin cathode films of the order of thickness of 5×10^{-8} cm., or with colored films of greater thickness, up to at least a half wave-length of sodium light.

¹ Holborn and Austin, loc. cit.

3. A sudden drop in potential (considered as a function of thickness of film) occurs with uncolored films at the same thickness at which Patterson found a rapid increase in the electrical conductivity of platinum films.

4. With colored films on quartz, and films on platinum the velocity of the projected electrons was found to increase with lapse of time, up to about 36 hours, independent of illumination and state of vacuum.

In conclusion I wish to acknowledge my indebtedness to Professor Mendenhall for his encouragement and numerous suggestions during the course of the work.

DEPT. OF PHYSICS, UNIVERSITY OF WISCONSIN,
June, 1911.

PROCEEDINGS
OF THE
AMERICAN PHYSICAL SOCIETY.

THE CALCULATION OF THE MAXIMUM VALUE OF THE FORCE BETWEEN TWO
COAXIAL CIRCULAR CURRENTS.¹

BY FREDERICK W. GROVER.

THE force between two parallel coaxial circular currents was given by Maxwell² in the form

$$f = \frac{\partial M}{\partial z} = -\pi \frac{z \sin \gamma}{\sqrt{Aa}} \{2F - (1 + \sec^2 \gamma)E\}, \quad (1)$$

where A and a are the radii of the two circles, z the distance between their planes, and F and E are the complete elliptic integrals of the first and second kind respectively, to the modulus

$$k = \sin \gamma = \frac{2\sqrt{Aa}}{\sqrt{(A+a)^2 + z^2}}.$$

Only for circles of equal radii is the force a maximum when the circles are in the same plane. When the radii are unequal, the force at first increases with the distance z , reaches a maximum, and then falls off indefinitely towards zero as the distance is indefinitely increased.

The maximum value of the force is a function, not of the actual values of the radii, but of their *ratio*, a fact which is taken advantage of in the Rayleigh current balance, the movable coil being placed at such a distance from the fixed coils, that the observed force is a maximum. The calculation of the maximum value of the force as a function of the ratio of the radii possesses therefore a practical as well as a theoretical interest.

The maximum force may be obtained with all the accuracy necessary by calculating the value of the force for various arbitrarily selected values of z in the neighborhood of the approximate value indicated by experiment, the maximum value of the force, and the distance z corresponding, being then derived by interpolation. For the purpose of these calculations, formula (1) may be employed, or some one of the expansions in q functions given by Naga-

¹ Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

² *Elect. and Mag.*, II., § 701.

oka.¹ Of these may be cited the following, which have been extended by the author of the present paper to include several extra terms.

$$f = \frac{192\pi^2 z}{\sqrt{Aa}} q^{\frac{1}{2}} (1 + 20q^2 + 225q^4 + 1,840q^6 + 12,120q^8 + 68,052q^{10} + 337,465q^{12} + 1,513,740q^{14} + 6,247,665q^{16} + \dots) \quad (2)$$

and

$$f = \frac{\pi z}{16q_1 \sqrt{Aa}} [(1 + 12q_1 - 192q_1^2 + 1,232q_1^3 - 5,634q_1^4 + 21,648q_1^5 - 73,600q_1^6 + 226,944q_1^7 - 648,189q_1^8 + \dots) - 12q_1 \log \frac{1}{q_1} (1 - 10q_1 + 60q_1^2 - 300q_1^3 - 1,300q_1^4 - 4,884q_1^5 + 16,320q_1^6 - 49,920q_1^7 + 142,500q_1^8)], \quad (3)$$

where

$$q = \frac{l}{2} + 2 \left(\frac{l}{2} \right)^5 + 15 \left(\frac{l}{2} \right)^9 + \dots,$$

$$\frac{l}{2} = \frac{1}{2} \frac{1 - \sqrt{k'}}{1 + \sqrt{k'}} = \frac{k'^2}{2(1 + k')(1 + \sqrt{k'})^2},$$

$$k^2 = \frac{2\sqrt{Aa}}{\sqrt{(A+a)^2 + z^2}}, \quad k'^2 = 1 - k^2 = \frac{\sqrt{(A-a)^2 + z^2}}{\sqrt{(A+a)^2 + z^2}},$$

$$q_1 = \frac{l_1}{2} + 2 \left(\frac{l_1}{2} \right)^5 + 15 \left(\frac{l_1}{2} \right)^9 + \dots,$$

$$\frac{l_1}{2} = \frac{1}{2} \frac{1 - \sqrt{k}}{1 + \sqrt{k}} = \frac{k'^2}{2(1 + k)(1 + \sqrt{k})^2}.$$

These formulas are in general very convergent, so that only two or three terms need be calculated. Formula (3) applies to the case of circles close together, where (2) is not sufficiently convergent. Tables recently published by Nagaoka² facilitate materially the use of these formulas.

As the above method of calculating the maximum force is indirect, the author was led to attempt the derivation of formulas which should reach the result more directly. Although the formulas given below do not materially simplify the calculation, they possess, apart from their academic interest, value in the means provided for rapidly checking the constants obtained by the indirect method.

The expression of Maxwell was first differentiated, and an expression involving z in terms of elliptic integrals derived. This expression seems, however, to be too complicated to admit of practical use. By following the

¹ Phil. Mag., 1903.

² Proc. Math. Phys. Soc. Tokyo, 6, p. 152; 1911.

method of Nagaoka, however, and expressing the results in q series, the two formulas given below were derived.

Nagaoka gives the force in the form

$$f = \frac{\partial M}{\partial z} = 4\pi Aaz \int_0^\pi \frac{\cos \theta d\theta}{(A^2 + a^2 + z^2 - 2Aa \cos \theta)^{\frac{3}{2}}}.$$

Differentiating this equation with respect to z we have the condition to be fulfilled by the distance z_0 corresponding to the maximum force

$$\begin{aligned} \frac{df}{dz} = 4\pi aA \int_0^\pi \frac{\cos \theta d\theta}{(A^2 + a^2 + z_0^2 - 2Aa \cos \theta)^{\frac{3}{2}}} \\ + 4\pi aAz_0^2 \int_0^\pi \frac{3 \cos \theta d\theta}{(A^2 + a^2 + z_0^2 - 2Aa \cos \theta)^{\frac{5}{2}}} = 0. \end{aligned}$$

The integration of this equation was obtained by introducing the p functions of Weierstrass. The result may then be expressed by means of the θ functions of Jacobi, and finally expanded in q series. As the work is long and tedious, only the final formulas can be here given.

$$\begin{aligned} z_0 = \sqrt{\frac{Aa}{20q}} (1 - 18q^2 + 199q^4 - 2,022q^6 + 21,547q^8 \\ - 242,522q^{10} + 2,829,828q^{12} \\ - 33,755,570q^{14} + 408,424,637q^{16} - \dots) \end{aligned} \quad (4)$$

and

$$\begin{aligned} z_0^2 [(1 - 16q_1 + 376q_1^2 - 4,672q_1^3 + 38,948q_1^4 - 252,192q_1^5 \\ + 1,365,888q_1^6 - 6,463,360q_1^7 + 27,500,946q_1^8 - \dots) \\ - 120q_1^2 \log \frac{1}{q_1} (1 - 16q_1 + 154q_1^2 - 1,120q_1^3 + 6,880q_1^4 \\ - 34,272q_1^5 + 156,268q_1^6 - \dots)] \\ = 32Aaq_1 [(1 + 24q_1 + 36q_1^2 + 384q_1^3 + 402q_1^4 + 3,456q_1^5 + 3,064q_1^6 \\ + 23,040q_1^7 + 18,351q_1^8 + \dots) \\ - 12q_1 \log \frac{1}{q_1} (1 + 2q_1 + 24q_1^2 + 28q_1^3 + 264q_1^4 + 252q_1^5 \\ + 2,016q_1^6 + 1,696q_1^7 + 12,264q_1^8 + \dots)]. \end{aligned} \quad (5)$$

Introducing the value of z_0 from (4) into (2) we find for the maximum force

$$\begin{aligned} f_m = \frac{96\pi^2}{\sqrt{5}} q^2 (1 + 2q^2 + 64q^4 - 252q^6 + 4,882q^8 \\ - 50,480q^{10} + 631,392q^{12} - \dots). \end{aligned} \quad (6)$$

Equation (5) is only to be used for circles relatively close together, where formulas (4) and (6) do not give a sufficiently convergent result.

Since in most cases only a few terms need to be calculated, these formulas

are by no means as elaborate as they may appear. The great difficulty under which they labor lies in the fact, that the quantities q and q_1 are functions of z and are only approximately known beforehand. The calculation of the maximum force by these formulas can therefore be carried through only by successive approximations. The work can however be very much shortened if the value of z_0 can previously be found *approximately* by some other means.

In the case of an actual current balance, this condition will be realized experimentally. If the ratio of the radii is not greater than one half, the approximate relation given by Rayleigh,

$$z_0 = \frac{A}{2} \left(1 - 0.9 \frac{a^2}{A^2} \right),$$

is of service. For the general case, the author has found the following empirical relation of value.

If through any element of the larger circle a plane be passed, including the common axis of the circles, then lines drawn from the element to the ends of that diameter of the smaller circle, which is determined by its intersection with the plane subtend an angle, which when multiplied by the ratio A/a of the radii, gives a value of about 46° , in no case being less than 45° nor greater than $46^\circ.5$. The author has prepared a table giving the value of this angle, for use in interpolating an approximate value of z_0 when no better method may be employed.

Having obtained an approximate value of z_0 , the calculation of a precise value may be carried out as follows. Applying formula (4) or (5), a second approximation for z_0 is derived. Denoting by $\Delta z/z$ the difference between the first and second approximations, two simple differential formulas give the value of $\Delta q/q$ corresponding, and the value of $\Delta z/z$ for a third approximation. By repeating this process, a very precise value of z_0 is quickly obtained, and this may be checked and any slight necessary correction found by a second calculation by formula (4) or (5).

DIRECT MEASUREMENT OF THE VELOCITY OF KATHODE RAYS AND THE VARIATION OF MASS WITH VELOCITY.¹

BY E. LEON CHAFFEE.

THE following is an outline of an investigation which is in progress, the object of which is to determine, by the direct measurement of the velocity of rapidly moving kathode particles together with the measurement of the deflection in a known magnetic field, the variation of the mass of the particles as a function of their velocity, and to add experimental evidence on the side of the Lorentz-Einstein theory or of the Abraham theory.

The radical difference between this investigation and other similar ones is in the method of measuring the velocity of the kathode rays, which method is free from assumptions and serious experimental errors.

¹ Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

Kathode rays are emitted in a high vacuum and are given a high velocity by differences of potential obtained from a large secondary battery. Potentials up to 40,000 volts have been used, and it is hoped to go to 140,000 volts by the same means. The kathode rays, after passing through a small aperture, enter a tube about 150 cm. long and finally impinge on a fluorescent screen at the end of the tube. The tube is inclined so that its axis is coincident with the direction of the earth's magnetic field. The kathode particles are timed in traversing a certain distance in the tube by means of electrical oscillations.

A new system for producing high-frequency electrical oscillations, which was described by the writer in the November issue of 1911 of the Proceedings of the American Academy of Arts and Sciences, supplies continuous undamped oscillations of 20 meters wave-length ($n = 15,000,000$ per sec.) to a coil about the tube, the coil being in such a position as to produce deflections of the kathode rays in a direction transverse to the axis of the tube. The coil is capable of rotation about the axis of the tube. About 70 cm. above this deflecting coil are electrostatic deflecting plates which are connected to the terminals of the condenser of the oscillatory circuit. The combined effect of the two deflecting fields on the kathode particles is to produce, on the fluorescent screen, an ellipse, the eccentricity of which depends upon the velocity of the kathode rays in traversing the distance between the two deflecting fields, upon the ratio of the deflecting powers of the two fields, and upon the angle between the planes of deflection of the two fields. Under *one* condition only does the ellipse become a circle, and when this is so the angle between the planes of the two deflecting fields is a function of the distance between the fields, the period of the oscillatory circuit, and the velocity of the kathode particles.

The measurements consist in varying the angle between the fields until a circle, as judged by the eye, is obtained. This adjustment can be made with surprising accuracy, and, by making several settings under the same conditions, the probable deviation of the mean velocity is of the magnitude of 4. per cent.

JEFFERSON PHYSICAL LABORATORY,
CAMBRIDGE, MASS.

A METHOD OF USING THE PHOTOELECTRIC CELL IN PHOTOMETRY.¹

BY EDWARD L. NICHOLS AND ERNEST MERRITT.

IT has been shown by Richtmyer that photoelectric cells containing sodium or potassium, made by the method first described by Elster and Geitel, give a current on illumination which is strictly proportional to the intensity of the exciting light through a very wide range, provided only that the quality of the light remains unchanged. We have recently used such a cell in determining the density distribution in the negatives obtained when photographing

¹ Abstract of a paper presented at the New York meeting of the Physical Society, March 2, 1912.

certain luminescence spectra, our purpose being to locate the crests of several faint and rather diffuse bands, and we find the method both convenient and reliable.

The photoelectric cell—a potassium cell obtained from Müller-Uri, Braunschweig—was mounted in a light-tight box, one side of which contained an adjustable slit, so placed that light entering the slit fell upon the sensitive surface of the cell. The negative was mounted immediately in front of the slit, and was carried by a micrometer screw. A mercury lamp was used as a source of light. To obtain sufficient detail the slit was made somewhat less than 0.1 mm. wide.

Under these circumstances the photoelectric current was too small to be measured by a galvanometer, and it was therefore necessary to use an electrometer. In order that the current might be indicated by the deflection of the electrometer rather than by the rate of change of the deflection the cell was connected as shown in the figure. The potassium terminal of the cell *K*

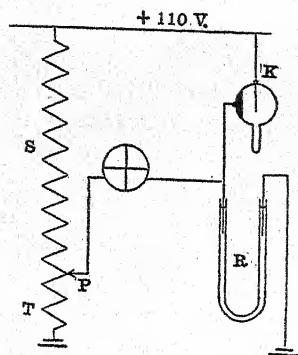


Fig. 1.

was connected to one pair of quadrants of the Dolezalek electrometer and also, through a resistance *R*, to earth, the other terminal of the cell being connected to the positive side of a 110-volt circuit. The resistance *R* consisted of a capillary tube containing absolute alcohol, and could be adjusted by varying the depth of immersion of the wire leading to earth. Since a small current passed through the cell even without illumination, it was found convenient to connect the other pair of quadrants to *P*, and by adjustment of the resistances *S* and *T* to bring the two pairs of quadrants to the same potential when the cell was in the dark. The deflection of the elec-

trometer upon illuminating the cell was then a measure of the intensity of the light entering the slit.

Readings could be taken with great rapidity. In one instance, for example, over four hundred readings were taken, covering the range from 4,338 A.U. to 5,790 A.U., in less than an hour. Since the quadrants of the electrometer are permanently connected to earth through the resistances *R* and *T* the instrument is almost entirely free from electrostatic disturbances. The fact that the quantity sought is measured by a deflection, rather than by the rate of change of a deflection, leads to a considerable increase in the convenience with which observations may be made; while the sensibility of the method may be made many times greater than that used in the work here described.

THE DIURNAL RANGE OF TEMPERATURE AND OTHER METEOROLOGICAL ELEMENTS AT DIFFERENT LEVELS ABOVE MOUNT WEATHER.¹

BY WM. R. BLAIR.

THE data for this study are being obtained by means of kites and captive balloons. A time is selected when the weather will probably be clear and the winds good for the following 30 to 36 hours, and a series of ascensions to a height of 3 kilometers or a little more above sea level is made. The ascensions of this series are in as close succession as it is possible to make them. The meteorograph carried by the head kite records air pressure, temperature, and humidity and wind velocity. Wind direction is determined to the nearest of 16 points by observation of the direction in which the kites are flying. The difference of electrical potential between the kite and the earth's surface is also observed. Three series have been made, two in the late summer, 1911, and one in the late winter, 1912.

These data have all been reduced and tabulated and charted so as to show the variation in each element recorded at levels 0.5 kilometer apart up to 3 kilometers in the summer series and up to 2.5 kilometers in the winter series. The departures of the hourly temperatures from the mean for the day are less at a level between 1.5 and 2 kilometers than either below or above it. Below this transition level the greatest positive and negative departures from the mean temperature for the day decrease and incline slightly forward with altitude. Above it the positions of these departures are very nearly transposed, the negative one occurring at about 1 to 2 P.M. and the positive one at 2 to 5 A.M. The observations of air pressure are used in computing the altitudes. Aside from this and the electrical potential of the atmosphere, the other elements observed have a less characteristic diurnal range than the temperature. The atmospheric potential is decidedly lower at all levels during the day than during the night hours. The minimum follows the time of maximum insolation.

MOUNT WEATHER OBSERVATORY,

BLUEMONT, VA.,

April 12, 1912.

ON THE EFFECT OF LIGHT ON A HELIX OF RAYS FROM THE WEHNELT CATHODE (EXPERIMENTAL).¹

BY CHAS. T. KNIPP.

THE paper is an account of an experimental attempt to test J. J. Thomson's corpuscular or pulse theory of light. His helical cathode beam indicator was employed. The principle of this method is that such a beam forms a sensitive indicator when struck, near its source, by radiation containing either positively or negatively (or both) charged carriers. The effect on the beam

¹ Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

would be to make it fuzzy, or if allowed to fall on a photographic plate the image should show blurred. A helical beam 47 cm. long, of pitch about 1 cm., and 4 to 8 mm. in diameter, was obtained by using the form of cathode and general mounting described in the PHYSICAL REVIEW, Vol. XXXIV., p. 58. Photographs were taken, first, of the beam alone, and second, of the beam when the end near the source, for a space of 1.5 cm., was illuminated by the light from a lantern. This was followed by another exposure without the lantern. Three dozen plates were thus exposed, some having six exposures on them—three without and three with the lantern. The conditions under which the different plates were exposed varied very widely, though for a given plate the conditions were practically constant, the six exposures occupying less than two minutes. The time of a single exposure was about one tenth of a second. The plates almost without an exception show an *effect*. On some it is more marked than on others. The effect thus far observed is to *reduce the size of the image* on the photographic plate. The circular image on a number of plates shows blurred towards the center, but not outwards. The effect is more marked when the image is not too clearly cut, *i. e.*, when the vacuum is not too high. Again, it also seems more marked when the tube is operated on low voltages. The incident light in general fell on but one turn of the helical beam. By adjustment of the field the helix could be wound up so as to present a convex or a concave arc towards the incident light. This, however, does not seem to alter the effect. For the want of data taken under sufficiently varying conditions it is impossible at this time to explain the phenomenon, and to state whether it agrees with Thomson's theory of light.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
April 12, 1912.

A THEORETICAL CORRECTION TO THE DROP METHOD OF DETERMINING " e ."

BY ARTHUR C. LUNN.

THE absolute determination of the elementary charge by the use of a drop of liquid as carrier of the ions depends on the measurement of the steady velocity of the drop through the surrounding medium under gravity, with and without the presence of an electrostatic field. The computation assumes that the factor of mobility of the drop is the same in both cases. If however the liquid has a dielectric constant different from that of the medium the Maxwell surface stresses produced by the field tend to distort a spherical drop into a somewhat ellipsoidal form up to the point where they are equilibrated by the normal component of surface tension forces which corresponds to the variation of curvature of the surface. In this paper a sufficiently

¹ Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

approximate formula is obtained for the amount of this ellipticity. The corresponding correction to Stokes' law for a sphere is shown to be inappreciable in the most extreme cases under the actual conditions of Millikan's measurements, as already inferred by him from the experimental data.

MAGNETIC INDUCTION IN A GROUP OF OBLATE SPHEROIDS OF IRON.¹

BY S. R. WILLIAMS.

THE magnetic induction in a group of twenty-seven oblate spheroids of soft iron was studied when the equatorial planes of the spheroids were oriented in different azimuths with respect to the imposed magnetic field. The twenty-seven spheroids were mounted so that they formed a cube with nine spheroids on a side. This group was tested by the ballistic method just as any specimen of iron would be. It was found that the induction was larger the more nearly the spheroids were turned with the equatorial planes parallel to the magnetic field. In other words if the spheroids were oriented so that their equatorial planes were all parallel to each other the induction varied in different directions in the cube.

This behavior of the group of spheroids is very much like the behavior of iron electrolytically deposited in a magnetic field, which shows different magnetic properties parallel to the field in which it was deposited from what it does at right angles to this direction.² Again it is similar to the behavior of pyrrhotine which Weiss³ investigated and found it possessed directions in which greater magnetic intensities occurred than in others.

In conjunction with the theory of magnetism proposed in the paper, "A Model of the Elementary Magnet,"⁴ this deportment of the oblate spheroids readily explains the cause of the Villari reversal effect, the behavior of certain crystals in a magnetic field and particularly the reason why the knee of the induction curve of hardened steel is flatter than that of soft iron.⁵

PHYSICAL LABORATORY, OBERLIN COLLEGE,
OBERLIN, OHIO,
March 21, 1912.

ABSOLUTE FORMULÆ FOR THE ATTRACTION OF TWO COAXIAL SOLENOIDS.¹

BY G. R. OLSHAUSEN.

THE mutual inductance of two coaxial solenoids is given by the expression

$$M = 4\pi n n' [I_1 - I_2 - I_3 + I_4],$$

¹ Abstract of a paper presented at the Cambridge meeting of the Physical Society, April 27, 1912.

² Maurain, Sci. Abstracts, No. 2235, 1900.

³ Weiss, Jour. d. Phys., Vol. IV., pp. 469, 829.

⁴ Williams, PHYS. REV., Vol. 34, Jan., 1912.

⁵ Hadley, Elec. and Mag., p. 390, 1908.

where n and n' are the respective numbers of turns per unit length of the two coils and the quantities in brackets are integrals of the form

$$I = a_1^2 a^2 \int_0^\pi \frac{\sin^2 \psi d\psi}{a_1^2 + a^2 - 2a_1 a \cos \psi} \sqrt{a_1^2 + a^2 + c^2 - 2a_1 a \cos \psi}.$$

In this expression a_1 and a are the respective radii of the coils and c has the following values:

$$c = d + l + l_1 \quad \text{for } I = I_1,$$

$$c = d + l_1 - l \quad \text{for } I = I_2,$$

$$c = d + l - l_1 \quad \text{for } I = I_3,$$

$$c = d - l - l_1 \quad \text{for } I = I_4,$$

where d is the distance between centers of the coils and $2l_1$ and $2l$ are their respective lengths.

The attraction of the two solenoids, when a unit current flows through each, is then

$$-\frac{\partial M}{\partial d} = -4\pi n n' \left[\frac{\partial I_1}{\partial c_1} - \frac{\partial I_2}{\partial c_2} - \frac{\partial I_3}{\partial c_3} + \frac{\partial I_4}{\partial c_4} \right].$$

The differential coefficients are found by differentiating the above value of I under the integral sign with respect to c . We obtain then

$$\frac{dI}{dc} = \frac{c(m\beta)^{\frac{1}{2}}}{2} \int_{\omega_3}^{\omega_2} \frac{(pu - e_2)(pu - e_3)}{pu - pw} du.$$

The constants entering into this and the following expressions and the variable u have the same significance as in my paper on "Absolute Formulæ for the Mutual Inductance of Two Coaxial Solenoids," *PHYS. REV.*, 31, pp. 617-636, December, 1910.

On integrating, we obtain the following four equivalent values of dI/dc .

$$\begin{aligned} (a) \quad \frac{dI}{dc} &= \frac{c(m\beta)^{\frac{1}{2}}}{2} \left[\frac{4(a_1^2 + a^2) + c^2}{3m\beta} \omega_1 - \eta_1 \right. \\ &\quad \left. + \frac{i(a_1^2 - a^2)}{|c| (m\beta)^{\frac{1}{2}}} \left\{ \eta_1 w_1 i - \omega_1 \frac{\sigma_1'}{\sigma_1} (w_1 i) \right\} \right], \\ (b) \quad \frac{dI}{dc} &= \frac{c(m\beta)^{\frac{1}{2}}}{2} \left[\frac{a_1^2 + a^2 + c^2 + 6a_1 a}{3m\beta} \omega_1 - \eta_1 \right. \\ &\quad \left. + \frac{i(a_1^2 - a^2)}{|c| (m\beta)^{\frac{1}{2}}} \left\{ \eta_1 w_1 i - \omega_1 \frac{\sigma_2'}{\sigma_2} (w_1 i) \right\} \right], \end{aligned}$$

where $|c|$ indicates that the absolute value of c is to be used in all cases.

Expressed in terms of Legendre's integrals these formulæ become

$$(c) \quad \frac{dI}{dc} = \frac{c}{2} \left[\frac{2(a_1^2 + a^2) + c^2}{\sqrt{(a_1 + a)^2 + c^2}} F(k) - \sqrt{(a_1 + a)^2 + c^2} E(k) \right. \\ \left. + \frac{a_1^2 - a^2}{|c|} \left\{ F(k) [F(\varphi, k') - E(\varphi, k')] - E(k) F(\varphi, k') \right\} \right]$$

and

$$(d) \quad \frac{dI}{dc} = \frac{c}{2} \left[\sqrt{(a_1 + a)^2 + c^2} \{ F(k) - E(k) \} \right. \\ \left. - \frac{(a_1^2 - a^2)}{|c|} \left\{ F(k) [F(\varphi, k') - E(\varphi, k')] - E(k) F(\varphi, k') + \frac{\pi}{2} \right\} \right].$$

In the first of these two formulæ

$$\sin^2 \varphi = \frac{c^2}{(a_1 - a)^2 + c^2},$$

in the second

$$\sin^2 \varphi = \frac{(a_1^2 - a^2)^2 + c^2(a_1 - a)^2}{(a_1^2 - a^2)^2 + c^2(a_1 + a)^2},$$

and in both of them

$$k^2 = \frac{4a_1a}{(a_1 + a)^2 + c^2} \quad \text{and} \quad k'^2 = \frac{(a_1 - a)^2 + c^2}{(a_1 + a)^2 + c^2}.$$

INDEX TO VOLUME XXXIV.

A.

- Absorbing Media, The Theoretical and Experimental Determination of Reflection Coefficients of, *J. T. Tate*, 240.
- Adams, Walter S., On the Relation between Pressure Shift and Wave-Length, 78.
- Adams, Walter S., The Spectra of Iron and Titanium at Moderate Pressures, 143.
- Adams, Walter S., The Spectrum of Titanium in a Partial Vacuum and the Proportionality of Displacement to Pressure at Moderate Pressures, 140.
- Allen, S. J., On the Passage of γ Rays of Radium through Matter, 296.
- Alternating E.M.F.'s in a Fournier's Series, On the Expansion of, *Geo. R. Ols-hausen*, 317.
- American Physical Society.
Abstracts, 68, 125, 228, 320, 381, 471.
Minutes, 62, 63, 226, 381.
Treasurer's Account, 67.
- Ammeter, A Compensating, Linear Scale, Hot-Wire, *A. H. Taylor*, 365.
- Anderson, S. Herbert, Effect of Frequency on the Capacity of a Condenser, with Kerosene for the Dielectric, 34.
- Atomic Weights, On the Family-Tree Arrangement of the Elements and Calculation of, on the Corpuscular Theory of the, *Albert C. Crehore*, 241.
- Atmosphere, The Vertical Temperature Gradient of the, *Wm. R. Blair*, 159.

B.

- Balloons, Note on the Ascensional Rate of the Free, Used for Meteorological Purposes, *Wm. R. Blair*, 159.
- Barker, Harold C., Note on the Measurement of the Peltier E.M.F., 224.
- Barnett, S. J., The Measurement of Inductances and Capacities with the Thomson Double Bridge, 74.
- Benedict, F. G., Magnetic Reactions Produced by a Copper Disc Rotating between the Poles of a Magnet, 315.

Beta Particles, Absorption of the, by Gases
Alois F. Kovarik, 142.

Binaural Localization, The Intensity Factors in the, of Sound, *G. W. Stewart*, 76.

Birchby, W. N., A Study of the Reversible Pendulum. Part II., Experimental Verifications, 110.

Black Body Spectra, Slit-Width Corrections in the Photometry of, *Edward P. Hyde*, 233.

Blair, Wm. R., The Vertical Temperature Gradient of the Atmosphere, 159.

Blair, Wm. R., Note on the Ascensional Rate of the Free Balloons Used for Meteorological Purposes, 159.

Blair, Wm. R., The Diurnal Range of Temperature and Other Meteorological Elements at Different Levels above Mount Weather, 477.

Book Reviews: 160, 320, 400.

Bridgman, P. W., The Collapse of Thick Cylinders under High Hydrostatic Pressure, 1.

Bronson, H. L., The Distribution of the Active Deposit of Radium in an Electric Field, 151.

Brown, F. C., An Electrical Method of Measuring Small Intervals of Time, 452.

Brown, F. C., The Effective Depth of Penetration of Selenium by Light, 201.

Buckley, O. E., The Electrical Resistance and the Polarization of Clay, Feldspar and Quartz, 141.

Bumstead, H. A., A New Radiation from Polonium, 234.

Butman, Chester A., The Photoelectric Effect of Phosphorescent Material, 158.

Butman, Chester A., The Electron Theory of Phosphorescence, 316.

C.

Cables, A Method of Measuring the Effective Capacity of High Tension, *G. W. Stewart* and *D. M. Terwilliger*, 76.

- Cady, W. G., Magnetic Reactions Produced by a Copper Disc Rotating between the Poles of a Magnet, 315.
- Candle-Power Measurements, International, *Clayton H. Sharp*, 313.
- Capacity of a Condenser, Effect of Frequency on, with Kerosene for the Dielectric, *S. Herbert Anderson*, 34.
- Capacities, The Measurement of Inductances and, with the Thomson Double Bridge, *S. J. Barnett*, 74.
- Carbon, The Thomson Effects in and the Thermal Conductivities of Tungsten, Tantalum and, at Glowing Temperatures, *A. G. Worthing*, 152.
- Cathode, On the Production of a Helix of Rays from the Wehnelt, 58.
- Cathode, Rays of Positive Electricity from the Wehnelt, *Chas. T. Knipp*, 71.
- Cathode, On the Effect of Light on a Helix of Rays from the Wehnelt (Experimental), *Chas. T. Knipp*, 477.
- Cathode Films, Photo-Electric Potentials of Thin, *Paul H. Dike*, 459.
- Cathode Rays, The Production of Light by, *Gordon S. Fulcher*, 68.
- Chaffee, E. Leon, Direct Measurement of the Velocity of Kathode Rays and the Variation of Mass with Velocity, 474.
- Change of State, Discussion of a General Case, *A. J. Lotka*, 235.
- Clay, The Electrical Resistance and the Polarization E.M.F. of a Mixture of, Feldspar and Quartz, *A. A. Somerville* and *O. E. Buckley*, 141.
- Compton, Karl T., The Photoelectric Effect 393.
- Concepts, The Primary, of Physics, *William Francis Magie*, 125.
- Conduction, The Electron Theory of Metallic, *O. W. Richardson*, 77.
- Conduction of Electricity at Contacts of Dissimilar Solids, *Robert H. Goddard*, 423.
- Crehore, Albert C., On the Family-Tree Arrangement of the Elements and Calculation of Atomic Weights on the Corpuscular Theory of the Atom, 241.
- Crystals, The Dielectric Constant, Resistivity and Electrostatic Absorption of Different, *H. L. Curtis*, 154.
- Curl, An Interpretation of, and Divergence in Terms of Vector Growth, *Frederick Slate*, 359.
- Current, The Distribution of, in Point-Plane Discharge, *Robt. F. Earhart*, 187.
- Current Produced by Light in Metallic Film, *P. A. Ross*, 72.
- Curtis, H. L., The Dielectric Constant, Resistivity and Electrostatic Absorption of Different Crystals, 154.
- Cylinders, The Collapse of Thick, under High Hydrostatic Pressure, *P. W. Bridgman*, 1.
- D.
- Dempster, A. J., On the Mobility of Ions in Air at High Pressures, 53.
- Dike, Paul H., Photo-Electric Potentials of Thin Cathode Films, 459.
- Dilution, Heats of, *William Francis Magie*, 396.
- Discharge of Electricity, Reaction Effects Produced by the, from Points in Gases and the Bearing of these Effects on the Theory of the Small Ion, *Edward J. Moore*, 81.
- Discharge, Electrical, from a Point to a Plane, *O. Hovda*, 25.
- Distortion in Spectral Luminosity Curves Produced by Variations in the Character of the Comparison Standard and in the Surroundings of the Photometric Field, *Herbert E. Ives*, 387.
- Divergence, An Interpretation of Curl and, in Terms of Vector Growth, *Frederick Slate*, 359.
- Drop Method, A Theoretical Correction to the, of Determining "e," *Arthur C. Lunn*, 478.
- E.
- Earhart, Robt. F., The Distribution of Current in Point-Plane Discharge, 187.
- Electric Condensers, The Effect of Temperature on the Absorbed Charge in, *Anthony Zeleny*, 141.
- Electrical Resistance, Variation of, with Temperature. V.—Oxides, *A. A. Somerville*, 399.
- Electrical Resistance, Variation of, with Temperature, 311.
- Electricity, Rays of Positive, from the Wehnelt Cathode, *Chas. T. Knipp*, 215.

Electron Theory of Conductors, The, O. W. Richardson, 383.

Electron Theory of Metallic Conduction, O. W. Richardson, 77.

Electron Theory of Phosphorescence, The, Chester A. Bulman, 316.

Elements, On the Family-Tree Arrangement of the, and Calculation of Atomic Weights on the Corpuscular Theory of the Atom, Albert C. Crehore, 241.

Ellipticity, The Magnetic Rotation and, Produced by Mirrors of Massive Metals, Paul D. Foote, 96.

Erikson, Henry A., The Absorption of Gamma Rays of Radium by Air at Different Pressures, 231.

Expansion, A New Type of Apparatus for Measuring Linear, Arthur W. Gray, 139.

F.

Feldspar, The Electrical Resistance and the Polarization E.M.F. of a Mixture of, Clay, and Quartz, A. A. Somerville and O. E. Buckley, 141.

Foote, Paul D., The Magnetic Rotation and Ellipticity Produced by Mirrors of Massive Metals, 96.

Force, The Calculation of the Maximum Value of the, Between Two Coaxial Circular Currents, 471.

Forsythe, W. E., Radiant Efficiency of Incandescent Filaments, 333.

Franklin, W. S., An Important Practical Problem in Gyrostatic Action, 48.

Fulcher, Gordon S., The Production of Light by Cathode Rays, 68.

Fulcher, Gordon S., The Stark-Doppler Effect for Hydrogen Canal Rays in Air, 231.

G.

Gale, Henry G., On the Relation between Pressure Shift and Wave-Length, 78.

Gale, Henry G., The Spectrum of Titanium in a Partial Vacuum and the Proportionality of Displacement to Pressure at Moderate Pressures, 140.

Gale, Henry G., The Spectra of Iron and Titanium at Moderate Pressures, 143.

Gamma Rays, On the Passage of, of Radium through Matter, S. J. Allen, 296.

Goddard, Robert H., A Study of Crystal Rectifiers, 149.

Goddard, Robert H., On the Conduction of Electricity at Contacts of Dissimilar Solids, 423.

Gray, Arthur W., A New Type of Apparatus for Measuring Linear Expansion, 139.

Grover, Frederick W., The Calculation of the Maximum Value of the Force Between Two Coaxial Circular Currents, 471.

Gyrostatic Action, An Important Practical Problem in, W. S. Franklin, 48.

H.

Heat, Convection and Conduction of, in Gases, Irving Langmuir, 401.

High Vacua, A Sensitive Black-Body-Vacuum Thermal-Junction and a Method of Producing, A. H. Pfund, 228.

Hovda, O., Electrical Discharge from a Point to a Plane, 25.

Hull, Albert W., The Effect of a Magnetic Field on Photoelectric Emission, 239.

Hyde, Edward P., Slit-Width Corrections in the Photometry of Black Body Spectra, 233.

I.

Incandescent Filaments, Radiant Efficiency of, W. E. Forsythe, 333.

Index, 482.

Inductances, The Measurement of, and Capacities with the Thomson Double Bridge, S. J. Barnett, 74.

Ingersoll, L. R., The Kerr Rotation for Transverse Magnetic Fields, 138.

Ionization, On Secondary, Produced by the Impact of Positive Ions on Solids, O. W. Richardson and Charles Sheard, 391.

Ions from Hot Salts, The, O. W. Richardson, 386.

Ions, On the Mobility of, in Air at High Pressures, A. J. Dempster, 53.

Iron, The Spectra of, and Titanium at Moderate Pressures, Henry G. Gale and Walter S. Adams, 143.

Ives, Herbert E., The Influence of Temperature on the Phenomena of Phosphorescence in Zinc Sulphide, 156.

Ives, Herbert E., Distortion in Spectral Luminosity Curves Produced by Variations in the Character of the Comparison Standard and in the

- Surroundings of the Photometric Field, 387.
- Ives, Herbert E., The Addition of Luminosities of Different Color, 389.
- J.
- Johnston, John, A New Form of Mechanical Vacuum Pump, 157.
- K.
- Kimura, Shunkichi, One-Waveness in Wire-less Telegraphy; Pseudo-Impact Excitation, 345.
- Knipp, Chas. T., On the Production of a Helix of Rays from the Wehnelt Cathode, 58.
- Knipp, Chas. T., Rays of Positive Electricity from the Wehnelt Cathode, 71.
- Knipp, Chas. T., Rays of Positive Electricity from the Wehnelt Cathode, 215.
- Knipp, Chas. T., On the Effect of Light on a Helix of Rays from the Wehnelt Cathode (Experimental), 477.
- Kathode Rays, Direct Measurement of the Velocity of, and the Variation of Mass with Velocity, *E. Leon Chaffee*, 474.
- Kovarik, Alois F., Absorption of the β -Particles by Gases, 142.
- L.
- Langmuir, Irving, Convection and Conduction of Heat in Gases, 401.
- Laub, Y., Note on the Optical Effects in Moving Media, 268.
- Law of Fall of a Drop through Air at Reduced Pressures and a Redetermination of e , *R. A. Millikan*, 398.
- Liquid Chlorides, The Selective Transmission and the Dispersion of the, *H. H. Marvin*, 161.
- Lotka, A. J., Change of State, Discussion of a General Case, 235.
- Luminosities, The Addition of, of Different Color, *Herbert E. Ives*, 389.
- Lunn, Arthur C., A Theoretical Correction to the Drop Method of Determining " e ," 478.
- Lyman, Theodore, Spark Spectra in the Schumann Region, 157.
- M.
- Magie, William Francis, Heats of Dilution, 396.
- Magie, William Francis, The Relation of Osmotic Pressure to Temperature, 397.
- Magie, William Francis, The Primary Concepts of Physics, 125.
- Magnet, A Model of the Elementary, *S. R. Williams*, 40.
- Magnetic Fields, The Kerr Rotation for Transverse, *L. R. Ingersoll*, 138.
- Magnetic Induction in a Group of Oblate Spheroids of Iron, *S. R. Williams*, 479.
- Magnetic Phenomena. II., Comparative Studies of, Relation between the Joule Effect and Permeability and Induction in the Same Specimens of Steel, *S. R. Williams*, 258.
- Magnetic Reactions Produced by a Copper Disc Rotating between the Poles of a Magnet, *F. G. Benedict and W. G. Cady*, 315.
- Marvin, H. H., The Selective Transmission and the Dispersion of the Liquid Chlorides, 161.
- McGougan, A. G., A New Radiation from Polonium, 234.
- Merritt, Ernest, A Method of Using the Photoelectric Cell in Photometry, *Edward L. Nichols and Ernest Merritt*, 475.
- Metallic Film, Current Produced by Light in a, *P. A. Ross*, 72.
- Millikan, R. A., The Effect of Prolonged Illumination on Photo-Electric Discharge in a High Vacuum, 68.
- Millikan, R. A., The Law of Fall of a Drop through Air at Reduced Pressures and a Redetermination of e , 398.
- Molby, F. A., Temperature Influence upon the Refraction of Quartz, Borosilicate Crown Glass, and Dense Flint Glass, from 100° C. to — 190° C., 232.
- Moody, H. W., A Determination of the Ratio of the Specific Heats and the Specific Heat at Constant Pressure of Air and Carbon Dioxide, 275.
- Moody, H. W., A Determination of the Ratio of the Specific Heats for Air and Carbon Dioxide, 75.
- Moore, Edward J., Reaction Effects Produced by the Discharge of Electricity from Points in Gases and the Bearing of these Effects on the Theory of the Small Ion, 81.

N.

- Naylor, J. P., A Simple Slit for the Spectroscope, 150.
 Nernst Filament, The Thermo E.M.F. of the, *J. S. Shearer*, 238.
 Nichols, Edward L., A Method of Using the Photoelectric Cell in Photometry, 475.

O.

- Optical Effects, Note on the, in Moving Media, *Y. Laub*, 268.
 Olshausen, Geo. R., On the Expansion of Alternating Current E.M.F.'s in a Fourier's Series, 317.
 Olshausen, G. R., Absolute Formulæ for the Attraction of Two Coaxial Solenoids, 479.
 Osmotic Pressure, The Relation of, to Temperature, *William Francis Magie*, 397
 Oxides, Variation of Electrical Resistance with Temperature, V., *A. A. Somerville*, 399.

P.

- Peltier E.M.F., Note on the Measurement of the, *Harold C. Barker*, 224.
 Pendulum, A Study of the Reversible, Part II., Experimental Verifications, *John C. Shedd* and *W. N. Birchby*, 110.
 Pfund, A. H., A Sensitive Black-Body-Vacuum Thermal-Junction and a Method of Producing High Vacua, 228.
 Pfund, A. H., Application of the Selenium Cell to Photometry, 370.
 Photoelectric Cell, A Method of Using the, in Photometry, *Edward L. Nichols* and *Ernest Merrill*, 475.
 Photoelectric Discharge in a High Vacuum, The Effect of Prolonged Illumination on, *R. A. Millikan* and *J. R. Wright*, 68.
 Photoelectric Effect of Phosphorescent Material, *Chester A. Butman*, 158.
 Photoelectric Effect, The, *O. W. Richardson* and *Karl T. Compton*, 393.
 Photoelectric Effects, The Application of Statistical Principles to, and Some Allied Phenomena, *O. W. Richardson*, 146.
 Photoelectric Emission, The Effect of a Magnetic Field on, *Albert W. Hull*, 239.

- Photoelectric Potentials of Thin Cathode Films, *Paul H. Dike*, 459.
 Phosphorescence, The Influence of Temperature on the Phenomena of, in Zinc Sulphide, *Herbert E. Ives*, 156.
 Phosphorescence, The Electron Theory of, *Chester A. Butman*, 316.
 Phosphorescent Material, The Photoelectric Effect of, *Chester A. Butman*, 158.
 Photometry, Application of the Selenium Cell to, *A. H. Pfund*, 370.
 Photometry, Slit-width Corrections in the, of Black Body Spectra, *Edward P. Hyde*, 233.
 Polonium, A New Radiation from, *H. A. Bumstead* and *A. G. McGougan*, 234.
 Pomeroy, J. C., The Charges on Thermions Produced in Air and Hydrogen at Atmospheric Pressure, 78.
 Potentiometer Method, Accurate, for Measuring Resistance, *Walter P. White*, 391.
 Pressure Shift, On the Relation between, and Wave-Length, *Henry G. Gale* and *Walter S. Adams*, 78.

R.

- Radiant Efficiency of Incandescent Filaments, *W. E. Forsythe*, 333.
 Radium, The Absorption of Gamma Rays of, by Air at Different Pressures, *Henry A. Erikson*, 231.
 Radium, The Distribution of the Active Deposit of, in an Electric Field, *E. M. Wellisch* and *H. L. Bronson*, 151.
 Radium, On the Passage of γ Rays of, through Matter, *S. J. Allen*, 296.
 Rectifiers, A Study of Crystal, *Robert H. Goddard*, 149.
 Reflection Coefficients, The Theoretical and Experimental Determination of, of Absorbing Media, *J. T. Tate*, 240.
 Reflection Coefficients, The Theoretical and Experimental Determination of, *J. T. Tate*, 321.
 Refraction of Quartz, Temperature Influence upon the, Boro-Silicate Crown Glass, and Dense Flint Glass from 100° C. to -190° C., *F. A. Molby*, 232.
 Richardson, O. W., The Electron Theory of Metallic Conduction, 77.
 Richardson, O. W., The Application of Statistical Principles to Photoelectric

- Effects and Some Allied Phenomena, 146.
- Richardson, O. W., The Electron Theory of Conductors, 383.
- Richardson, O. W., The Ions from Hot Salts, 386.
- Richardson, O. W., On Secondary Ionization Produced by the Impact of Positive Ions on Solids, 391.
- Richardson, O. W., The Photoelectric Effect, 393.
- Ross, P. A., Current Produced by Light in a Metallic Film, 72.
- Rotation, The Kerr, for Transverse Magnetic Fields, *L. R. Ingersoll*, 138.
- Rotation, The Magnetic, and Ellipticity Produced by Mirrors of Massive Metals, *Paul D. Foote*, 96.
- S.**
- Scientific Observation, A Quantitative Measure of Development in, *Otto Stuhlmann, Jr.*, 145.
- Schumann Region, Spark Spectra in the, *Theodore Lyman*, 157.
- Selenium, The Effective Depth of Penetration of, by Light, *F. C. Brown*, 201.
- Selenium Cell, Application of the, to Photometry, *A. H. Pfund*, 370.
- Sharp, Clayton H., International Candle-Power Measurements, 313.
- Shearer, J. S., The Thermo E.M.F. of the Nernst Filament, 238.
- Sheard, Charles, On Secondary Ionization Produced by the Impact of Positive Ions on Solids, 391.
- Shedd, John C., A Study of the Reversible Pendulum. Part II. Experimental Verifications, 110.
- Slate, Frederick, An Interpretation of Curl and Divergence in Terms of Vector Growth, 359.
- Solenoids, Absolute Formulæ for the Attraction of Two Coaxial, *G. R. Olshausen*, 479.
- Somerville, A. A., Variation of Electrical Resistance with Temperature. V.—Oxides, 399.
- Somerville, A. A., The Electrical Resistance and the Polarization E.M.F. of a Mixture of Clay, Feldspar and Quartz, 141.
- Somerville, A. A., Variation of Electrical Resistance with Temperature, 311.
- Sound, The Intensity Factors in the Binaural Localization of, *G. W. Stewart*, 76.
- Spark Spectra in the Schumann Region, *Theodore Lyman*, 157.
- Specific Heats, A Determination of the Ratio of the, for Air and Carbon Dioxide, *H. W. Moody*, 75.
- Specific Heats, A Determination of the Ratio of the, and the Specific Heat at Constant Pressure of Air and Carbon Dioxide, *H. W. Moody*, 275.
- Spectral Luminosity Curves, Distortion in, Produced by Variations in the Character of the Comparison Standard and in the Surroundings of the Photometric Field, *Herbert E. Ives*, 387.
- Spectroscope, A Simple Slit for the, *J. P. Naylor*, 150.
- Spheroids, Magnetic Induction in a Group of Oblate, of Iron, *S. R. Williams*, 479.
- Stark-Doppler Effect for Hydrogen Canal Rays in Air, *Gordon S. Fulcher*, 231.
- Steel, Relation between the Joule Effect and the Induction and Permeability in the Same Specimens of, *S. R. Williams*, 145.
- Stewart, G. W., The Intensity Factors in the Binaural Localization of Sound, 76.
- Stewart, G. W., A Method of Measuring the Effective Capacity of High Tension Cables, 76.
- Stuhlmann, Jr., Otto, A Quantitative Measure of Development in Scientific Observation, 145.
- T.**
- Tantalum, The Thomson Effects in and the Thermal Conductivities of Tungsten, and Carbon at Glowing Temperatures, *A. G. Worthing*, 152.
- Tate, J. T., The Theoretical and Experimental Determination of Reflection Coefficients of Absorbing Media, 240.
- Tate, J. T., The Theoretical and Experimental Determination of Reflection Coefficients, 321.
- Taylor, A. H., A Compensating, Linear Scale, Hot-Wire Ammeter, 365.
- Temperature, The Diurnal Range of, and Other Meteorological Elements at Different Levels above Mount Weather, *Wm. R. Blair*, 477.
- Temperature Influence upon the Refraction

- of Quartz, Boro-Silicate Crown Glass, and Dense Flint Glass from 100° C. to — 190° C., *F. A. Molby*, 232.
- Terwilliger, D. M., A Method of Measuring the Effective Capacity of High Tension Cables, 76.
- Thermal Conductivities of Tungsten, The Thomson Effects in and the, Tantalum and Carbon at Glowing Temperatures, *A. G. Worthing*, 152.
- Thermal-Junction, A Sensitive Black-Body-Vacuum, and a Method of Producing High Vacua, *A. H. Pfund*, 228.
- Thermions, The Charges on, Produced in Air and Hydrogen at Atmospheric Pressure, *J. C. Pomeroy*, 78.
- Thermo E.M.F. of the Nernst Filament, The, *J. S. Shearer*, 238.
- Thomson Double Bridge, The Measurement of Inductances and Capacities with the, *S. J. Barnett*, 74.
- Time, An Electrical Method of Measuring Small Intervals of, *F. C. Brown*, 452.
- Titanium, The Spectra of Iron and, at Moderate Pressures, *Henry G. Gale and Walter S. Adams*, 143.
- Titanium, The Spectrum of, in a Partial Vacuum and the Proportionality of Displacement to Pressure at Moderate Pressures, *Walter S. Adams and Henry G. Gale*, 140.
- Tungsten, The Thomson Effects in and the Thermal Conductivities of, Tantalum and Carbon at Glowing Temperatures, *A. G. Worthing*, 152.
- V.
- Vacuum Pump, A New Form of Mechanical, *John Johnston*, 157.
- W.
- Wave-Length, On the Relation between Pressure Shift and, *Henry G. Gale and Walter S. Adams*, 78.
- Wehnelt Cathode, Rays of Positive Electricity from the, *Chas. T. Knipp*, 215.
- Wellisch, E. M., The Distribution of the Active Deposit of Radium in an Electric Field, 151.
- White, Walter P., Accurate Potentiometer Method for Measuring Resistance, 391.
- Williams, S. R., A Model of the Elementary Magnet, 40.
- Williams, S. R., Relation between the Joule Effect and the Induction and Permeability in the Same Specimens of Steel, 145.
- Williams, S. R., Comparative Studies of Magnetic Phenomena. II. Relation between the Joule Effect and Permeability and Induction in the Same Specimens of Steel, 258.
- Williams, S. R., Magnetic Induction in a Group of Oblate Spheroids of Iron, 479.
- Wireless Telegraphy, One-Waveness in; Pseudo-Impact Excitation, 345.
- Worthing, A. G., The Thomson Effects in and the Thermal Conductivities of Tungsten, Tantalum and Carbon at Glowing Temperatures, 152.
- Wright, J. R., The Effect of Prolonged Illumination on Photo-electric Discharge in a High Vacuum, 68.
- Z.
- Zinc Sulphide, The Influence of Temperature on the Phenomena of Phosphorescence in, *Herbert E. Ives*, 156.
- Zeleny, Anthony, The Effect of Temperature on the Absorbed Charge in Electric Condensers, 141.

ERRATUM.

Page 44. A Model of the Elementary Magnet. Figures 6 and 7 should be interchanged.